

## **A Manufacturing–Remanufacturing Supply Chain Model With Learning And Forgetting Including Inspection Errors**

<sup>1</sup>Jyothi Ummidi , <sup>2</sup>M. Srinivasa Rao , <sup>3</sup>V.V.S. Kesavarao

### **Abstract**

*A producing–reproducing system is considered to make new product as well as recycled product keeping same quality to keep it environmental sustainable. The manufacturer and the retailer Inventories are controlled by considering consignment stock policy. The receiving of spent products is getting increased focus for its reusability with proper inspection and performing remanufacturing to convert it into usable product of same as fresh product quality. The received used products will be subjected extensive quality inspection process in accordance with remembering and forgetting theory and exposed to inspection errors. At the time of production, the impact of Greenhouse gases is also taken into one of the important constraint in addition to money spent for disposing the non friendly eco material used for packaging and chemicals.*

*A model is developed for optimizing the economic order quantity with number of batches. A mixed-integer nonlinear programming model along with, genetic algorithm is applied on the model, cost reduction of 29 percent is attainable when learning, forgetting and inspection errors are considered in the model instead of ignoring the same.*

**Keywords:** *Reproducing, Errors found during checking, Stock policy which is consignment in nature, Learning and forgetting, Supply chain.*

### **1. Introduction**

End-of-life commodities that pose an environmental risk due to waste increase, land fill as well as depletion of resources poses challenges in supply chain. Rapid utilization will lead to rapid exhaust of resources. Which draws attention of scarcity of resources, forward supply chain focuses economic and efficient use of resources whereas reverse logistic deals with reuse of Life Products whose life span is nearing to completion, a supply chain system which address design and carrying out the activities for improving the useful utility/value for entire e product life cycle while taking out value from the returned products is called as closed loop supply chain (CLLSC).

Learning curves in lot sizing models are introduced by various authors such as Jaber [1]. The performance of workers enhances by repeated training on the work is amounts to learning by workers. The outcome of this is improvement in the quality of products by decreasing the rejections and process time reduction. When the workers are idle they will be in rest inertia and they will be losing tenacity of working and it is again a painful process to bring them into work inertia. This was marked as ‘forgetting effect’. Imposing a cost on carbon emission has been proved to be the most effective way to control the greenhouse gas emission. The used product returned from retailer often comes into packages which are not environmental friendly and items shall be prepared for inspection using chemicals and abrasive tools for surface preparation. Hence these wastages are to be

---

<sup>1</sup> Part-time Ph.D. Scholar in Department of Mechanical Engineering, College of Engineering, Andhra University, Visakhapatnam-3.

<sup>2</sup> DGM (Maintenance), Department of MMSM, Visakhapatnam Steel Plant.

<sup>3</sup> Professor, Department of Mechanical Engineering, College of Engineering, Andhra University, Visakhapatnam.

disposed after doing the inspection. A single-vendor single-buyer supply chain is considered where the vendor ships every production batch in a number of lots to the buyer's warehouse, inspection errors into either type I (a good item is classified as defective) or type II (a defective item is classified as good). Figure 1 depicts the production flow.

Remanufacturing /repairing activity is mainly supported by the rejected items from the customers, return of used items is considered as random to make our model more realistic. Not only at the time of production but also at the time of inspection, learning and forgetting effects the output in terms of quantity.

During inspection activity, the inspectors gain some experience which accelerates the inspection process and Reduces occurrence of unwanted errors. Similarly, at the time of non-inspection, the experience slowly drops resulting in slow down of the inspection process in addition to this there is always a possibility of inspection errors.. The main motivation behind this study is the consideration of learning and forgetting effects together in production and inspection with errors in inspection in a closed-loop supply chain model with consignment stock policy for inventory optimization. The major concern is to get the proper explanations so that it is useful decision maker to sustain in competitive environment. The insight made into the importance of learning and forgetting nature of human beings during manufacturing, remanufacturing and consideration of cost spent for environmental sustenance on inventory control has motivated to consider the current topic for research purpose. It is also important to know the influence of inspection errors on remanufacturing. The economic order quantity and number of batches to be made is the metric for inventory control.

Section 2 examines the relevant literature articles. Section 3 deals with assumptions, symbols, and models followed by formulation procedures in Section 4. In Section5 results are analyzed and lastly conclusions and future scope are mentioned in Section. 6.

## **2. Literature Review**

Network design, lot-sizing and inventory management, etc. are the different fields which are encapsulating the CLLSC. VMI contract optimizes the bullwhip effect, learning curve effects the performance in the organization. Clubbing of VMI and learning curve may increase the performance of CLLSC which is studied here.

Govindan and Solleimani [2], Govindan et al. [3] conducted evaluations on CLLSC production planning and control including inventory control are important in the study of CLLSC. VMI contract aids CLLSC due to its effectiveness in the control of inventory, Petruzzi and Dada [4]. VMI is used to integrate producers, remanufacturers, repairers, warehouses and retail customers to make a reasonable decision on the size of order as per . Govindan et al. [3]. The VMI method mimics inventory costs and response time to client requirements including improvement of cooperation among different sections as per Le, and Chu, [5], Ru and Wang, Y. [6], Wang et al. [7], Disney and Towill [8]. Bazan et al. [9] investigated the VMI policy and determined production batch size, remanufacturing times and cycle time for minimizing the costs as a part of inventory control policy. Bazan et al. [10] has found the costs of transportation plays a role in manufacturing whether it is classical inventory or VMI. Bazan et al. [11] proposed consignment stock contract (VMICS) with the principles of VMI and lone producer/retailer, even though it is beneficial compared to traditional inventory policy but not ecofriendly. Qiu et al. [12] made a comparison by changing reproduction parameters on the total costs under VMI contracts. Taleizadeh and Moshtagh [13] presented a CLLSC model. They dealt with acceptable quality levels for returned products as a variable factor upon which the exchange rate in VMI contracts is dependent. Learning effects have been seen in a variety of fields, including price, charge, demand, sales as well as lot-size learning. Jaber [1] stated that system cost reduces due to lessening of manufacturing time as a result. Bhatnagar et al. [14] examined the effect of learning on permanent or temporary workers on learning in the distinct production units on sub-processes and it is further considered in lot sizing decisions.

Jaber and Bonney [15,16] contributed on by reviews on the effect of learning on lot sizing inventory Jaber and Saadany [17] demonstrated utilization of available capacity in reverse logistics by the application of learning curve technique. Zaroni et al. [18] applied various consignment agreement policies and applied learning curve effects on them under a VMI technique for controlling the inventory. Khan et al. [19] explored the reasons for defective products and found human error is one reason and mitigated by applying learning effects on supply chain model, the model is applied on EOQ and EBQ models and Manna et al. [20] allowed shortages on an EBQ model including reworking of imperfect items, and the effect of learning and disremembering in a CLLSC model with stochastic return rate is reduction of profit, which is substantiated by Giri and Glock [21]. Giri and Masanta [22] considered price and quality-dependent demand., learning effect including stochastic return rate in their CLLSC model. Zaroni et al. [23] explored role of energy efficiency and found influential on lot-sizing issue including learning in production units for EOQ quantity.

In this paper function of probability density is considered defective product rates, standard deviation of costs for minimizing over all costs in CLLSC. Studying the effect of production rate and length of production interval and learning curve effects on predicting the charges current and for future purpose on multi product and multi-level.

Consignment stock (CS) is helpful for vendors who has limited space for storage of finished goods and suggested by Braglia and Zavanella [24]. This policy beneficial than conventional inventory issue as per Hill [25]. Later, Zavanella and Zaroni (2009) removed the limitation of single purchaser and suggested how to get benefit for vendors and buyers. Jaber et al. [10] studied models of Hill [25] and Braglia and Zavanella [24] and inserted entropy charge in supply chain, similar to second law of thermodynamics as a CS policy case and claimed most profitable. Zaroni et al. [23] suggested an entirely novel CS policy with implications for learning and forgetting in the manufacturing process with varied delivering quantity and delivery time.

Jaber et al. [10] considered disposal of wastages for benefit on the environment by remanufacturing in consignment stock policy. The positive impact on medical field in the consignment stock policy is studied by Giri and Masanta [22] is considered. Many consumers to ship things from a single provider, Zahran and Jaber [26] explained the same on many customers. Hemmati et al. [27, 28] guessed that demand is directly related to product price as well as the final stock levels. Nanda and Sen [29] applied GA on CS policy for deteriorating products with space restriction in a vendor-buyer inventory, the next issue to be studied is remanufacturing.

Qiu et al. [12] examined centralized and decentralized approaches involving price as well as carbon emission based remanufacturer demand. Talleizadeh and Moshtagh [13] discussed reproduction and the impact of quality of rejected products on CS scheme. Kazemim [30], Yang Liu and Zhenyu Yang [31] established a two-route collaboration for selling novel as well as replicated products within a closed loop distribution network, pricing decision in a reproducing considering carbon trade is explored by . Yuan, et al. [32].

Recently, Mondal et al. [33] considered issue of corporate social responsibility by utilizing used products in CLLSC. Jaber and Bonney [34] studied effect of learning and forgetting. On CLLSC and usefulness of these human characteristics on real life situations of inventory systems. Zaroni et al. [18] examined the efficiency of a consignment stock policy model with the impact of the learning-forgetting phenomenon. Jaber and Bonney [15] considered human exhaustion as a result of intense work, with lesser and extra rest which is more than adequate on the work performance with simultaneously learning and forgetting effect on the cost of manufacturing. Human factors such as fatigue, learning effect etc., play a role during inspection in the environment of perfect manufacturing process. As per the work of Giri and Glock [21] on imperfect manufacturing processes, even batch wise inspection gets influenced by learning effect as per Dey and Giri[35]. Konstantaras et al. [36] applied learning during inspection procedure on EOQ model for imperfect manufacturing process. Asadkhani et al. [37] investigated the role of studying for minimizing inspection errors, carbon emission reduction is considered presently by various

organizations in the CLLSC. Qiu et al. [12] created a reproduction/reworking that takes into account price as well as carbon emission mitigation.

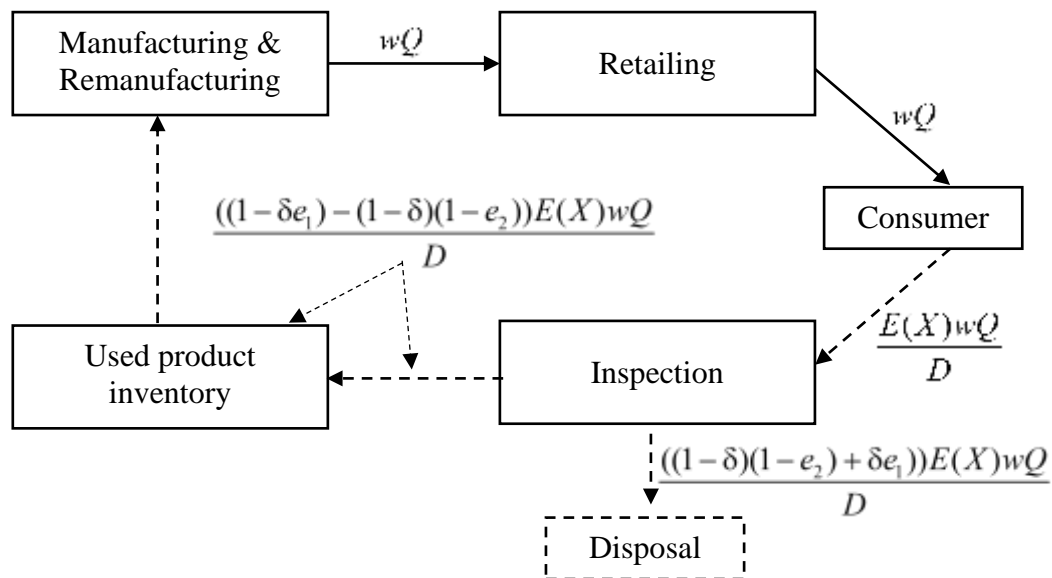
The prior literature does not properly collaborate on the influence of learning-forgetting during inspection and carbon emission. Zanoni et al. [18] has demonstrated consignment mechanism without reproducing environment.

### 2.1 Literature gap

Inspection errors and effect of disposal of waste product care not considered in the paper of Masanta and Giri [22]. Heuristic method is better method for solving which is considered as extension.

### 2.2 Objective of the paper

Minimizing the economic order quantity for a production cycle involving production and reproduction lots while considering learning and forgetting effects together during production and inspection in a closed-loop supply chain model with consignment stock policy with impact of inspection errors and environment sustainability .



**Figure 1: Flow of goods for manufacturing/remanufacturing**

### 3. Assumptions and Notation

#### 3.1 Assumptions considered in the model

1. Production process generally uses raw material for making fresh product whereas reproduction process uses recollected used products. Each unit of new material or used product will result after processing one unit of finished product all the products are used same marketing channels, here only one retailer is considered.
2. In the beginning production rate and demand rate are static. Production rate changes and improves as learning takes place and decreases when forgetfulness increases. Forgetfulness takes place when production hampers.
3. Inventories levels are determined as per agreed consignment stock policy finished products are distributed instantly after production and reproduction.
4. Production and reproduction activities are subject same rate of learning
5. Random return rate of used items is assumed

6. The reproduction of used items takes place after an inspection activity involves learning and forgetting where it is decided to go either for reproduction or dispose of production and reproduction products are made with same quality and effect of greenhouse gas, disposal of eco-unfriendly material during inspection process and is taken into account.
7. Inspection activity is subjected for errors.

### 3.2 Model

As shown in Figure 1, inventory sequence of products as follows .production and reproduction units make products called as goods .the same are sent to retailer/consigner. The consigner keeps goods . some of the products that are sold. It often happens when used products are returned by customers as used products for the purpose of remanufacturing . Defective items are distributed at random throughout reproduction processes . manufacturing of new products as well as reworking performed during same cycle. Batched final goods are delivered to retailers in multiple lots of  $Q$ .

### 3.3 Notations considered for unit

- $A_m$  producer's setup charge per setup
- $A_r$  reproducers 's set up charge
- $A_b$  purchaser's set up charge
- $h_m$  producer's holding charge for finished goods / unit / unit time
- $h_r$  reproducers r's carrying charge of finished products / unit /unit time
- $h_{mr}$  producers' and reproducers' carrying charge for finished goods / unit time at retailer's place
- $h_b$  carrying charge by the retailer for the final product / unit at retailer's location
- $h_{mu}$  carrying charge for items returned per unit time
- $cm$  production charge
- $cr$  remanufacturing charge
- $ci$  inspection charge per unit time
- $cu$  buying charge of used items
- $cd$  disposing charge of rejected products upon inspection
- $cep$  carbon emission charges because of production
- $cer$  carbon emission charge because of reproduction
- $P$  rate of production
- $D$  rate of demand
- $I$  inspection rates
- $T_1$  production /reproduction period for producing the first unit
- $ex_i$  corresponding producing quantity for experience at start of inventory cycle  $i$ , where  $i = 1,2,\dots$
- $ex_{i,j}$  corresponding quantity assessed for experience at start of inventory cycle  $i$ , where  $i = 1,2,\dots$
- $FI_1$  disremembering (forgetting) the time of production/reproduction processes
- $FI_2$  forgetting (disremembering) the time of inspection procedure
- $le_1$  learning the exponent production/reproduction procedure, ( $0 < le_1 < 1$ )
- $le_2$  learning the exponent inspection procedure, ( $0 < le_2 < 1$ )
- $X$  return rate for used products ( $0 < X < 1$ )
- $FI_X()$  probability density function,  $X$
- $T_{pi}(T_{di})$  producers producing(non producing ) period for inventory cycle  $i$ , where  $i = 1,2,\dots$
- $\delta$  inspected items good fraction( $0 < \delta < 1$ )
- $e_1$  Chances of a correct item to be testified as defective (type I error)
- $e_2$  Chances of a bad item to be testified as correct (type II error)
- $cer1$  Waste disposal cost per unit cost

#### Decision variable:

- $v$  No. of shipments from reproducer (remanufacturer) to retailer

w No. of shipments from producer to retailer Q batch sizes

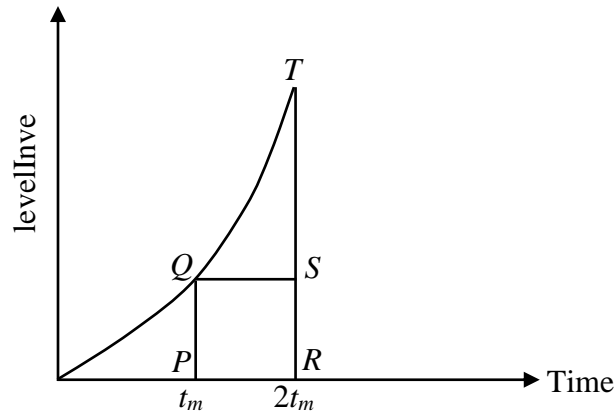
The mistakes in checking process can lead even the good items and the bad items be treated as bad or fault free items. The effective rejection proportion, say,  $p_e$ , is the sum of correctly rejected items,  $(1 - \delta)(1 - e_2)$  as wrongly treated as bad items,  $\delta e_1$ :

$$p_e = (1 - \delta)(1 - e_2) + \delta e_1 \quad (1)$$

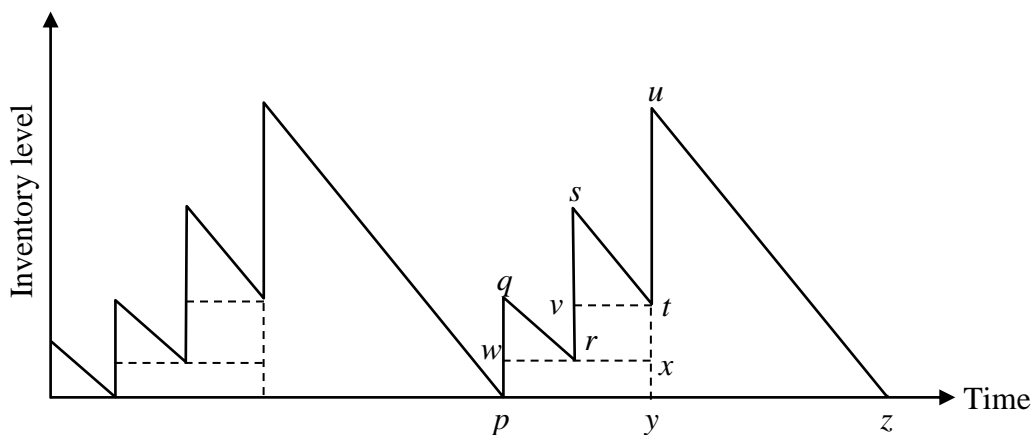
If the amount of bad units is  $p_e$ , then only  $q_i((1 - \delta e_1) - (1 - \delta)(1 - e_2))$  be considered as good items and their proportion will be of size say  $q_i$  from every order.

#### 4. Mathematical Formulation

The producer and reproducer produce ‘ $vQ$ ’ and ‘ $wQ$ ’ units during a single producing and reproducing cycle to satisfy the requirement of purchaser, all activities are subjected to learning and forgetting. Same quality finished products are marketed at same price. end-customers return the products within expected return date  $E(X)D$ . inspection process is influenced by learning and has beginning inspection rate. Producer ships finished products to retailers in the same period of time and does not retain any inventory on hand Zanoni et al. [19]. All the below mentioned carries at unit.



**Figure 2: Area of Holding for two batches at the time of Production** (source Giri [23])



**Figure 3: Inventory cycle at retailer side** (source Giri [23])

**4.1 Charge of production and reproduction**

As per initial consideration, batches v as well as w corresponding to size Q are dispatched to retailer during production as well as reproduction units, considering the effect of learning and forgetting. Power function for time and unit produced, as  $t_i T_x = T_1 x^{-le_1}$ ,  $t_i$  w  $T_x$  denotes time required to manufacture  $x^{th}$  quantity,  $T_1$  represents time needed to manufacture initial quantity in present production run,  $le_1$  the learning curve index, forgetting function considered by Carlson and Rowe [38]:

$$\hat{T}_x = \hat{T}_1 x^{f_1} \tag{2}$$

$x$  indicates unit produced discontinuation cannot happen,  $\hat{T}_1$  is relevant time period of forgetting curve during initial unit,  $\hat{T}_x$  denotes time needed for  $x^{th}$  unit of lost experience.  $f_1$ , computed by Jaber and Bonney [35]:

$$f_1 = \frac{le_1(le_1 - 1) \log(ex_i + kQ)}{\log\left(1 + \frac{F}{T(kQ)}\right)}, \quad \text{where } k = v, w \tag{3}$$

$ex_i$  is knowledge gained from previous cycle ( $(i-1)^{th}$ ). Forgetting time is endless when experience transfer occurs. The worker never forgets experience acquired,  $ex_i$  Jaber and Bonney [35]:

$$ex_{i+1} = (ex_i + kQ)^{1 + \frac{f_1}{le_1}} U_i^{-\frac{f_1}{le_1}} \tag{4}$$

$U_i$  is obtained

$$U_i = \left[ \frac{1 - le_1}{T_1} \left( \frac{kQ}{D} - T(kQ) + T(ex_i + kQ) \right) \right]^{\frac{1}{1 - le_1}} \quad \text{where } k = v, w \tag{5}$$

**4.1.1 Production charge**

We suppose that producers dispatches units available in same period  $t_m$ . inventory is influenced by learning and forgetting theory, produced goods do not remain same in successive periods.

Every production cycle produces  $vQ$  units: Time required is calculated using Eq.(6) developed by Zanoni et al. [19].

$$T_m = \frac{T_{i1,m}}{1 - le_1} (vQ)^{1 - le_1} \tag{6}$$

Taken into account consignment stock policy, producer would dispatch the available stock to retailer during cycle time  $t_m$ ,

$$t_m = \frac{T_{i1,m} (vQ)^{1 - le_1}}{v(1 - le_1)} \tag{7}$$

Figure 2 depicts carrying charges for first as well as thee second batch. The inventory for initial batch is given by

$$H_{m1} = \int_0^{t_m} \left( \frac{1 - le_1}{t_{i1,m}} \right)^{\frac{1}{1 - le_1}} dx \tag{8}$$

where  $T_{i1,m}$  denotes time for producing initial product in the  $i^{th}$  manufacturing cycle.

Inventory time interval  $t_m$  is moved to retailer.

$$\text{inventory holding area} = \text{Area PQTR} - \text{Area of PQSR} \tag{9}$$

Inventory carrying  $k^{th}$  batch of the production and reproduction cycle where ( $k = 1, \dots, v$ ) is developed by Zanoni et al. [19], Eq.(10).

$$H_{mk} = \int_{(k-1)t_m}^{kt_m} \left( \frac{1-le_1}{t_{il,m}} x \right)^{\frac{1}{1-le_1}} dx - (kt_m - (k-1)t_m) \left( \frac{1-le_1}{t_{il,m}} t_m \right)^{\frac{1}{1-le_1}} \quad (10)$$

So, the holding charge of the Producer is

$$h_m \sum_{k=1}^v H_{mk} \quad (11)$$

Producers 's charge is the sum of setup charge as well as holding charge, and represented as

$$TC_m = A_m + h_m \sum_{k=1}^v H_{mk} \quad (12)$$

#### 4.1.2 Reproduction charge

wQ units of remanufactured product, resembling freshly produced units, are dispatched to retailers via n shipments. Time of reproducing  $T_r$  is calculated by

$$T_r = \frac{T_{il,r}}{1-le_1} (wQ)^{1-le_1} \quad (13)$$

where  $T_{il,r}$  is the time reproducing initial units in the  $i^{th}$  reproduction cycles. Reproduced goods are dispatched to retailers within recycling time frame as in producers 's case. length of same time pause given as

$$t_r = \frac{T_r}{w}$$

holding area is calculated by

$$H_{rk} = \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{2-le_1}{1-le_1}} \frac{T_{il,r}}{2-le_1} \left( k^{\frac{2-le_1}{1-le_1}} - (k-1)^{\frac{2-le_1}{1-le_1}} \right) - \left( \frac{1-le_1}{T_{il,r}} (k-1) \right)^{\frac{1}{1-le_1}} t_r^{\frac{2-le_1}{1-le_1}} \quad (14)$$

So, the reproducers charge which is sum of setup charge and carrying charge

$$TC_r = A_r + h_r \sum_{k=1}^v H_{rk} \quad (15)$$

#### 4.2 Total charge of the retailers

The retailer's carrying area is computed as shown in Figure 3 is developed by Zanoni et al. [19].

Producers cycle, the area which is covered by retailer ( $H_{bm}$ ) is the total areas of pwxv and rvtx, areas of triangles qwr, vst, as well as triangle yuz.

$$\begin{aligned} H_{bm} &= \sum_{k=1}^{v-1} (Q(kt_m) - Dt_m) + \frac{1}{2} (v-1)Dt_m^2 + \frac{1}{2} \frac{\left( \sum_{k=1}^v Q(kt_m) - (v-1)Dt_m \right)^2}{D} \\ &= \left( \frac{1-le_1}{T_{il,m}} t_m \right)^{\frac{1}{1-le_1}} \sum_{k=1}^v (k-1)^{\frac{1}{1-le_1}} - \frac{1}{2} (v-1)^2 Dt_m^2 \\ &\quad + \frac{1}{2D} \left[ \left( \frac{1-le_1}{t_{il,m}} vt_m \right)^{\frac{1}{1-le_1}} - (v-1)Dt_m \right] \end{aligned} \quad (16)$$

by minimizing producing length of time period.



$$\begin{aligned}
 H_{br} = & \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{1}{1-le_1}} \sum_{k=1}^w (k-1) - \frac{1}{2} (w-1)^2 Dt_r^2 \\
 & + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,r}} wt_r \right)^{\frac{1}{1-le_1}} - (w-1)Dt_r \right]^2
 \end{aligned} \tag{17}$$

The total charge (TC<sub>b</sub>) of the retailer is the value comprising of set up charges and holding charges occurred for production as well as reproduction:

$$\begin{aligned}
 TC_b = & (v+w)A_b + h_b \left[ \left( \frac{1-le_1}{T_{il,m}} t_m \right)^{\frac{1}{1-le_1}} \sum_{k=1}^v (k-1)^{\frac{1}{1-le_1}} + \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{1}{1-le_1}} \sum_{k=1}^w (k-1)^{\frac{1}{1-le_1}} \right. \\
 & \left. - \frac{1}{2} (v-1)^2 Dt_m^2 - \frac{1}{2} (w-1)^2 Dt_r^2 + \frac{1}{2D} \left[ \left( \frac{1-le_2}{T_{il,m}} vt_m \right)^{\frac{1}{1-le_1}} - (v-1)Dt_m \right] \right. \\
 & \left. + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,r}} wt_r \right)^{\frac{1}{1-le_1}} - (w-1)Dt_r \right]^2 \right]
 \end{aligned} \tag{18}$$

**4.3 Charge of products which are used**

The rate of products which are returned is denoted as XD before reproduction, used products are offered inspection which is susceptible to learning and forgetting. Beginning rate of inspection, I is not less than starting production rate. time to inspect the y<sup>th</sup> unit:

$$T_y = \frac{1}{I} y^{-le_2}$$

le<sub>2</sub> denotes learning index of the inspection activity.

Time for inspecting returned items may be computed using [similar to Eq. (6)]:

$$T_1 = \frac{T_{il,I}}{1-le_2} (E((X)(v+w)Q))^{1-le_2}$$

T<sub>il,I</sub> indicates time taken to review i<sup>th</sup> cycle of process which is inspected, and is denoted as

$$T_{il,I} = \frac{1}{I} (ex_{i,I} + 1)^{1-le_2},$$

ex<sub>i,I</sub> implies gained experience (comparable quantity for evaluated units) at the starting of the i<sup>th</sup> cycle.

$$ex_{i+1,I} = (ex_{i,I} + E(X)(v+w)Q)^{1+\frac{f_2}{le_2}} V_i^{\frac{f_2}{le_2}} \tag{19}$$

where V<sub>i</sub> is given by

$$V_i = \left[ \frac{1-le_2}{\frac{1}{I}} \left( \frac{(v+w)Q}{D} - T_I \right) + T(ex_{i,I} + E(X)(v+w)Q) \right]^{\frac{1}{1-le_2}} \tag{20}$$

$V_i$  is gained experience or quantity verified in case of forgetting occurred during verification process, the proportion of remanufactured units is  $p_e$  inventory position reduces to 0 in time  $T_r$  (see Figure 4). Overall carrying charge

$$H_u = h_{mu} \left[ \int_0^{T_i} ((E(X)(v+w)Q) - Q(T))dt + \int_{T_i}^{T_r} (((1 - \delta e_1) - (1 - \delta)(1 - e_2))(E(X)(v+w)Q) - Q(t))dt + \frac{1}{2} \frac{((v+w)Q)^2 E(X)}{D} \right] \quad (21)$$

$Q(t)$  defines as produced item when at time  $t$  in the  $i^{\text{th}}$  reproduction cycle and

$$Q(t) = \left( \frac{1 - le_1}{T_{i1,r}} t_r \right)^{\frac{1}{1-le_1}}$$

Hence for products returned, charge is the sum of inspecting, carrying, purchasing and disposing charges.

For producing  $n_i Q_{i1}$  units in every cycle the time taken

$$T_{CU} = c_i \frac{T_{i1,l}}{1-le_2} (E((X)(v+w)Q))^{1-le_2} + c_d (((1 - \delta)(1 - e_2) + \delta e_1)) E(X)(v+w)Q + c_r ((1 - \delta e_1) - (1 - \delta)(1 - e_2)) E(X)(v+w)Q + h_{mi} \left[ \left( 1 - ((1 - \delta e_1) - (1 - \delta)(1 - e_2)) \frac{((v+w)Q)^{2-le_2} T_i}{1-le_2} (E(X))^{2-le_2} + ((1 - \delta e_1) - (1 - \delta)(1 - e_2)) \frac{(v+w)QT_{i1,r} E(X)}{1-le_1} (wQ)^{1-le_1} \right) - \frac{T_{i1,r}}{2-le_1} (wQ)^{2-le_1} \right] \quad (22)$$

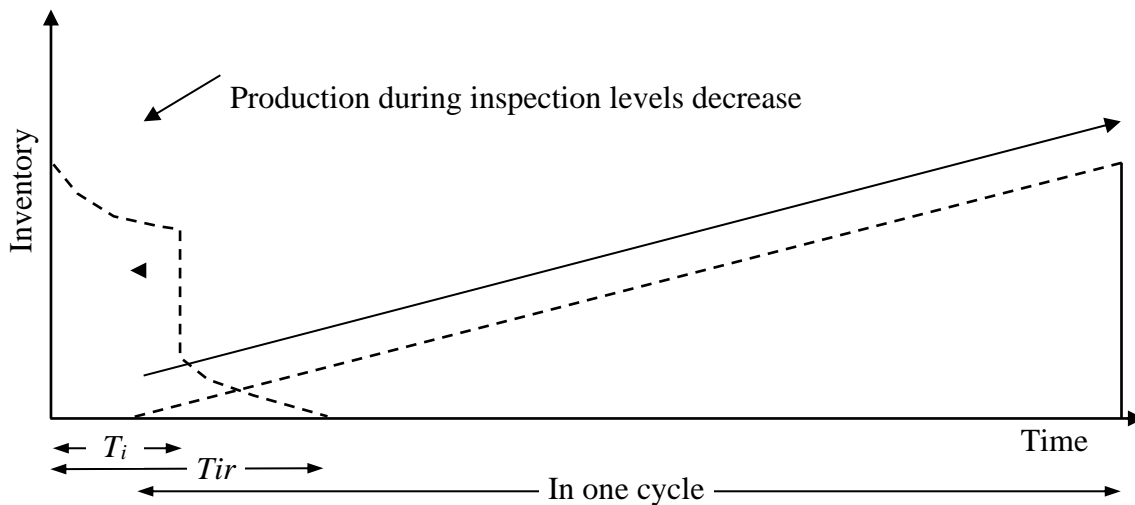


Figure 4: Used inventory at inspection premises (source Giri [23])

**4.4 Charge for carbon emission**

Charge for carbon emission charge is accepted by producer as well as reproducer, carbon emission charge is given by the inspection process involves removal of packaging material ,cleaning cost which are environmental effecting elements. Hence the cost is also considered in the cei per unit cost

$$TC_e = c_{ep}vQ + c_{er}WQ + (cerl *E(x))*((v + w)Q)$$

**4.5 Producers and reproducers stock carrying charge at retailers' end**

The finished products charge which are preserved at retailers' end, the carrying charge is given by

$$\begin{aligned}
 TC_{mr} = h_{mr} & \left[ \left( \frac{1-le_1}{T_{il,m}} t_m \right)^{\frac{1}{1-le_1}} \sum_{k=1}^v (k-1)^{\frac{1}{1-le_1}} + \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{1}{1-le_1}} \sum_{k=1}^v (k-1)^{\frac{1}{1-le_1}} \right. \\
 & - \frac{1}{2} (v-1)^2 Dt_m^2 - \frac{1}{2} (w-1)^2 Dt_r^2 + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,m}} vt_m \right)^{\frac{1}{1-le_1}} - (v-1)Dt_m \right]^2 \\
 & \left. + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,r}} wt_r \right)^{\frac{1}{1-le_1}} - (w-1)Dt_r \right]^2 \right] \tag{23}
 \end{aligned}$$

**4.6 The average value of the expected total charge of the CLLSC**

The average value of the expected charge is calculated by adding the supplier's total charge, the producers' anticipated charge, the reproducers' projected charge, the total charge of utilized goods, the carbon emission charge, and the producer - reproducers carrying charge on the retailer's end. Hence, average anticipated charge(cost) of thee CLLSC is represented by

$$\begin{aligned}
 ETC_i(v, w, Q) &= \frac{D}{(v+w)Q} \{TC_m + TC_r + TC_b + TC_U + TC_{mr} + TC_e\} \\
 &= \frac{D}{(v+w)Q} \left[ A_m + A_r + (v+w)A_b + h_m \sum_{k=1}^v H_{mk} \right. \\
 & \quad \left. h_r \sum_{i=1}^w H_{rk} + h_b \left[ \left( \frac{1-le_1}{T_{il,m}} t_m \right)^{\frac{1}{1-le_1}} \sum_{k=1}^v (k-1)^{\frac{1}{1-le_1}} \right. \right. \\
 & \quad \left. \left. + \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{1}{1-le_1}} \sum_{k=1}^n (k-1)^{\frac{1}{1-le_1}} - \frac{1}{2} (v-1)^2 Dt_m^2 - \frac{1}{2} (w-1)^2 Dt_r^2 \right. \right. \\
 & \quad \left. \left. + \frac{1}{2D} \left( \left( \frac{1-le_1}{T_{il,m}} vt_m \right)^{\frac{1}{1-le_1}} - (v-1)Dt_m \right)^2 \right] \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{2D} \left[ \left( \left( \frac{1-le_1}{T_{il,r}} wt_r \right)^{\frac{1}{1-le_1}} - (w-1)Dt_r \right)^2 \right] \\
 & + h_{mu} \left[ \left( ((1-\delta)(1-e_2) + \delta e_1) \frac{(v+w)Q}{1-le_2} T_l (E(X))^{2-le_2} \right. \right. \\
 & \left. \left. + ((1-\delta e_1) - (1-\delta)(1-e_2)) \frac{(v+w)QT_{il,r}E(X)}{1-le_1} (wQ)^{1-le_1} - \frac{T_{il,r}}{2-le_1} (wQ)^{2-le_1} \right) \right] \\
 & + c_i \frac{T_{il,l}}{1-le_2} (E(X)(v+w)Q)^{1-le_2} + c_d \left( ((1-\delta)(1-e_2) + \delta e_1) E(X)(v+w)Q \right. \\
 & \left. + c_{ep}vQ + c_{er}Wq + (cer1 * E(x)) * ((v+w)Q) \right. \\
 & \left. + \left( \frac{1-le_1}{T_{il,r}} t_r \right)^{\frac{1}{1-le_1}} \sum_{k=1}^w (k-1)^{\frac{1}{1-le_1}} - \frac{1}{2} (v-1)^2 Dt_m^2 - \frac{1}{2} (w-1)^2 Dt_r^2 \right. \\
 & \left. + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,m}} vt_m \right)^{\frac{1}{1-le_1}} - (v-1)Dt_m \right] \right. \\
 & \left. + \frac{1}{2D} \left[ \left( \frac{1-le_1}{T_{il,r}} wt_r \right)^{\frac{1}{1-le_1}} - (w-1)Dt_r \right]^2 \right] \quad (24)
 \end{aligned}$$

As the mathematical model deals with simultaneous optimization of 3 variables it is proposed meta heuristic to solve. That is GA algorithm.

1. Fitness function is considered total cost.
2. The variables n, m, q for each experience status is considered as chromosomes.

Selection: This process carried by awarding ranks depending on fitness value of existing chromosome and depending on the best ranked chromosome is chosen for next operation..

Crossover: Two point cross over is chosen and after completion of cross operation the next operation that is interchanging of genes takes place. Two cross over points C1 and C2 are chosen and the genes which are existing before C1 and after C2 are unaltered even after cross over operation.

Mutation: The execution of mutation is carried only on a selected gene which is available at mutation points on the particular gene present at the mutation points. The location of mutation points also done randomly. The four mutation points may select any of the seven genes.

## 5. Model Data and Results

Aim is to calculate optimum values for w, q, v by minimizing the overall cost.

### 5.1 Data

$c_m =$  Rs. 40,000 per day;  $A_m =$  RS16000;  $le_1 = 0.1$ ;  $c_r =$  RS2000;  $A_b =$  RS1000;  
 $le_2 = 0.2$ ;  $c_d =$  RS40;  $h_{m11} =$  RS20;  $\beta = 0.75$ ;  $c_i =$  RS600;  $h_{r11} =$  RS10;  
 $P =$  1800 per year;  $c_u =$  RS80;  $h_{mr} =$  RS4;  $D = 1200$ ;  $c_{ep} =$  RS80;  $h_b =$  RS10;

$F_1 = 200$  days;  $c_{er} = RS40$ ;  $h_{mu} = RS8$ ;  $FI_2 = 200$  days;  $e_1 = 0.02$ ;  
 $e_2 = 0.02$ ;  $cer1 = 40$

**5.2 Results**

The solution for 6 cycles presented in Table-2 as the number of cycles increases due to learning in production and experience, charges or expenses will come down, after some cycles the rate of such change is negligible because of limitations in human intelligence. At this point some motivations are required.

the cost is found less because of consideration of human factors in learning/forgetting, it provides 29% decrease in cost while considering learning (Table-3) compared to old model where learning effect is not considered.

**Significance of change in parameters.:** Table-4 denotes that learning exponent has a positive influence on the cost function. The expected total cost and learning exponent are indirectly proportional to each other. When the value of  $le_1$  ranges as 0.1 to 0.5, the average cost is decreased by 24%, 18%, 13% and 8.9% respectively, and since the inspection time is very less its influence is very less on cost function.

The inspection rate and the  $le_2$  are directly proportional and it leads to reduction in the holding cost. The returned item purchasing cost, inspection cost and disposal cost of rejected item, emission costs and environmental costs are examined. Their costs paves gradual increase in the expected total cost when they are increasing from low level to high level. As per Table-5. the purchasing cost of used items have greater impact on expected cost of used items. Table-1 shows the main parameters of algorithms with their corresponding three levels of values and optimum values obtained for parameters

**Table-1: Parameters considered for algorithm**

Sl. No.	Parameter	Model a	Model b	Model c	Optimum
1	Population size	50	100	150	150
2	Mutation probability	0.1	0.25	0.4	0.25
3	Cross over probability	0.5	0.7	0.9	0.9
4	Max iteration	50	100	150	150
5	ETC	25235	24120	23400	22933

**Table-2: Optimum results for cycle**

i	cx <sub>i</sub>	v*	w*	Q*	ETC <sub>i</sub> *
1	0	3	4	48	28760
2	70.1	3	3	55	23360.8
3	89	3	3	54	23027.2
4	95.63	4	3	53.5	22933.6
5	96	4	3	53.2	22902.4

**Table-3: Comparison of old and new model**

Sl. No.	Description	v	w	q	ETC
1	Old model	4	4	42	37200
2	New model	3	3	51	24600

**Table-4: Results for total cost for different  $le_1$  and  $le_2$**

Parameters	Value	Parameters	Values	v*	w*	Average Q*	Average ETC
le <sub>2</sub>	0.5	le <sub>1</sub>	0.1	2.2	3.21	52	24196.8
			0.25	2.23	2.25	63	18322.4
			0.3	1.5	2.2	72	15127.2
			0.45	1.5	1.5	102	13193.6
			0.5	1	1.2	110	12032

le1	0.5	le2	0.2	2.3	3.3	50	24196.8
			0.35	2.25	3.2	51.83	24180.8
			0.4	2.2	3.2	52	24172.8
			0.5	2.15	3.2	52.1	24168
			0.6	2.1	3.2	54	24164.8

**Table-5: Results which are optimum in nature for different types of charges  $c_u$ ,  $c_i$  and  $c_d$**

Par a-met ers	Val ues	ET C	Par a-met ers	Val ues	ET C	Par a-met ers	Val ues	ET C	Par a-met ers	Val ues	ET C	Par a-met ers	Val ues	ET C	Par a-met ers	Val ues	ET C
$c_u$	40	23944	$c_i$	400	23472	$c_d$	40	24192	cep	40	24196.8	cer	40	23656.8	cer1	40	23840
	80	24192		600	24192		80	24376		80	25156.8		80	24196.8		80	24276.8
	120	24736		800	24912		120	24552		120	26116.8		120	24736.8		120	24816
	160	25272		100	25632		160	24736		160	27076.8		160	25276.8		160	25352
	200	25816		120	26352		200	24896		200	28036.8		200	25816.8		200	25896

Taking into effect the variation of forgetting time in both production and inspection at a time and forgetting time in inspection, the workers' experience grows in terms of equivalent gained experience for bigger volume of production as depicted in Table-6

**Table-6: Experience vs. forgetting time**

Equivalent quantity (only inspection considered)	(F1 and F2) value	Equivalent quantity (production and manufacturing considered)
5	150	40
9	200	70
10	250	90
12	450	100
15	500	110

### 6. Conclusions and Future Scope

An increase in learning effect in manufacturing tends to a lowering the system cost, though the impact of forgetting phenomena - does not brings up that cost line . Hence it can concluded that if learning only assumed in production it will be better from overall angle. Hence the production managers must search the methods and procedures to eliminate idle time. The managers must search all provisions to enhance the learning rate in their operations by training.

It is also evident that higher the quantity of reproduced items lesser the generation of greenhouse gases because the use of fresh raw material is minimized.

The variation of learning component can help the production managers to determine the correct learning rate and reduce the forgetting time. These measures help in the extension of the threshold level of worker learning and reduces the effect of forgetting.

The reduction of inspection time leads to decrease of holding time which in return leads to price drop in holding used items. On the contrary very fast inspection will make reduction in learning time and increase the forgetting time. Hence the inspection rate shall

be a reasonable one so that inspection learning rate will improve, which in turn improves inspection rate gradually. The return of items shall be packaged in lots and proper bio degradable material shall be used on return products transport so that the cost of disposal will be minimum which in turn helps environment sustainability. if inspection error is minimized by proper training ,the cost of disposal will reduce.

Consideration of multi product and multi retailers in inventory control can be considered in future scope .it can be extended for assemblies instead of individual components. different heuristics can be tried

## References

1. Mohamad Y. Jaber (2011), Learning Curves Theory, Models, and Applications, <https://doi.org/10.1201/b10957>, eBook ISBN9780429150142
2. Govindan K. Soleimani H (2017) A review of reverse logistics and closed-loop supply chains: a Journal of Cleaner Manufacturing focus. *J Clean Prod* 142:371- 384
3. Govindan K, Soleimani H. Kannan D (2015) Reverse logistics and closed-loop supply chain: a comprehensive review to explore the future. *Eur J Oper Res* 240:603-626
4. Petruzzi, N.C. Dada. M. Pricing and newsvendor issue: a review with extensions. *Oper. Res.* 47. 183-194(1999)
5. Le, C., Chu, W. (2005) Who should control the inventory in a supply chain? *Eur. J. Oper. Res.* 164,158-172
6. Ru, J., Wang, Y. (2010) Consignment contracting: Who should control inventory in the supply chain? *Eur. J. Oper. Res.* 201,760-769
7. Wang. Y, Jiang, L., Shen. Z.J. (2004) Channel performance under consignment contract with revenue sharing. *Manag. Sci.* 50. 34-47
8. Disney SM, Towill DR (2003) The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains. *Int J Prod Econ* 85:199-215
9. Bazan E, Jaber MY, El Saadany AM (2015) Carbon emissions and energy effects on manufacturing-remanufacturing inventory models. *Comput Ind Eng* 88:307-316
10. Bazan E, Jaber MY, Zanoni S (2015) Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions. *Appl Math Model* 39:5131-5151
11. Bazan E, Jaber MY. Zanoni S (2017) Carbon emissions and energy effects on a two- level manufacturer-retailer closed-loop supply chain model with remanufacturing subject to different coordination mechanisms. *Int J Prod Econ* 1 83:394 408
12. Qiu Y, Ni M. Wang L, Li Q. Fang X, Pardalos PM (2018) Manufacturing routing issues with reverse logistics and remanufacturing. *Transp Res Part E Logist Transp Rev* 1 11:87-100
13. Taleizadeh AA, Moshtagh MS (2019) A consignment stock scheme for closed loop supply chain with imperfect manufacturing processes, lost sales, and quality dependent return: multi levels structure. *Int J Prod Econ* 217:298-316
14. Bhatnagar R. Saddikutti V, Rajgopalan A (2007) Contingent manpower planning in a high clock speed industry. *Int J Prod Res* 45:2051-2072
15. Jaber MY, Bonney M (2003) Lot sizing with learning and forgetting in set-ups and in product quality. *Int J Prod Econ* 83:95-11 1
16. Jaber MY, Bonney M (2011) The lot sizing issue and the learning curve: a review. *Learning curves. Theory Models Appl* 21:267-293
17. Jaber MY, El Saadany AM (2011) An economic manufacturing and re manufacturing model with learning effects. *Int J Prod Econ* 131:115-127
18. Zanoni S, Jaber MY. Zavanella LE (2012) Vendor managed inventory (VMI) with consignment considering learning and forgetting effects. *Int J Prod Econ* 140:721- 730
19. Khan M, Jaber MY, Ahmad A-R (2014) An integrated supply chain model with errors in quality inspection and learning in manufacturing. *Omega* 42:16-24
20. Manna AK. Das B. Dey JK. Mondal SK (2017) Multi-item EPQ model with learning effect on imperfect manufacturing over fuzzy- random planning horizon. *J Manag Anal* 4:80-110
21. Giri B, Glock CH (2017) A closed-loop supply chain with stochastic product returns and worker experience under learning and forgetting. *Int J Prod Res* 55:6760-6778
22. Giri B. Masanta M (2021) A manufacturing–remanufacturing supply chain model with learning and forgetting in inspection ,Operational research. <https://doi.org/10.1007/s12351-021-00662-1>
23. Zanoni S, Marchi B, Jaber MY (2019) Economic manufacturing quantity model with learning in manufacturing, quality, reliability and energy efficiency. *Comput Ind Eng* 129:502-511

24. Braglia M., Zavanella L (2003) Modeling an industrial strategy for inventory management in supply chains: the Consignment Stock case. *Int J Prod Res* 41:3793- 3808
25. Hill RM (1999) The optimal manufacturing and shipment policy or the single- vendor single-buyer integrated manufacturing-inventory issue. *Int J Prod Res* 37:2463-2475
26. SK Zahran, MY Jaber (2017), Investigation of a consignment stock and a traditional inventory policy in a three-level supply chain system with multiple-suppliers and multiple-buyers - *Applied Mathematical Modelling*, Elsevier, Sciencedirect.com
27. Hemmati M. Flatemi Ghomi SMT. Sajadieh MS (2017) Vendor managed inventory with consignment stock for supply chain with stock-and price-dependent demand. *Int J Prod Res* 55(18):5225-5242
28. Hemmati M. Flatemi Ghomi SMT. Sajadieh MS (2018) Inventory of complementary products with stock-dependent demand under vendor-managed inventory with consignment policy. *Sci Iran* 25(4):2347-2360
29. A Nanda, S Sen, (2021), A complex network theory based approach to better understand the infiltration-excess runoff generation thresholds, *Journal of Hydrology*, Vol. 603, Part-B, <https://doi.org/10.1016/j.jhydrol.2021.127038>
30. Nima Kazemi (2018) A review of reverse logistics and closed loop supply chain management studies published in IJPR: a bibliometric and content analysis, *International Journal of Production Research*, Vol. 57, Issue 15-16. <https://doi.org/10.1080/00207543.2018.1471244>
31. Yang Liu and Zhenyu Yang, (2021) Information provision and congestion pricing in a risky two-route network with heterogeneous travelers, *Transportation Research Part C: Emerging Technologies*, Vol. 128, <https://doi.org/10.1016/j.trc.2021.103083>
32. Yaqin Yuan, Chung-Lien Pan, Zhiying Wu, Weiting Lu, Qianman Lu (2023) Proceedings of the 2nd International Academic Conference on Blockchain, Information Technology and Smart Finance (ICBIS 2023), DOI 10.2991/978-94-6463-198-2\_134.
33. Chirantan Mondal, Bibhas C. Giri & Sanjib Biswas, (2022) Integrating Corporate Social Responsibility in a closed-loop supply chain under government subsidy and used products collection strategies, *Flexible Services and Manufacturing Journal*, 34, pages65–100.
34. Jaber MY. Bonney M (1998) The effects of learning and forgetting on the optimal lot size quantity of intermittent manufacturing runs. *Prod Plan Control* 9:20-27
35. Dey O. Giri BC (2019) A new approach to deal with learning in inspection in an integrated vendor-buyer model with imperfect manufacturing process. *Comput Ind Eng* 131:515-23
36. K. Skouri, I. Konstantaras, A.G. Lagodimos, S. Papachristos (2014) An EOQ model with backorders and rejection of defective supply batches, *International Journal of Production Economics*, Vol. 155, Pages 148-154. <https://doi.org/10.1016/j.ijpe.2013.11.017>
37. Asadkhani J., Hadi Mokhtari, Saman T., (2022) Optimal lot-sizing under learning effect in inspection errors with different types of imperfect quality items, *Operation Research* 22(3-4), DOI: 10.1007/s12351-021-00624-7.