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Πg - Locally Closed Sets And Πgl-Continuous Functions

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Abstract: -
In this paper, we explore the concept of generalized πg-locally closed sets and investigate their properties within the framework of ideal topological spaces. We introduce a new class of functions, namely πgl-continuous functions, by leveraging the interplay between closed sets and specific kernels. Our study delves into the intricate relationships between these novel sets and functions, shedding light on their behaviour and applicability within the realm of ideal topological spaces, through rigorous analysis, we establish fundamental properties and theorems, providing a comprehensive understanding of the generalized πg-locally closed sets and πgl-continuous functions in this context. This research contributes to the border field of topology by extending the existing knowledge base and paving the way for further exploration in this area.

Keywords and Phrases: - *πg-closed sets, generalized locally closed LC-continuous, πgl continuity and πgl-irresolutenes*

Mathematics Subject Classification, 54A05

1. INTRODUCTION

The initiation of the study of generalized closed sets was done by Levine[1] in 1970. The notion of πg-closed sets a[1]s a weak form of generalized closed sets was introduced by Dontchev & Noiri [2] . The notion of locally closed sets in a topological space was introduced by. Ganster & Reilly[3] , further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. Balachandran et al[4] introduced the Concepts of generalized locally closed sets and GLC-continuous functions and investigated some of their properties. In 1997, Arockiarani et al. [5] studied regular generalized locally closed sets and RGL-continuous functions in a topological space. The aim of this chapter is to continue the study of generalizations of locally closed sets and investigate the classes of πgl-continuous functions and πgl-irresolute functions in a topological space. Throughout this thesis, a space (X, τ) denotes a topological space with a topology τ on which no separation axioms are assumed unless explicitly stated. For a subset A of X, cl (A) and int (A) denote the closure of A and the interior of A with respect to (X, τ) respectively.

2.1 Preliminaries

Definition 2.1.1 Power Set (P(X)): For any set X, the power set P(X) is the set of all subsets of X, including the empty set and X itself.

Definition 2.1.2 Topology (X, τ): Here, X represents a set, and τ is a collection of subsets of X that satisfy certain properties defining the concept of open sets. These open sets are drawn from τ, forming a topology on X.

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Definition 2.1.3 Closure (clθ(A)): The closure of a set A, denoted clθ(A), consists of all elements x in the space X such that for every open set U in the topology τ containing x, the intersection of the closure of U and A is not empty. In simpler terms, it includes all the points that are adherent to A under the θ-topology.

Definition 2.1.4 θ-Closed Set: A set A contained within the space (X, τ) is considered θ-closed if it equals its closure under the θ-topology. This implies that every point in A is adherent to A itself.

Definition 2.1.5 Complement of a θ-Open Set: The complement of a θ-open set is called θ-closed.

Definition 2.1.6 A subset A of a space (X, τ) is called

1. Regular open (Stone) [6] if A = int (cl (A)).
2. π-open (Zaitsav) [7] if the finite union of regular open sets.
3. Generalized closed (g-closed) (Levine)[1] if cl (A) ⊂ U whenever A ⊂ U and U is open in X.
4. πg-closed (Dontchev & Noiri) [2] if cl (A) ⊂ U whenever A ⊂ U and U is π-open in X.
5. θ -generalized closed (θ-g-closed) (Dontchev & Maki) [8] if $cl_\theta(A) \subset U$ whenever A ⊂ U and U is open in X.
6. Locally closed (Ganster & Reilly) [3] if A = S ∩ F where S is open and F is closed in X.
7. Generalized locally closed (glc) (Balachandran et al.) [9] if A = S ∩ F where S is g-open and F is g-closed in X.
8. θ-generalized locally closed (θglc) (Arockiarani & Balachandran) [10] if A = S ∩ F where S is θ-g-open and F is θ-g-closed in X.
9. θ-locally closed (θlc) (Arockiarani & Balachandran) [10] if A = S ∩ F where S is θ-open and F is θ-closed in X.
10. θlc^*-set (Arockiarani et al.) [10] if A = S ∩ F where S is θ-open and F is closed in X.
11. θlc^{**} -set (Arockiarani et al. [10] if A = S ∩ F where S is open and F is θ-closed in X.
12. glc^*-set (Balachandran et al) [9] if A = S ∩ F where S is g-open and F is closed in X
13. glc^{**}-set (Balachandran et al. [9] if A = S ∩ F where S is open and F is g-closed in X.
14. θ-glc^*-set (Arockiarani et al.) [10] if A = S ∩ F where S is θ-g-open and F is closed in X.
15. θ-glc^{**}-set (Arockiarani et al.) [10] if A = S ∩ F where S is open and F is θ-g-closed in X.

The complements of the above mentioned closed (open) sets are called their respective open (closed) sets.

Remark 2.1.7 (Dontchev & Maki) [8]
The following diagram holds in a topological space.

θ-closed → θ-g-closed
↓ ↓
closed →g-closed →πg-closed

Figure 2.1 Implication diagram

Definition 2.1.8

A function f: (X, τ) →(Y, σ) is called

1. LC-continuous (Ganster & Reilly) [3] if $f^{-1}(V)$ is locally closed in (X, τ) for every V∈σ.
2. GLC-continuous (Balachandran et al.) [9] if $f^{-1}(V)$ is glc-set in (X, τ) for every V∈σ.
3. θ GLC-continuous (Arockiarani & Balachandran) [10] if f^{-1} (V) is θglc-set in (X, τ) for every V∈σ.
4. θ -LC-continuous (Arockiarani & Balachandran) [10] if f^{-1} (V) is θlc-set in (X, τ) for every V∈σ.

Theorem 2.1.9 (Ekici & Baker) [11] If A is π-open and πg-closed in a space (X, τ), then A is closed.

Lemma 2.1.10 (Dontchev & Noiri [2]

For πg-closed sets of a space X, the following properties hold:

1. Every finite union of πg-closed sets is always a πg-closed set.
2. Even a countable union of πg-closed sets need not be a πg-closed set.
3. Even a finite intersection of πg-closed sets may fail to be a πg-closed set.

Lemma 2.1.11 (Ekici & Baker 2007) [11]

A set A of X is πg-open if and only if F ⊂ int(A) whenever F ⊂ A and F is π- closed.

2.2 πg-LOCALLY CLOSED SETS

Definition 2.2.1

A subset S of a space (X, τ) is said to be πg-locally closed (πglc) if S = G ∩ F where G is πg-open and F is πg-closed in (X, τ).

Definition 2.2.2

A subset S of a space (X, τ) is called πglc^* if there exists a πg-open set G and a closed set F of (X, τ) such that S = G ∩ F.

Definition 2.2.3

A subset B of a space (X, τ) is called πglc^{**} if there exists an open set G and a πg-closed set F of (X, τ) such that B = G ∩ F.

The collection of all πg-locally closed (resp. πglc^*, πglc^{**}) sets of a space (X, τ) will be denoted by πGLC(X, τ) (resp. $\pi GLC^*(X, \tau)$, $\pi GLC^{**}(X, \tau)$).

From the above definitions we have the following results.

Theorem 2.2.4

1. Each locally closed set is πglc.
2. Each θ-locally closed set is πglc.
3. Each θglc-set is πglc.
4. Each πglc^*-set or πglc^{**} is πglc.
5. Each glc-set is πglc.
6. Each θlc-set is πglc^* or πglc^{**}.
7. Each glc^*-set is πglc^*.
8. Each θlc^*-set is πglc^*.
9. Each y θlc^{**}-set is πglc^{**}.
10. Each θglc^*-set is πglc^*.
11. Each locally closed set is πglc^* and πglc^{**}.

However the converses of the above are not true may be seen by the following Examples.

Example 2.2.5

Let X = {a, b, c, d} and τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}}. Then locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b,d} and πglc-sets are P(X). It is clear that {a, c} is πglc-set but it is not locally closed.

Example 2.2.6

In Example 2.2.5, θ-locally closed sets are ϕ, X and πglc-sets are P(X). It is clear that {a, b} is πglc-set but it is not θ-locally closed set.

Example 2.2.7

In Example 2.2.5, θglc-sets are ϕ, X, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c,d} and πglc-sets are P(X). It is clear that {b, c} is πglc-set but it is not θglc-set.

Example 2.2.8

Let X = {a, b, c, d, e} and τ = {ϕ, X, {a}, {e}, {a, e}, {c, d}, {a, c, d}, {c, d, e}, {a, c, d, e}, {b, c, d, e}}. Then πglc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a,d}, {a, e}, {b, e}, {c, d}, {c, e}, {d, e}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {c, d, e}, {a, b, c, d}, {a, c, d, e}, {b, c, d, e} and πglc-sets are P(X). It is clear that {b, c} is πglc-set but it is not πglc^*-set.

Example 2.2.9

In Example 2.2.5, πglc-sets are P(X) and glc-sets are ϕ, X, {a}, {b}, {c},{d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}. It is clear that {b, c} is πglc-set but it is not glc-set.

Example 2.2.10

In Example 2.2.5, θlc-sets are ϕ, X and πglc^* (or) πglc^{**}-sets are P(X). It is clear that {a, b} is πglc^* (or) πglc^{**}-set but it is not θlc-set.

Example 2.2.11

In Example 2.2.5, glc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {c, d}, {a, b,c}, {a, b, d} and πglc^*-sets are P(X). It is clear that {b, c} is πglc^*-set but it is not glc^*-set.

Example 2.2.12

In Example 2.2.5, θlc^*-sets are ϕ, X, {c}, {d}, {c, d} and πglc^*-sets are P(X). It is clear that {a, d} is πglc^*-set but it is not θlc^*-set.

Example 2.2.13

In Example 2.2.5, θlc^{**}-sets are ϕ, X, {a, b}, {a, b, c}, {a, b, d} and πglc^{**}-sets are P(X). It is clear that {a} is πglc^{**}-set but it is not θlc^{**}-set.

Example 2.2.14

In Example 2.2.5, θglc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {c, d} and πglc^*-sets are P(X). It is clear that {b, c} is πglc^*-set but it is not θglc^*-set.

Example 2.2.15

In Example 2.2.5, locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d} and πglc^* and πglc^{**}-sets are P(X). It is clear that {a, c} is both πglc^* and πglc^{**}-set but it is not locally closed set.

Theorem 2.2.16

For a subset S of a space (X, τ) the following are equivalent:

1. $S \in \pi GLC^*(X, \tau)$.
2. $S = P \cap cl(S)$ for some πg-open set P.
3. $cl(S)-S$ is πg-closed.
4. $S \cup (X-cl(S))$ is πg-open.

Proof

(1) $\Rightarrow$ (2) Let $S \in \pi GLC^*(X, \tau)$. Then there exists a πg-open set P and a closed set F such that $S = P \cap F$. Since $S \subset P$ and $S \subset cl(S)$ we have $S \subset P \cap cl(S)$. Conversely, since $cl(S) \subset F$, $P \cap cl(S) \subset P \cap F = S$ which implies that $S = P \cap cl(S)$.

(2) $\Rightarrow$ (1): Since P is πg-open and cl(S) is closed $P \cap cl(S) \in \pi GLC^*(X, \tau)$.

(3) $\Rightarrow$ (4): Let $F = cl(S) - S$. Then F is πg-closed by the assumption and $X-F = X \cap (cl(S) - S)^c = S \cup (X-cl(S))$. But X−F is πg-open. This shows that $S \cup (X-cl(S))$ is πg-open.

(4) $\Rightarrow$ (3): Let $U = S \cup (X-cl(S))$. Then U is πg-open. This implies that X− U is πg-closed and $X-U = X-(S \cup (X-cl(S))) = cl(S) \cap (X-S) = cl(S)-S$. Thus cl(S)−S is πg-closed.

(4) $\Rightarrow$ (2): Let $U = S \cup (X-cl(S))$. Then U is πg-open. Hence we prove that $S = U \cap cl(S)$ for some πg-open set U. $U \cap cl(S) = (S \cup (X-cl(S))) \cap cl(S) = (cl(S) \cap S) \cup (cl(S) \cap X-cl(S)) = S \cup \phi = S$. Therefore $S = U \cap cl(S)$.

(2) $\Rightarrow$ (4): Let $S = P \cap cl(S)$ for some πg-open set P. Then we prove that $S \cup (X-cl(S))$ is πg-open. $S \cup (X-cl(S)) = (P \cap cl(S)) \cup (X-cl(S)) = P \cap (cl(S) \cup X-cl(S)) = P \cap X = P$ which is πg-open. Thus $S \cup (X-cl(S))$ is πg-open.

Definition 2.2.17

A topological space (X, τ) is called πg-submaximal (resp. g-submaximal if every dense subset is πg-open (resp. g-open).

Theorem 2.2.18

A topological space (X, τ) is πg-submaximal if and only if $P(X) = \pi GLC^*(X, \tau)$.

Proof

Necessity: Let $S \in P(X)$ and let $V = S \cup (X-cl(S))$. Then V is πg-open and $cl(V) = cl(S) \cup (X-cl(S)) = X$. This implies that V is a dense subset of X. By the above Theorem $S \in \pi GLC^*(X, \tau)$. Therefore, $P(X) = \pi GLC^*(X, \tau)$.

Sufficiency: Let S be a dense subset of (X, τ). Then $S \cup (X-cl(S)) = S \Rightarrow S \in \pi GLC^*(X, \tau)$ and S is πg-open. This proves that X is πg-submaximal.

Remark 2.2.19

It follows from definitions that if (X, τ) is g-submaximal, then it is πg-submaximal. But the converse is not true as seen by the following Example.

Example 2.2.20

In Example 2.2.5, dense sets are X, {a}, {b}, {a, b}, {a, c}, {a,d}, {b, c},{b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, g-open sets are ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a,b,d} and πg-open sets are P(X). Then it is πg-submaximal but not g-submaximal.

Theorem 2.2.21

For a subset S of (X, τ) if $S \in \pi GLC^{**}(X, \tau)$ then there exists an open set P such that $S = P \cap \pi g\text{-}cl(S)$ where πg-cl(S) is the πg-closure of S.

Proof

Let $S \in \pi GLC^{**}(X, \tau)$. Then there exists an open set P and a πg-closed set F such that S P $\cap$ F. Since $S \subset P$ and $S \subset \pi g\text{-}cl(S)$, we have $S \subset P \cap \pi g\text{-}cl(S)$. Conversely since $\pi g\text{-}cl(S) \subset F$, we have $P \cap \pi g\text{-}cl(S) \subset P \cap F = S$. Thus $S = P \cap \pi g\text{-}cl(S)$.

Theorem 2.2.22

Let A and B be subsets of (X, τ). If $A \in \pi GLC^*(X, \tau)$ and $B \in \pi GLC^*(X,\tau)$ then $A \cap B \in \pi GLC^*(X, \tau)$.

Proof

Let A and B ∈ πGLC* (X, τ). Then there exist πg-open sets P and Q such that A = P ∩ cl(A) and B = Q ∩ cl(B). Therefore A ∩ B = P ∩ cl(A) ∩ Q ∩ cl(B) = P ∩ Q ∩ cl(A) ∩ cl(B) where P ∩ Q is πg-open and cl(A) and cl(B) is closed. This shows that A ∩ B ∈ πGLC*(X, τ).

Theorem 2.2.23
If A ∈ πGLC** (X, τ) and B is open, then A ∩ B ∈ πGLC** (X, τ).

Proof
Let A ∈ πGLC** (X, τ). Then there exists an open set G and a πg-closed set F such that A = G ∩ F. So A ∩ B = G ∩ F ∩ B = G ∩ B ∩ F. This proves that A ∩ B ∈ πGLC** (X, τ).

Theorem 2.2.24
If A ∈ πGLC(X, τ) and B is πg-open, then A ∩ B ∈ πGLC (X, τ).

Proof
Let A ∈ πGLC(X, τ). Then A = G ∩ F where G is πg-open and F is πg-closed. So A ∩ B = G ∩ F ∩ B = G ∩ B ∩ F. This implies that A ∩ B ∈ πGLC (X, τ).

Theorem 2.2.25
If A ∈ πGLC*(X, τ) and B is πg-closed π-open subset of X, then A ∩ B ∈ πGLC∗ (X, τ).

Proof
Let A ∈ πGLC*(X, τ). Then A = G ∩ F where G is πg-open and F is closed.
A ∩ B = G ∩ (F ∩ B) where G is πg-open and F ∩ B is closed. Hence A ∩ B ∈ πGLC*(X,τ).

Theorem 2.2.26
Let A and Z be subsets of (X, τ) and let A ⊆ Z. If Z is πg-open in (X,τ) and A ∈ πGLC*(Z, τ/Z), then A ∈ πGLC*(X, τ).

Proof
Suppose A is πglc*-set, then there exists a πg-open set G of (Z, τ / Z) such that A = G ∩ cl_Z(A). But cl_Z(A) = Z ∩ cl (A). Therefore, A = G ∩ Z ∩ cl (A) where G ∩ Z is πg-open. Thus A ∈ πGLC*(X, τ).

Remark 2.2.27
The following Example shows the assumption that Z is πg-open cannot be removed from the above Theorem.

Example 2.2.28
Let X = {a, b, c, d, e}, τ = {ϕ, X, {a}, {e}, {a, e}, {c, d}, {a, c, d}, {c, d, e}, {a, c, d, e}, {b, c, d, e}}. Let V be the collection of all πg-open sets of (X, τ). Then V = {ϕ, X, {a}, {c}, {d}, {e}, {a, c}, {a, d}, {a, e}, {c, d}, {c, e}, {d, e}, {a, c, d}, {a, c, e}, {a, d, e}, {c, d, e}, {a, c, d, e}, {b, c, d, e}}. Put Z = A = {a, b, c}. Then Z is not πg-open and A ∈ πGLC *(Z, τ/Z). However A ∉ πGLC * (X, τ).

Theorem 2.2.29
If Z is πg-closed, π-open set in (X, τ) and A ∈ πGLC*(Z, τ/Z) then A ∈ πGLC*(X, τ).

Proof
Let A ∈ πGLC *(Z, τ/Z). Then A = G ∩ F where G is πg-open and F is closed in (Z, τ/Z). Since F is closed in (Z, τ/Z), F = B ∩ Z for some closed set B of (X, τ). Therefore A = G ∩ B ∩ Z. Then B ∩ Z is closed. Hence A ∈ πGLC*(X, τ).

Theorem 2.2.30
If Z is closed and open in (X, τ) and A ∈ πGLC(Z, τ/Z), then A ∈ πGLC(X, τ).

Proof

Let A ∈ πGLC (Z, τ/Z). Then there exists a πg-open set G and a πg-closed set F of (Z, τ/Z) such that A = G ∩ F. Then by the above Theorem A ∈ πGLC(X, τ).

Theorem 2.2.31

If Z is πg-closed, π-open subset of X and A ∈ πGLC**(Z, τ/Z), then A ∈ πGLC**(X, τ).

Proof

Let A ∈ πGLC **(Z, τ/Z). Then A = G ∩ F where G is open and F is πg-closed in (Z, τ/Z). Since Z is πg-closed π-open subset of (X, τ), then F is πg-closed in (X, τ). Therefore A ∈ πGLC**(X, τ).

Theorem 2.2.32

If A is πg-open and B is open, then A ∩ B is πg-open.

Proof

Let A be πg-open. Then int (A) ⊃ F whenever A ⊃ F and F is π-closed set.
Suppose A ∩ B ⊃ F, then we prove that int (A ∩ B) ⊃ F. Since B is open,
int (B) = B ⊃ F.

Therefore by assumptions int (A ∩ B) = int (A) ∩ int (B) F. This proves that A ∩ B is πg-open.

Theorem 2.2.33

Suppose that the collection of all πg-open sets of (X, τ) is closed under finite unions. Let ∈ πGLC*(X, τ) and B ∈ πGLC*(X, τ). If A and B are separated, then A ∪ B ∈ πGLC*(X, τ).

Proof

Let A, B ∈ πGLC*(X, τ). Then there exist πg-open sets G and S of (X, τ) such that A = G ∩ cl (A) and B = S ∩ cl (B). Put V = G ∩ (X−cl (B)) and W = S ∩ (X−cl (A)). Then V and W are πg-open sets and A = V ∩ cl (A) and B =W∩ cl (B). Also V ∩ cl (B) =ϕ and W∩ cl (A) =ϕ. Hence it follows that V and W are πg-open sets of (X, τ). Therefore A ∪ B = (V ∩ cl (A)) ∪ (W ∩ cl (B)) = V ∪ W ∩ cl (A) ∪ cl (B). Here V ∪ W is πg-open by assumption. Thus A ∪ B ∈ πGLC*(X, τ).

Remark 2.2.34

The assumptions that A and B are separated cannot be removed from Theorem 2.2.33.

Example 2.2.35

Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then {a,b} and {a, d} ∈ πglc∗(X, τ) but {a, b, d} ∉ πglc∗(X, τ), since they are not separated. For we have {a, b} ∩ cl({a, d}) = {a} ≠ ϕ and {a, d} ∩ cl({a, b}) = {a, d} ≠ ϕ.

Theorem 2.2.36

Let {Z_i : i ∈ I} be a finite πg-closed cover of (X, τ) and let A be a subset of (X, τ). If A ∩ Z_i ∈ πGLC**(Z_i, τ/Z_i) for every i ∈ I, then A ε πGLC**(X, τ).

Proof

For every i ∈ I there exists a set U_i ∈ τ and a πg-closed set F_i of (Z_i, τ/Z_i) such that A ∩ Z_i = U_i ∩ (Z_i ∩ F_i). Then A = ∪ {A ∩ Z_i : i ∈ I} = ∪ {U_i : i ∈ I} ∩ (∪ {Z_i ∩ F_i : i ∈ I}). This shows that A ∈ πGLC**(X, τ).

Theorem 2.2.37

Let (X, τ) and (Y, σ) be topological spaces.

1. If $A \in \pi GLC(X, \tau)$ and $B \in \pi GLC(Y, \sigma)$, then $A \times B \in \pi GLC(X \times Y, \tau \times \sigma)$.
2. If $A \in \pi GLC^*(X, \tau)$ and $B \in \pi GLC^*(Y, \sigma)$, then $A \times B \in \pi GLC^*(X \times Y, \tau \times \sigma)$.
3. If $A \in \pi GLC^{**}(X, \tau)$ and $B \in \pi GLC^{**}(Y, \sigma)$, then $A \times B \in \pi GLC^{**}(X \times Y, \tau \times \sigma)$.

Proof

Let $A \in \pi GLC(X, \tau)$ and $B \in \pi GLC(Y, \sigma)$. Then there exist πg-open sets V and V′ of (X, τ) and (Y, σ) and πg-closed sets W and W′ of X and Y respectively such that $A = V \cap W$ and $B = V' \cap W'$. Then $A \times B = (V \times V') \cap (W \times W')$ holds. Hence $A \times B \in \pi GLC(X \times Y, \tau \times \sigma)$. Similarly the other results follow from the definition.

2.3 <u>πgl-CONTINUITY AND πgl-IRRESOLUTENESS</u>

Definition 2.3.1

A function f: (X, τ) → (Y, σ) is called πgl-continuous (resp. πgl^*- continuous and πgl^{**}-continuous) if $f^{-1}(V) \in \pi GLC(X, \tau)$ (resp. $f^{-1}(V)\ \varepsilon\ \pi GLC^*(X, \tau)$, $f^{-1}(V)\ 2\ \pi GLC^{**}(X, \tau)$) for every $V \in \sigma$.

Definition 2.3.2

A function f : (X, τ) →(Y, σ) is said to be πgl-irresolute (resp. πgl^*-irresolute, πgl^{**}-irresolute) if $f^{-1}(V) \in \pi GLC(X, \tau)$ (resp. $f^{-1}(V) \in \pi GLC^*(X, \tau)$, $f^{-1}(V) \in \pi GLC^{**}(X, \tau)$) for every $V \in \pi GLC(Y, \sigma)$ (resp. $V \in \pi GLC^*(Y, \sigma)$, $V \in \pi GLC^{**}(Y, \sigma)$).

Theorem 2.3.3

Let f: (X, τ) →(Y, σ) be a function.

1. If f is LC-continuous then it is πgl-continuous.
2. If f is θ-LC-continuous then it is πgl-continuous.
3. If f is θGLC-continuous then it is πgl-continuous.
4. If f is πgl^*-continuous then it is πgl-continuous.
5. If f is θ-LC-continuous then it is πgl^* or πgl^{**}-continuous.
6. If f is GLC-continuous then it is πgl-continuous.
7. If f is LC-continuous then it is GLC-continuous.
8. If f is θGLC-continuous then it is GLC-continuous.
9. If f is θ-LC-continuous then it is θGLC-continuous.
10. If f is θ-LC-continuous then it is LC-continuous.

The proof follows from the definitions and Theorem 2.2.4. However the converses of the above need not be true as shown from the following.

Example 2.3.4

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a}, {b}, {a, b}}. Let f: (X, τ) → (Y, σ) be the identity function. Then πglc-sets are P(X) and locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}. It is clear that $f^{-1}(\{a\}) = \{a\}$ is not locally closed set. We conclude that f is πgl-continuous but not LC-continuous.

Example 2.3.5

Consider Example 1.3.4. Let f: (X, τ) →(Y, σ) be the identity function. Then πglc-sets are P(X) and θ-locally closed sets are ϕ, X. It is clear that $f^{-1}(\{a, b\}) = \{a, b\}$ is not θ-locally closed set. We conclude that f is πgl-continuous but not θ-LC-continuous.

Example 2.3.6

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a, c}, {a, c, d}}. Let f: (X, τ) → (Y, σ) be the identity function. Then πglc-sets are P(X) and θglc-sets are ϕ, X, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}. It is clear that $f^{-1}(\{a, c\}) = \{a, c\}$ is not θglc-set. We conclude that f is πgl-continuous but not θGLC-continuous.

Example 2.3.7

Let X = Y = {a, b, c, d, e}, τ = {ϕ, X, {a}, {e}, {a, e}, {c, d}, {a,c,d}, {c, d, e}, {a, c, d, e}, {b, c, d, e}} and σ = {ϕ, Y, {b, c}. Let f: (X, τ) → (Y, σ) be the identity function. Then πglc-sets are P(X) and πglc*-sets are ϕ, X, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, e}, {c, d}, {c, e}, {d, e}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {c, d, e}, {a, b, c, d}, {a, c, d, e}, {b, c, d, e}. It is clear that f^{-1}({b, c}) = {b, c} is not πglc*-set. We conclude that f is πgl-continuous but not πgl*-continuous.

Example 2.3.8

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a, b}}. Let f: (X, τ) → (Y, σ) be the identity function. Then πglc* (or) πglc**-sets are P(X) and θ-locally closed sets are ϕ, X. It is clear that f^{-1}({a, b}) = {a, b} is not θ-locally closed set. We conclude that f is πgl* or πgl**-continuous but not θ-LC-continuous.

Example 2.3.9

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a}, {b}, {a, b}}. Let f: (X, τ) →(Y, σ) be the identity function. Then πglc-sets are P(X) and glc-sets are φ, {a}, {b}, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X. It is clear that f^{-1}({a}) = {a} is not glc-set. We conclude that f is πgl-continuous but not GLC-continuous.

Example 2.3.10

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a, c}, {a, b, c}}. Let f: (X, τ) →(Y, σ) be the identity function. Then glc-sets are P(X) and locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}. It is clear that f^{-1}({a, c}) = {a, c} is not locally closed set. We conclude that f is GLC-continuous but not LC-continuous.

Example 2.3.11

Let X = Y = {a, b, c, d}, τ = {ϕ, X, {a, b}, {a, b, c}, {a, b, d}} and σ = {ϕ, Y, {a, c}, {a, c, d}}. Let f: (X, τ) →(Y, σ) be the identity function. Then glc-sets are P(X) and θglc-sets areϕ, X, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}. It is clear that f^{-1}({a, c}) = {a, c} is not θglc-set. We conclude that f is GLC-continuous but not θGLC-continuous.

Example 2.3.12

Consider Example 2.3.8. Let f: (X, τ) →(Y, σ) be the identity function. Then θ glc-sets are ϕ, X, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d} and θ-locally closed sets are ϕ, X. It is clear that f^{-1}({a, b}) = {a, b} is not θ-locally closed set. We conclude that f is θGLC-continuous but not θ-LC-continuous.

Example 2.3.13

Consider Example 2.3.8. Let f: (X, τ) →(Y, σ) be the identity function. Then locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d} and θ-locally closed sets are {ϕ, X}. It is clear that f−1({a, b}) = {a, b} is not θ-locally closed set. We conclude that f is LC-continuous but not θ-LC-continuous.

From the above definitions, results and Examples we have the following implications.

Remark 2.3.14

We obtain the following diagram from the above discussions.

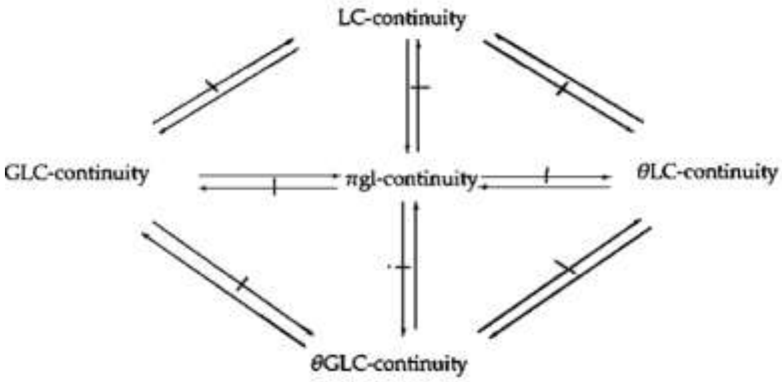

Figure 2.2 Implication diagram

Theorem 2.3.15

1. If f: X → Y is πgl-continuous and g: Y → Z is continuous, then g ∘ f:: X → Z is πgl-continuous.
2. If f: X → Y is πgl-irresolute and g: Y → Z is πgl-continuous, then g ∘ f:: X → Z is πgl-continuous.
3. If f: X → Y is πgl-irresolute and g: Y → Z is GLC-continuous, then g ∘ f: X →Z is πgl-continuous.
4. If f: X → Y and g: Y → Z are πgl-irresolute, then g ∘ f:: X → Z is also πgl-irresolute.

The proof follows from the definitions and Theorem 2.2.4.

2.4 CONCLUSION

In image processing and digital topology, the concept of πg locally closed sets in ideal topological spaces helps in defining and analysing the connectivity and structure of objects represented digitally. These sets contribute to characterizing shapes and patterns, aiding in tasks such as object recognition, shapes analysis and image segmentation. They provide a foundation for understanding the topological properties of digital images, which is crucial for various computer vision.

In recent trends, continuous functions between spaces endowed with the πgl continuous functions in ideal topology, have garnered attention due to their significance in studying topological properties. This specialized framework offers a nuanced understanding of continuity within ideal topological spaces, enabling deeper insights into the behaviour of functions and their preservation of structural properties. The exploration of πgl continuous functions within ideal topologies stands as a promising are, festering advancements in topology and its applications.

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