

## A Basic Pareto Distribution To Solve The Length Of Stay At Hospital By Using Fuzzy Stochastic Transportation Problem

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### Abstract :

*In this paper a fuzzy time algorithm is applied using stochastic transportation problem with the aspect of Pareto distribution in the rearrangement of hospital bed and length of long stay at hospital. The LOS at hospital have the major three objectives – reduction of expenses, effectiveness of treatment, and the lack of available beds. The LOS measure is used to make the decisions based on the given objectives because it is dependent on the variations of the losses, determining the number of reassigned in each department's bed might thus be viewed as stochastically. Computational results with numerical.*

**Keywords :** Stochastic transportation problem, fuzzy set, Pareto distribution, LOS.

### 1. Introduction

In general, the length of stay plays a key role in hospital administration. By based on the length of stay patients the hospitality morals, efficiency, service and finance are affected and reached directly to the publications. So, the basic equipment's and availability of the services are two sides of the coins. In the based on the number of inpatients are mainly affect the new patient's admission, waiting period, emergency cases and transforming to the other departments, or hospitals etc., The main objective of this paper is calculated length of stay based on the basic pareto distribution with the objectives are rate on occupation bed, increasing the admissions, minimizing the waiting period and different staying periods based on different diseases. The LOS at hospital have the major three objectives – reduction of expenses, effectiveness of treatment, and the lack of available beds. Among these objectives frame the multiobjective fuzzy time algorithm for solving the length of stay. The length of stay patients is not stable and clearly mentioned because the situations may be clear, faith, good, well treated, unsatisfactory, financial issues, time imbalance, or any. That's the main factor the length of stay values differs at any time. Therefore, the values are situation handled by taking the fuzzy multi-objective time algorithm on the basic pareto distribution to solve the length of stay at hospital by using stochastic transportation problem.

In this paper we discussed in section 2, as the Literature Reviews of the length of the stay, in section 3, as the pdf and cdf of the basic pareto distribution and the basic pareto distribution LOS description, as well in section 4, the algorithm of the problem, in section 5, the real-life hospital data we solved by using the given algorithm, in section 6 conclusion.

### 2. Literature Reviews on LOS

The length of stay at hospital is the very important issues in hospital management based on this we had some future recommendations and compact ability. In the below we mention

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the some of the papers from last decades. Many of the researchers can be worked out the problems of the length of stay at hospital since 1963. WHO declared that length of stay has major role in the hospital management. Some of the papers are given here

1. 2006 – Akkerman R and Knip M [1]– Reallocation of beds to reduce waiting time for cardiac surgery – The waiting time is a significant problem in the heart operations, solving by case studies.
2. 2007 – Bachouch R, Giomet A and Hajri-Gabouj S [2]–Gestion des lits mutualises dun etablissement hospitalier – The problem of the bed consists of demand of the bed, costly patients transfer.
3. 2008 – Ciesla D, Save J and Kennedy S [3]– Trauma Patients: you can get them in, but you can't get them out – Comparing the bed consumption and classified in terms of stay – long, middle and short.
4. 2009 – Li, Beullens P, Jones D and Tamiz M [4]– An integrated queuing and multiobjective bed allocation model with application to a hospital in china – Bed allocation in each department wise with refused patients and non-utilized bed.
5. 2010 – Qualls M, Daniel JP, and Jeremiah DS [5]– Parametric versus nonparametric statistical tests: the length of stay example – Assuming inappropriate will lead to errors in estimating the stay which in turn affect the model.
6. 2011 – Sanjeev Kumar J, Dwivedi AKD, and Amod T [6]– Necessity of goodness of fit tests in research and development – Finding the p value in different distributions.
7. 2012 – Tang X, Zhehui L, and Gardiner JC [7]– Modelling hospital length of stay by coxian phase-type regression with heterogeneity – LOS giving importance for healthcare practitioners and administrators in by using to strategic planning.
8. 2013 – Hachesu PR, Maryam A, Somayyeh A, Farahnaz S [8]– Use of data mining techniques to determine and predict length of stay of cardiac patients. – Stay of patient as the extent of a single episode of hospitalization of the patients for which the healthcare providers are interested to determine the distribution.
9. 2015 – Gul M, Guneri AF [9]– Forecasting patient length of stay in an emergency department by artificial neural networks – LOS and forecasting is crucial for hospital since it helps in appropriate allocation of Resources.
10. 2016 – Papi M, Luca P, and Roberto S [10]– A new model for the length of stay of hospital patients – Study of the quantify and LOS is asymmetric.
11. 2017 – Specogna AV, Tanvir CT, Scott BP, and Michael DH[11] – Hospital treatment costs and length of stay associated with hypertension and multimorbidity after haemorrhagic stroke – Critical decisions which is helpful for forecasting, allocation and improving occupancy of beds for future.
12. 2018 – Harini S, Subbiah M, and Srinivasan M R [12]– Fitting Length of Stay by Multi Stage Classification of Covariates Using Transformed Gamma–Pareto Distribution – Transformed distributions to study length of stay with other health problems and also, for practitioners in decision making process.
13. 2019 – Doupe P, Faghmous J, and Basu S.[13] – Machine learning for health services researchers – Hospital management issued problems solved by machine learning.
14. 2020 – Badreddine Jerbi [14]– A fuzzy multiobjective polynomial time algorithm to solve the stochastic transportation formulation of a hospital bed rearrangement problem –

Bi-objective bed rearrangement problem, treating the increasing admission and the bed occupation.

15. 2021 – Thang s. Han, et.,al., [15]– Evaluation of the association of length of stay in hospital and outcomes – Timely discharging based on the NHO, death.

16. 2022 – Mehrnoosh Eskandari, et.,al.,[16] – Evaluation of factors that influenced the length of hospital stay using data mining techniques – The longer old patients than younger patients in critical ward using data mining.

17. 2023 – Nassim Dehouche, and et.,al. [17]– Hospital length of stay: A cross-specialty analysis and Beta-geometric model – Beta geometrical model to reproduce empirical of LOS.

### 3. Basic Pareto Distribution

The Pareto distribution is named after Vilfredo Pareto [25], an economist. The Pareto distribution is a skewed, heavy-tailed distribution that is sometimes used to simulate income and other financial variable distributions. Recently the pareto distribution had wide applications in the medical department. Decision making, forecasting, multi-objective, stochastic decisions, uncertainty, different cases studies etc., problem solving by using distributions. Random variable  $X=bZ$  has the Pareto distribution with shape parameter  $a$  and scale parameter  $b$ . Here  $Z$  is the special case, the Basic Pareto Distribution.

#### 3.1 Continuous Distribution Function

The basic Pareto distribution with shape parameter  $a \in (0, \infty)$  is a continuous distribution on  $[1, \infty)$  with distribution function  $G$  given by

$$G(z)=1-\frac{1}{z^a}, z \in [1, \infty)$$

$G$  is increasing and continuous on  $[1, \infty)$ , with  $G(1)=0$  and  $G(z) \rightarrow 1$  as  $z \rightarrow \infty$ .

#### 3.2 Probability Density Function

The basic Pareto distribution with shape parameter  $a \in (0, \infty)$  is a continuous distribution on  $[1, \infty)$  with distribution function  $g$  given by

$$g(z) = \frac{a}{z^{(a+1)}}, z \in [1, \infty)$$

#### 3.3 Basic Pareto Distribution of LOS variable measure

The LOS has a special feature based on the several reasons it exits the curve based on log normal, Weibull, gamma distribution (2009). Recently the power law distribution is also used to arrange the beds in hospital (2020). Now we defined in the basic pareto distribution because nowadays pareto distribution with single parameter used in real life decision making modelling in all the fields. We had derive the probability density function at length of stay as follows, the parameter  $a$  here shape parameter of the basic pareto distribution, as

$$g(z) = \frac{a}{z^{(a+1)}}, z \in [1, \infty)$$

$$\begin{aligned} E(z) &= \int_1^{\infty} z \frac{a}{z^{(a+1)}}, 0 < n < a \\ &= \frac{a}{a-1}, a > 1 \end{aligned}$$

$$ALOS, \frac{a}{a-1} = \frac{\text{length of stay at hospital}}{\text{number of annual admissions}} = \frac{1}{p}$$

Therefore,  $a = 1/1-p$ ,

where  $p=(\text{annual admissions})/(\text{length of stay at hospital})$ .

The probability distribution function of the LOS variable calculated by using the above formula in basic pareto distribution.

$$P(\text{LOS}) = \sum_{z=\min}^{365} \frac{a}{z^{(a+1)}} = \alpha \sim (\text{capacity} * 10\%) + \alpha$$

Here  $\min= 5$  days of the stay at hospital and also consider the not to exceed 10% of the patients stay at hospital in each department.

#### 4. Procedure of the LOS

For solving the LOS by taking the basic pareto distribution we need to follow the following procedure

##### 4.1 Measuring Variables

The basic variables are measured to calculate length of stay in stochastic transportation problem. They are the ALOS, excess of bed, average occupation percentage, demanding departments (above the 100%) and supply departments (below the 100%) of the beds [24].

1.  $ALOS = (\text{length of stay at hospital})/(\text{annual admissions at hospital})$ .
2.  $\text{Excess of beds} = \text{beds capacity} - (\text{length of stay at hospital} / 365)$ .
3.  $\text{Average occupation percentage} = \text{length of stay at hospital} / 365 / \text{beds capacity} * 100$ .
4.  $\text{Demanding Department} = \text{above the 100\% of average occupation percentage}$ .
5.  $\text{Supplying Department} = \text{below the 100\% of average occupation percentage}$ .

##### 4.2 Probability on LOS

The probability on LOS are measured by using the basic pareto distribution. For this, we calculate the shape parameter  $a$  and  $p$ . By observing these  $P(\text{LOS})$  values we determine the demanding department and supplying department.

1.  $p=(\text{annual admissions})/(\text{length of stay at hospital})$ .
2.  $a = 1/1-p$ .
3. probability density of the length of stay at hospital by taking the pareto distributions i.e.,  $P(\text{LOS}) = \sum_{z=\min}^{365} \frac{a}{z^{(a+1)}} = \alpha \sim (\text{capacity} * 10\%) + \alpha$
4.  $\text{Demanding Department} = \text{positive values of } (\text{admissions at hospital})/365 * P(\text{LOS}) - \text{beds capacity}$ .
5.  $\text{Supplying department} = \text{negative values of } (\text{admissions at hospital})/365 * P(\text{LOS}) - \text{beds capacity}$ .

##### 4.3 Measuring new costs for TP

For solving stochastic linear programming transported we have to calculate the new cost values from supplying department to demanding department. It depends on both staying of hospital departments of the needed bed so we convert into imprecise. The triangular fuzzy number is in the form of

$$\tilde{X} = (\min = 5, \min(D, S), \max(D, S))$$

Now we easily calculated the relating of these departments successfully. Also, for easy evaluate the measuring LOS we convert the crisp data into real data by using the ranking method [18],

$$x = \frac{a + 8 * b + c}{10}$$

Now we get the new cost value of stochastic transportation. Now we go on the next procedure for that we modelling the transportation problem.

#### 4.4 Stochastic Transportation Problem

It is special linear programming transported [19] from m sources with supplies and n destinations with demands. Let  $a_i$  is the supply of the source,  $i=1,2, \dots, m$  and  $b_j$  is the demand of destination,  $j=1,2, \dots, n$  and  $x_{ij}$  is the number of transport from source to destination and  $c_{ij}^k$  is the coefficients of the objective function.

The objective function of the k as

$$\min z_k = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^k x_{ij}, k = 1, 2, \dots, K$$

under the constraints,

$$\sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \leq b_j, j = 1, 2, \dots, n$$

and  $x_{ij} \geq 0 \forall i$  and  $j$ .

and also  $a_i > 0, b_j > 0$  then  $\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$ .

#### 4.5 Solving TP

We all know that the there are three ways for determining the initial basic feasible solution. They are

1. NorthWest Corner Cell Method.
2. Vogel's Approximation Method (VAM).
3. Least Call Cell Method.

Among these [20-23] the least cost method we used here as the most preferable method and for the optimal solution we prefer the MODI method. By using these we calculate the objective solution and determine the reassignment of each department.

#### 4.6 LOS Measure

Now by getting the objective solution of the transportation problem, we found the new capacity of the beds in each department as well as New ALOS, changes of admissions, new admission.

1. New capacity = old capacity +/- assigned value from supply department to demand department.
2. New ALOS = (New-Old)capacity of beds \*(365/Old ALOS).
3. Changes of Admissions = length of stay at hospital/New ALOS.
4. New Admissions = Old admissions +/- changes of admissions.

#### 5. Discussion

For this we take the real hospital data on the admitted the second term covid patients on having different illness, Aardhya hospital, Vellore at the year 2022. Now by using the given hospital data we calculate the preliminary measurable variables based on step-1 as shown in table 1.

Dept.	Beds	NA	NS	ALOS	Excess	AO%
OR	178	4186	89625	21.41	-67.55	137
ENT	52	1084	18671	17.22	0.85	98
SG	30	841	7320	8.70	9.95	66
UR	60	1228	12931	10.53	24.57	59
NS	32	4252	12699	2.99	-2.79	108
EM	43	2055	10383	5.05	14.55	66
CD	15	2903	7072	2.44	-4.38	129
ICM	21	4896	8796	1.8	-3.1	114

Table 1. Covid on different illness data

Here NA - number of admissions and NS - number of days staying at hospital and ALOS - average of length of stay, AO% - average of occupational in percentage, and also describe from the table AO% is above 100 are Demand departments and below 100 are supply departments.

As well as by using the hospital data we calculate the probabilities on LOS by using the step-2 as follows, in table 2. By comparing the P(LOS) we get needed of beds.

Dept.	p	a	P(LOS)	NBN	D/S
OR	0.05	1.05	17.8	26	D
ENT	0.06	1.06	6.5	-32	S
SG	0.11	1.13	4.3	-20	S
UR	0.09	1.10	7.3	-35	S
NS	0.33	1.50	4.8	23	D
EM	0.19	1.24	5.6	-10	S
CD	0.41	1.69	3.3	11	D
ICM	0.55	2.25	4.3	37	D

Table 2. P(LOS) on covid patients on different illness

Here p and a are the parameters of pareto distribution, P(LOS) - probabilities of the length of stay at hospital, NBN - number of new beds needed by using pareto distribution. When we know the demand and supply departments, based on these departments we have to

calculate the cost of the transportation. For this we take the help of crisp information for exact adjustments. By using step-3 formulas then we get the TP as follows, in table 3.

Dept.	OR	NS	CD	ICM	Demand
ENT	23899	12026	7525	8904	32
SG	14819	7126	6390	6736	20
UR	19307	11452	6951	8330	35
EM	17269	9576	6696	8075	10
Supply	26	23	11	37	97

Table 3. Stochastic Transportation Problem

Now we get the initial basic feasible solution as well as Least cost method and verified by MODI method. We got the optimal solution as mentioned 1058925. The reassigned of the beds in each department as well as below shows

$$\begin{pmatrix} 0 & 19 & 11 & 2 \\ 16 & 4 & 0 & 0 \\ 0 & 0 & 0 & 35 \\ 10 & 0 & 0 & 0 \end{pmatrix}$$

By using reassigned values we can calculate the remaining variables as step-6 as follows,

Dept.	Old	New	New ALOS	NA	Changes	N Ad
OR	178	204	443	4186	202	4388
ENT	52	20	-678	1084	-27	1056
SG	30	10	-838	841	-8	832
UR	60	25	-1213	1228	-10	1217
NS	32	55	2810	4252	4	4256
EM	43	33	-722	2055	-14	2040
CD	15	26	1648	2903	4	2907
ICM	21	58	7517	4896	1	4897

Table 4. Reassigned beds in each department

By observing the above values we reach the reassigned number of beds in each department and also increase the number of admissions from 21445 to 21596, i.e., 151.

## 6. Conclusions

The basic pareto distribution had been used to reassigned the beds in each department which need. The estimated parameters in basic pareto distribution depends on the number of staying in hospital and annual admissions. The probabilities of the length of stay gave the demand and supply departments. The relation of the demand and supply departments' length of stay we used here the crispy rule. The triangular fuzzy number has been converted into real by taking ranking formula, which gives approximate value. The optimal solution gets by using the MODI method, the value of the TP path gives assigning beds from supply department to demand department. After reassigning the beds the admissions of the hospital increase and also length of stay hospital arranging in each department beds.

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