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Advancing Quality Assurance In Manufacturing: A Weighted Exponential Distribution Control Chart For Enhanced Production Monitoring

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ABSTRACT

Over the last few decades, weighted distributions have received substantial attention from researchers due to their utility across a range of research fields. With an emphasis on the relevance of weighted distributions, a new control chart for a weighted exponentially distributed characteristic using an exponentially weighted moving average (EWMA) has been proposed in this manuscript. The average run length of both in-control and out-of-control processes has been derived. The efficiency of the proposed control chart has been compared with existing chart based on the unweighted exponential distribution in terms of Average Run Length (ARL) and the percentage decrease in ARL (ΔARL) using Monte Carlo Simulations. Furthermore, the proposed control chart has been applied to various real-life applications to highlight its applicability in industrial and manufacturing processes.

KEYWORDS *TBE; exponential distribution; weighted exponential distribution; adjustment bias; ARL; ΔARL.*

1. Introduction

Modern industrial processes rely on advanced fault detection technology to guarantee both process safety and product quality [1]. A critical challenge in quality management involves overseeing and [1]diagnosing processes, commonly achieved through the application of control charts. Quality engineering is like a science that follows a structured approach. It's all about making things better and ensuring that customers are happy. Imagine a company as a kind of puzzle where different parts work together. Each part, or process, follows a set of steps to turn something in the beginning into a finished product. We keep an eye on this process using feedback loops and control variables, which help us, make adjustments if needed. Sometimes, things can go a little off track, this could be because of random changes (we call it natural process variability or "noise") or more organized changes that we can fix by adjusting how we control the process.

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SPC is crucial in monitoring processes across multiple service and industrial sectors using statistical tools. Control charts are an integral part of SPC. The theory behind SPC and control charts in manufacturing was first established by Walter A. Shewhart in the 1920s.

A control chart typically consists of two lines, UCL and LCL, referred to as decision lines. These lines are used to monitor the process, and if a sample point falls outside of them, the process is deemed out of control. If it falls within the lines, the process is considered stable or in control [2]. These limits are effective in reducing defective items and therefore increase the profitability of an organization.

The Exponential Weighted Moving Average (EWMA) is a powerful tool for analyzing trends in data introduced by [3]. EWMA has two main applications including smoothing trend curves and building statistical control charts for quality monitoring. Unlike simple moving averages, EWMA considers all data points but assigns them decreasing weights based on their age. The most recent data points have the highest influence, while older ones have less impact on the calculated average. This weighting scheme makes EWMA particularly effective for capturing small changes in the process.

If the quality characteristic of any random variable denoted by $\mathrm{Y_i}$, then we can define the EWMA statistic $(\mathrm{Q_i})$ as follows

$$\mathrm{Q_i} = \gamma \mathrm{Y_i} + (1-\gamma)\mathrm{Q_{i-1}} \; 0 \le \gamma \le 1,$$

Where γ represents the weight assigned to the current observation of $\mathrm{Y_i}$ and $(1-\gamma)$ represent the weight assigned to the previous observations $\mathrm{Q_{i-1}}$. Hence, at $\mathrm{i} = 1$, we have the first value $\mathrm{Q_1}$ can be obtained as

$$\mathrm{Q_1} = \gamma \mathrm{Y_1} + (1-\gamma)\mathrm{Q_0}.$$

The mean of the process μ_{Y} can be assumed as $\mathrm{P_o}$ or it can be taken as zero [4].

The Shewhart control chart and the Exponential Weighted Moving Average (EWMA) chart differ notably in terms of the values plotted and the criteria for determining process stability. In the Shewhart control chart, an assessment of the process being out of control is made when the last point, representing the most recent measurement in the last sample, falls outside the given control limits.

In contrast, the EWMA chart takes a more comprehensive view, incorporating not only the latest measurement but also the past data points. It assigns decreasing weights to older measurements, ensuring that recent trends hold greater part in its assessment. This unique approach allows the EWMA chart to excel in detecting minor shifts and gradual deviations that might elude the Shewhart chart's focus on the immediate present.

The EWMA control procedure offers a unique ability to fine-tune its sensitivity to process deviations. By carefully selecting the value of lambda, you can adjust it to detect even small or gradual shifts in the process, ensuring timely identification of the shift in the process [1, 5-9].

When making a control chart, people usually assume that the quality of a product follows a normal distribution. However, this assumption doesn't always hold true in real-world experiments. Sometimes, the control chart may not detect changes in the process, especially if the quality characteristic follows an unknown or non-normal distribution [10]. This can lead to misleading results, and engineers might miss identifying shifts in the process. When the data is skewed, the exponential distribution is considered a better choice, especially for modeling the time between events. These control charts are called "t charts," and you can learn more about them in references [11]. A substitute for t chart using normal approximation was proposed by [12]. [13] developed a new EWMA t chart assuming that the characteristic of interest follows the exponential distribution.

In the last couple of decades, weighted distributions took a considerable attention from the scientists due to its practical application in daily life. Considering that importance of weighted distributions a new control charting plan for weighted exponentially distributed characteristic using exponential weighted moving average (EWMA) have been suggested in this research article. The efficiency of the proposed control chart based on weighted exponential distribution will be compared to the existing control charting scheme proposed by [13] in terms of ARL and ΔARL.

2. Formulation

In this section, the control charting schemes of the proposed and existing model will be represented.

2.1. Control Chart Based On Transformation.

The quality characteristic being studied, such as the time between events represented by Y, follows an independent and identically distributed exponential distribution with a parameter α. The probability density function (pdf) of this distribution is given by:

$$f(y) = \frac{1}{\alpha} e^{-\frac{y}{\alpha}} \qquad \alpha > 0 \tag{1}$$

[14] proposed that the transformed random variable $y^* = y^{\frac{1}{\delta}}$ follows the Weibull distribution with $\alpha^{\frac{1}{\delta}}$ and δ as scale and shape parameter respectively. [15] pointed out that the Weibull distribution follows the approximate normal distribution when $\delta = 3.6$. [13] applied the above transformation to suggest the following EWMA control chart with the following control limits

$$\mathrm{LCL} = \left[\Gamma\left(1+\frac{1}{3.6}\right)\right] - \mathrm{k}\sqrt{\frac{\lambda}{2-\lambda}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)} \quad (2)$$

$$\mathrm{UCl} = \left[\Gamma\left(1+\frac{1}{3.6}\right)\right] + \mathrm{k}\sqrt{\frac{\lambda}{2-\lambda}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)} \quad (3)$$

2.2. **Proposed control chart based on weighted exponential distribution.**

The exponential distribution is very popular in statistical quality control. Weighted exponential distribution is a generalization of exponential distribution. The pdf of weighted exponential distribution with a scale parameter $\alpha > 0$ is

$$\mathrm{f(y)} = \frac{1}{\alpha^2}\mathrm{y}e^{-\frac{y}{\alpha}} \qquad y \geq 0 \quad (4)$$

The corresponding CDF is

$$\mathrm{F(Y)} = 1 - \frac{e^{-\frac{y}{\alpha}}[y+\alpha]}{\alpha} \quad (5)$$

The mean and the standard deviation of weighted exponential distribution are $\mathrm{E(Y)} = 2\alpha$ and $\mathrm{SD(Y)} = \sqrt{2\alpha^2}$ respectively.

Hence, the control limits of the proposed chart by using its mean and standard deviation are as follows

$$\mathrm{UCL} = 2\alpha(1 + \mathrm{k}\sqrt{\frac{c}{2}}) \quad (6)$$

$$LCL = 2\alpha(1 - k\sqrt{\frac{c}{2}}) \quad (7)$$

Where constant k has been used to determine the width of control limits and $c = \frac{\gamma}{2-\gamma}$.

The efficiency of a control chart is often evaluated using ARL, which is the average number of sample points taken before the process goes out of control. The in-control ARL (usually represented byARL_o) refers to the stable process under normal variations, while the out of-control ARL (denoted by ARL_1) relates to the process experiencing abnormal variations. To assess the performance of the control chart, a performance measure called the percentage decrease in ARL (ΔARL) is also included along with ARL.

3. Simulation Study

R language 4.1.0 is used for the simulation study and necessary measures have been calculated given in Tables (1-4) to assess the performance of the proposed control chart. The value of ARL_0 has been fixed at 370,300 and 200. The corresponding risk of type-I error is 0.0027, 0.0033 and 0.005 respectively.

The complete algorithm of the simulation study is outlined in the following steps.

3.1. Algorithm

i. Control Limit Calculation: This step involves deriving the control limits for the proposed chart using the known parameters of the underlying weighted exponential distribution.

ii. Sample Generation: To evaluate the performance of the control chart, 40,000 samples of size $n = 1$ are generated from the weighted exponential distribution provided in equation (Eq. 4).

iii. Out-of-Control Detection: Each generated sample is compared against the predetermined control limits until the sample falls outside these limits.

iv. Run Length Recording: When an out-of-control observation occurs, the sample number at which it occurs is recorded as a value of the random variable "RL". The process then returns to step 2 again to draw another sample.

v. Repetition and Analysis: This process of sample generation, out-of-control detection, and RL recording is repeated 100,000 times. This generates a set of 100,000 RL values for further analysis.

vi. Performance Evaluation: The final step involves analyzing the distribution of the RL values. Descriptive statistics like mean, median, standard deviation, minimum, and maximum are calculated. Additionally, the 25th, 75th, and 99th percentiles of the RL distribution are determined. Finally, the percentage decrease in the Average Run Length (ΔARL) are also computed with other measures.

3.2. Performance assessment of proposed Weighted-TBE chart

To determine the efficiency of any control chart average run length is considered as one of the most suitable measures. To access the performance in detail, we also calculated some other important measures of RL by using Monte Carlo Simulations, like SDRL, Min RL, Median RL, Max RL, MRL and some percentiles. Graphical representation of RL is also included for the further comparisons. In Table 1-6 the trend of these measures can be seen at different combinations of ARL_0 and λ.

Table-1: RL measures for shifted process at $ARL_0 = 200$ and $\lambda = 0.2$

$K = 2.839$ $\lambda = 0.2$								
$SHIFT$	ARL	MRL	$SDRL$	$MIN\ RL$	$MAX\ RL$	$P25$	$P75$	$P99$
1	200.02	139	199.42	1	2648	59	277	918.01
1.1	82.33	58	79.53	1	895	26	113	369
1.2	42.87	31	40.19	1	378	14	59	187
1.3	26.56	19	24.18	1	267	10	36	114
1.4	18.22	14	15.86	1	183	7	24	74.01
1.5	13.65	10	11.28	1	105	6	18	54
1.6	10.83	8	8.59	1	89	5	14	41
1.7	8.89	7	6.82	1	97	4	12	32
1.8	7.61	6	5.75	1	58	4	10	27
1.9	6.61	5	4.86	1	52	3	9	24
2	5.87	5	4.24	1	50	3	8	21
2.5	3.86	3	2.55	1	29	2	5	13
3	2.96	3	1.84	1	16	2	4	9

Table-2: RL measures for shifted process at $ARL_0 = 300$ and $\lambda = 0.8$

$K = 4.102$ $\lambda = 0.8$								
$SHIFT$	ARL	MRL	$SDRL$	$MIN\ RL$	$MAX\ RL$	$P25$	$P75$	$P99$
1	299.95	207	298.36	1	3372	86	418	1356
1.1	153.76	107	152.69	1	1589	44	213	700
1.2	89.01	62	88.49	1	927	26	124	410
1.3	56.4	39	55.92	1	637	17	78	256
1.4	38.57	27	37.94	1	440	12	53	174
1.5	27.78	20	27.29	1	305	8	38	127
1.6	20.94	15	20.43	1	238	6	29	96
1.7	16.56	12	15.97	1	193	5	23	74
1.8	13.32	9	12.62	1	138	4	18	59

1.9	11.06	8	10.34	1	110	4	15	48
2	9.38	7	8.73	1	97	3	13	40
2.5	5.15	4	4.54	1	51	2	7	22
3	3.55	3	2.9	1	27	1	5	14

Table-3: RL measures for shifted process at $ARL_0 = 370$ and $\lambda = 0.4$

$K = 3.7595,$				$\lambda = 0.4$				
$SHIFT$	ARL	MRL	$SDRL$	$MIN\ RL$	$MAX\ RL$	$P25$	$P75$	$P99$
1	370.04	256	370.34	1	3384	107	510	1721.01
1.1	159.67	113	157.59	1	2193	48	220	729.01
1.2	81.92	57	80.75	1	969	25	112	374.01
1.3	48.38	34	47.3	1	574	15	67	219
1.4	31.45	22	30.09	1	381	10	43	140
1.5	22.34	16	20.94	1	215	8	30	97
1.6	16.72	12	15.25	1	172	6	23	72
1.7	13.03	10	11.57	1	133	5	18	54
1.8	10.66	8	9.28	1	134	4	14	44
1.9	9.05	7	7.62	1	92	4	12	36
2	7.76	6	6.43	1	67	3	10	31
2.5	4.56	4	3.45	1	38	2	6	17
3	3.31	3	2.33	1	31	2	4	11

The following pattern can be identified from the Tables 1-6

A decreasing tendency has been observed in ARL_1 as the value of δ increases. This decline is quick when ARL_0 is set as higher and it is faster if λ is set at lower value. The percentage decrease in ARL_1 is higher for the larger value of ARL_0. For example from Table 4-6 it can be observed that the decline in ARL_1 is 58.84% when $\lambda = 0.2, ARL_o = 200$ and $\delta = 1.1$. on the other hand, keeping the other parameters fixed, if we set the $ARL_0 = 300$ the decline has been observed as 61.98% and if the value of ARL_0 set at 370 with the same values of δ and λ the value is drop down to 63.69%. The difference between the declining trend is getting smaller as the value of δ increases. For example, when $\lambda = 0.8, ARL_o = 200$ and $\delta = 2$, the value of ΔARL is 97.07%, on the same values of δ and λ, ΔARL has been observed to be 96.87% and 97.23% for $ARL_0 = 300$ and $ARL_0 = 370$ respectively.

A control chart can be assessed by its detection ability of the shifted process. early detection is a sign of more efficient control chart. A control chart having smaller values of ARL_1 for the fixed value of ARL_0 is considered more reliable and effective.

3.3. Comparative study

It is desirable that the control chart used to monitor any process signal as early as possible if the underlying process shifted from its target value. in this section a comparison between the existing and proposed control charting techniques with the help of ARL have been discussed.

3.3.1. Advantages of Proposed charting technique over the existing control chart in terms of Monte Carlo Simulations

[13] proposed a control charting technique by using EWMA. From Table 4-6, the values of ARL_1 of the existing EWMA t chart and the proposed control chart have been included at various combinations of ARL_0, δ and λ. It can be clearly observed that the proposed control charting technique outclass the existing control chart at all values of shift parameters with different combinations of ARL_0 and λ by producing smaller values of ARL_1.

Larger differences between the ΔARL have been observed between the existing and proposed techniques at smaller shift sizes as desired. For example, when $\lambda = 0.2, ARL_o = 370$ and $\delta = 1.1$, the existing technique detected the shift at 219th sample while the proposed control chart detected it at 135th sample which is almost 84 sample earlier. As for as the ΔARL is concerned when $\lambda = 0.2, ARL_o = 370$ and $\delta = 1.1$, the ARL has decreased 40.88% for the existing and 63.69 for the proposed control charts respectively. The values of ARL_1 are lower at different combinations of δ, λ and ARL_o. The same patterns have been observed at different combinations which can be seen from Table 4-6. It creates a significant difference. It proved the detection ability of the proposed charting technique, and this comparison demonstrated that the proposed control charting scheme is more efficient and effective for monitoring the manufacturing process. From the above discussion we can conclude that applying usual probability distributions on the data without monitoring the process deeply may mislead the results by producing extra faulty items. Also, the unwanted variations may also be noticed due to loss of original information. The control limits based on the exact probability distribution enhance the performance of the control chart. The underlying process of TBE will be settled according to the original information.

Table-4: The ARL for the proposed and existing control chart at different combinations of λ and δ (ΔARL in parenthesis)

$\lambda = 0.2$	$\lambda = 0.4$	$\lambda = 0.6$	$\lambda = 0.8$

Shift	Existing	proposed	Existing	proposed	Existing	Proposed	Existing	Proposed
1	200.01	200.02	200	200.73	200	199.81	200	200.35
1.1	125.3	82.33	142.52	93.9	149.24	102.03	152.82	107.31
	(37.35)	(58.83)	(28.74)	(53.22)	(25.38)	(48.94)	(23.59)	(46.44)
1.2	66.02	42.87	92.32	51.55	105.63	59.23	113.64	64.46
	(66.99)	(78.57)	(53.84)	(74.32)	(47.18)	(70.36)	(43.18)	(67.83)
1.3	36.33	26.56	59.87	51.55	74.59	37.85	84.62	42.3
	(81.84)	(86.72)	(70.06)	(74.32)	(62.70)	(81.06)	(57.69)	(78.89)
1.4	21.78	18.22	40.3	22.16	53.82	26.27	64.01	29.41
	(89.11)	(90.89)	(79.85)	(88.96)	(73.09)	(86.85)	(67.99)	(85.32)
1.5	14.15	13.65	28.36	16.45	39.98	19.49	49.47	21.97
	(92.93)	(93.18)	(85.82)	(91.80)	(80.01)	(90.25)	(75.26)	(89.03)
1.6	9.85	10.83	20.83	12.66	30.59	14.72	39.10	16.63
	(95.08)	(94.59)	(89.58)	(93.69)	(84.70)	(92.63)	(80.45)	(91.70)
1.7	7.25	8.89	15.88	10.15	24.06	11.86	31.57	13.19
	(96.38)	(95.56)	(92.06)	(94.94)	(87.97)	(94.06)	(84.21)	(93.42)
1.8	5.59	7.61	12.5	8.52	19.40	9.78	26..00	10.97
	(97.21)	(96.20)	(93.75)	(95.76)	(90.30)	(95.11)	(87.00)	(94.52)
1.9	4.48	6.61	10.12	7.34	15.98	8.19	21.79	9.20
	(97.76)	(96.70)	(94.94)	(96.34)	(92.01)	(95.90)	(89.10)	(95.41)
2	3.70	5.87	8.39	6.35	13.42	7.13	18.55	7.88
	(98.15)	(97.07)	(95.80)	(96.84)	(93.29)	(96.43)	(90.72)	(96.07)
2.5	1.99	3.86	4.24	3.94	6.93	4.22	9.94	4.55
	(99.00)	(98.07)	(97.88)	(98.04)	(96.53)	(97.89)	(95.03)	(97.73)
3	1.45	2.96	2.79	2.95	4.50	3.03	6.51	3.20
	(99.28)	(98.52)	(98.60)	(98.53)	(97.75)	(98.48)	(96.74)	(98.40)

Table-5: The ARL for the proposed and existing control chart at different combinations of λ and δ (ΔARL in parenthesis)

	$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
Shift	Existing	proposed	Existing	proposed	Existing	Proposed	Existing	Proposed
1	300.01	299.78	300.00	299.96	300.00	298.97	300.01	299.95
1.1	180.89	113.99	207.77	133.48	218.37	143.26	224.04	153.76
	(39.71)	(61.98)	(30.74)	(55.50)	(27.21)	(52.08)	(25.32)	(48.74)
1.2	91.08	56.07	130.08	70.35	150.25	81.01	162.55	89.01
	(69.64)	(81.30)	(56.64)	(76.55)	(49.92)	(72.90)	(45.82)	(70.32)
1.3	48.25	32.98	81.77	42.16	103.30	49.77	118.20	56.4
	(83.92)	(89.00)	(72.74)	(85.94)	(65.57)	(83.35)	(60.60)	(81.20)
1.4	28.02	22.06	53.58	28.1	72.77	33.47	87.51	38.57
	(90.66)	(92.64)	(82.14)	(90.63)	(75.74)	(88.80)	(70.83)	(87.14)
1.5	17.73	16.25	36.85	20.18	52.93	24.08	66.33	27.78
	(94.09)	(94.58)	(87.72)	(93.27)	(82.36)	(91.95)	(77.89)	(90.74)
1.6	12.05	12.45	26.52	15.22	39.76	18.00	51.54	20.94
	(95.98)	(95.85)	(91.16)	(94.93)	(86.75)	(93.98)	(82.82)	(93.02)
1.7	8.69	10.15	19.86	12.09	30.77	14.31	40.98	16.56
	(97.10)	(96.61)	(93.38)	(95.97)	(89.74)	(95.21)	(86.34)	(94.48)
1.8	6.58	8.57	15.40	9.99	24.45	11.6	33.29	13.32
	(97.81)	(97.14)	(94.87)	(96.67)	(91.85)	(96.12)	(88.90)	(95.56)
1.9	5.19	7.44	12.29	8.4	19.89	9.65	27.56	11.06
	(98.27)	(97.52)	(95.90)	(97.20)	(93.37)	(96.77)	(90.81)	(96.31)
2	4.23	6.56	10.06	7.24	16.50	8.26	23.20	9.38
	(98.59)	(97.81)	(96.65)	(97.59)	(94.50)	(97.24)	(92.27)	(96.87)
2.5	2.16	4.2	4.84	4.31	8.15	4.65	11.90	5.15
	(99.28)	(98.60)	(98.39)	(98.56)	(97.28)	(98.44)	(96.03)	(98.28)
3	1.53	3.2	3.09	3.18	5.13	3.34	7.56	3.55
	(99.49)	(98.93)	(98.97)	(98.94)	(98.29)	(98.88)	(97.48)	(98.82)

Table:6 The ARL for the proposed and existing control chart at different combinations of λ and δ (ΔARL in parenthesis)

	$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
Shift	Existing	proposed	Existing	proposed	Existing	Proposed	Existing	Proposed
1	370	370.93	370.00	370.04	370.00	370.87	370.01	369.63
1.1	218.72	134.7	252.47	159.67	265.86	174.04	273.05	184.79
	(40.89)	(63.69)	(31.76)	(56.85)	(28.15)	(53.07)	(26.20)	(50.01)
1.2	107.61	64.41	155.33	81.92	180.28	95.6	195.58	105.27
	(70.92)	(82.64)	(58.02)	(77.86)	(51.28)	(74.22)	(47.14)	(71.52)
1.3	55.91	37.07	96.09	48.38	122.25	57.9	140.50	65.35
	(84.89)	(90.01)	(74.03)	(86.93)	(66.96)	(84.39)	(62.03)	(82.32)
1.4	31.96	24.29	62.11	31.45	85.06	38.49	102.87	44.23
	(91.36)	(93.45)	(83.21)	(91.50)	(77.01)	(89.62)	(72.20)	(88.03)
1.5	19.94	17.51	42.21	22.34	61.21	27.37	77.21	31.58
	(94.61)	(95.28)	(88.59)	(93.96)	(83.46)	(92.62)	(79.13)	(91.46)
1.6	13.4	13.53	30.07	16.72	45.55	20.37	59.46	23.66
	(96.38)	(96.35)	(91.87)	(95.48)	(87.69)	(94.51)	(83.93)	(93.60)
1.7	9.56	10.96	22.32	13.03	34.96	15.85	46.91	18.34
	(97.42)	(97.05)	(93.97)	(96.48)	(90.55)	(95.73)	(87.32)	(95.04)
1.8	7.17	9.17	17.16	10.66	27.57	12.75	37.84	14.77
	(98.06)	(97.53)	(95.36)	(97.12)	(92.55)	(96.56)	(89.77)	(96.00)
1.9	5.61	7.85	13.60	9.05	22.27	10.58	31.13	12.04
	(98.48)	(97.88)	(96.32)	(97.55)	(93.98)	(97.15)	(91.59)	(96.74)
2	4.54	6.9	11.06	7.76	18.38	8.94	26.06	10.22
	(98.77)	(98.14)	(97.01)	(97.90)	(95.03)	(97.59)	(92.96)	(97.24)
2.5	2.25	4.36	5.19	4.56	8.86	4.97	13.07	5.50
	(99.39)	(98.82)	(98.60)	(98.77)	(97.61)	(98.66)	(96.47)	(98.51)

3	1.57 (99.58)	3.29 (99.11)	3.26 (99.12)	3.31 (99.11)	5.49 (98.52)	3.48 (99.06)	8.18 (97.79)	3.75 (98.99)

3.3.2. Performance evaluation by using ARL graphs

The performance of the proposed control chart is presented in figure 1-4, focusing on a specific value of ARL_0=370, and for different values of γ. The graphical representation in Figure 1-4 demonstrates a consistent trend, the ARL curve of the proposed control chart consistently outperforms that of the existing control chart. This observation implies that the proposed control chart effectively detects scale shifts at a faster rate compared to the existing technique as desired.

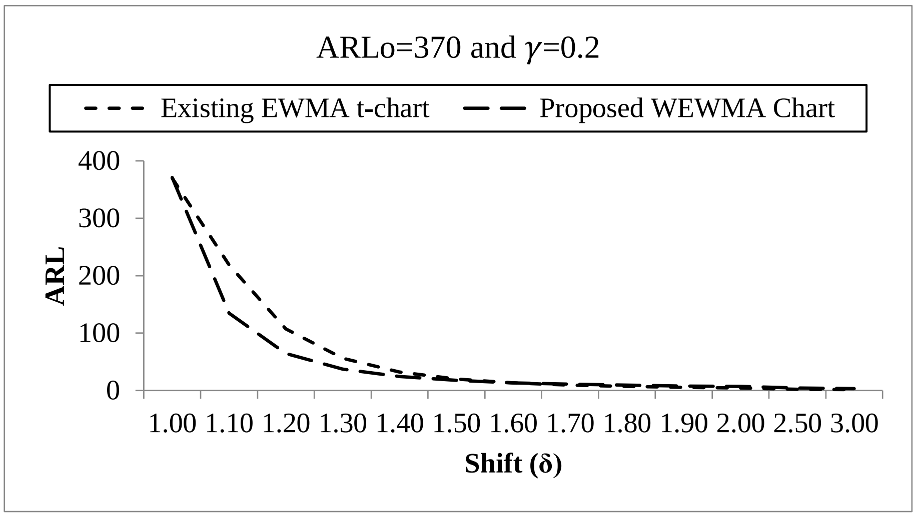

Figure 1: ARL curves of Proposed and existing control charts at $ARL_0 = 370$ and $\gamma = 0.2$

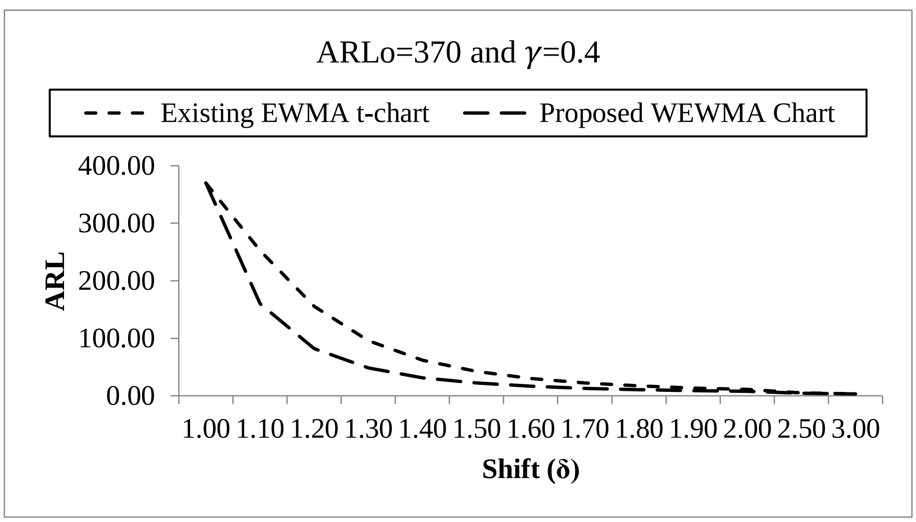

Figure 2: ARL curves of Proposed and existing control charts at $ARL_0 = 370$ and $\gamma = 0.4$

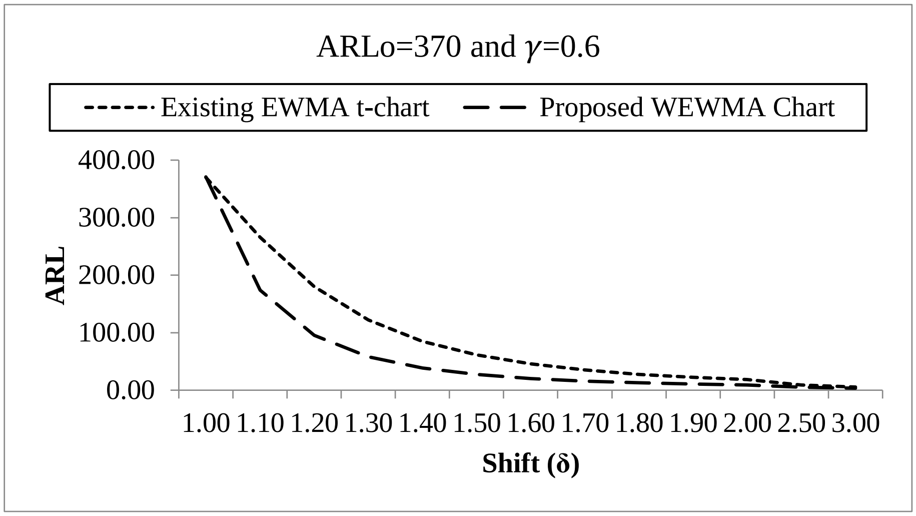

Figure 3: ARL curves of Proposed and existing control charts at $\mathrm{ARL_0} = 370$ and $\gamma = 0.6$

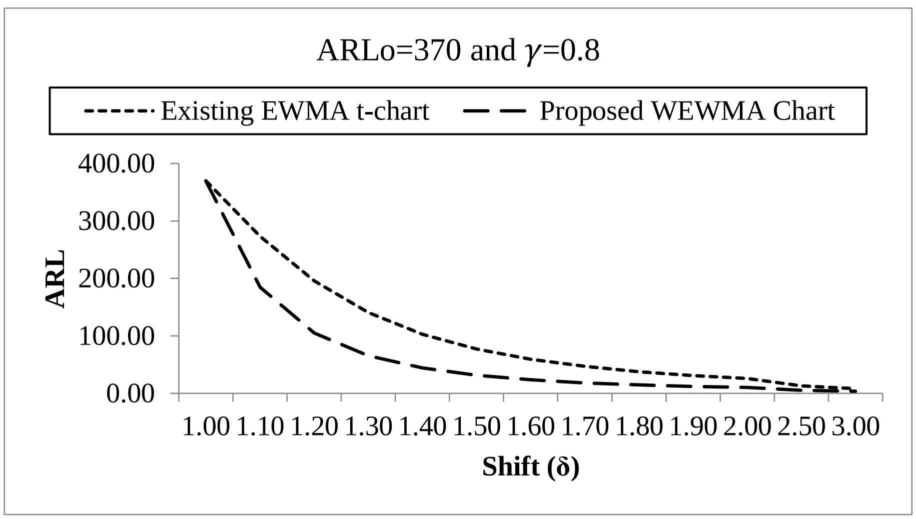

Figure 4: ARL curves of Proposed and existing control charts at $\mathrm{ARL_0} = 370$ and $\gamma = 0.8$

4. Application

In this section, we aim to provide the application of weighted exponential distribution in various real-life scenarios, particularly within the realms of reliability engineering and medicine. To illustrate its practical utility, we have utilized the proposed Weighted-TBE chart to analyze three distinct datasets taken from the field of reliability engineering and medical.

4.1. Application to Monitoring Urinary Tract Infection Data

In our first example, we applied the proposed control chart to monitor urinary tract infection (UTI) data, as previously utilized by [13]. This dataset records the time between discharge for male UTI patients. The hospital's objective is to monitor the frequency of discharged patients who develop a UTI during their hospital stay, enabling swift detection of any increase in infection rates.

Parameter estimation was conducted using R 4.1.0 software [16], and the resulting charting statistics were plotted against control limits. A comparative analysis between the proposed Weighted-TBE chart and the existing chart based on unweighted exponential distribution was performed. Figures 5 and 6 display the results, indicating that the existing control chart failed to identify any out-of-control processes at any sample point. However, the proposed Weighted-TBE chart successfully detected an out-of-control process at the 49^{th} sample value.
This observation suggests that employing a weighted distribution, as opposed to an unweighted one, may enhance the detection probability of control chart.

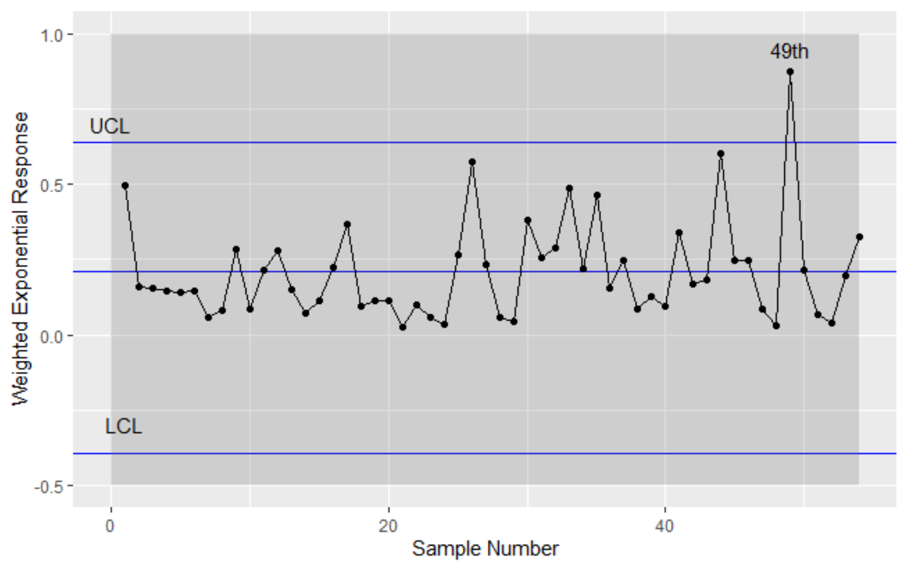

Figure 5: The proposed control chart applied to urinary tract infection (UTI) data.

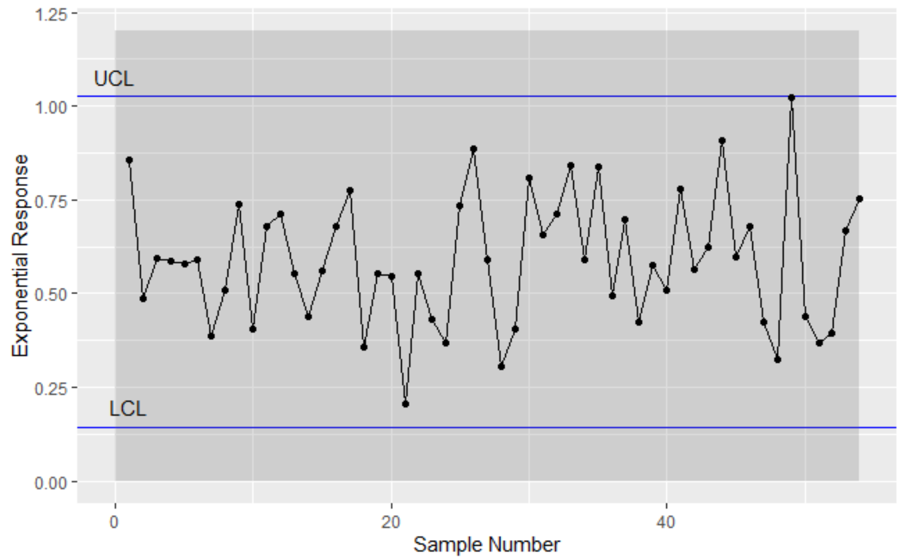

Figure 6: The existing control chart applied to urinary tract infection (UTI) data.

4.2. Application To Survival Time of Breast Cancer Patients

In this example we have applied our proposed control chart to data set represents the survival times of 121 patients with breast cancer obtained from a large hospital during the time of 1929 to 1938. This data set has recently been used by [17].

We have selected a random sample of 40 patients for the purpose of illustration and the effectiveness of the proposed control chart for detecting the shift in the process.

For stable process, the first 20 observations of random sample have been selected. To assess the detection capabilities of both the proposed and existing charts, a shift of size $\delta = 1.5$ was introduced in the scale parameter $\hat{\theta}_o = 23.16$ for the last 20 observations, using $\hat{\theta}_1 = \delta\hat{\theta}_o$ Subsequently, the charting statistics were plotted against control limits, facilitating a comparative analysis between the proposed and existing charts.

It can be noticed from Figures 7-8 that the Weighted-TBE chart detected the shift overall on 12 out of shifted 20 sample points at 26th, 27th, 31st, 32nd. 33rd, 34th, 35th, 36th, 37th, 38th ,39th and 40th samples. Whereas the unweighted-based chart detected the shift only at 31st, 32nd and 38th sample point. Here it can be seen that the proposed control chart outperforms the existing control chart in many ways. The proposed control chart detected the shift earlier as compared to the existing chart also the existing control chart detected the shift only at 3 sample points, whereas the proposed control chart based on weighted distribution detected the shift at 12 points. it shows that the Weighted-TBE chart can be very useful in identifying unwanted variations.

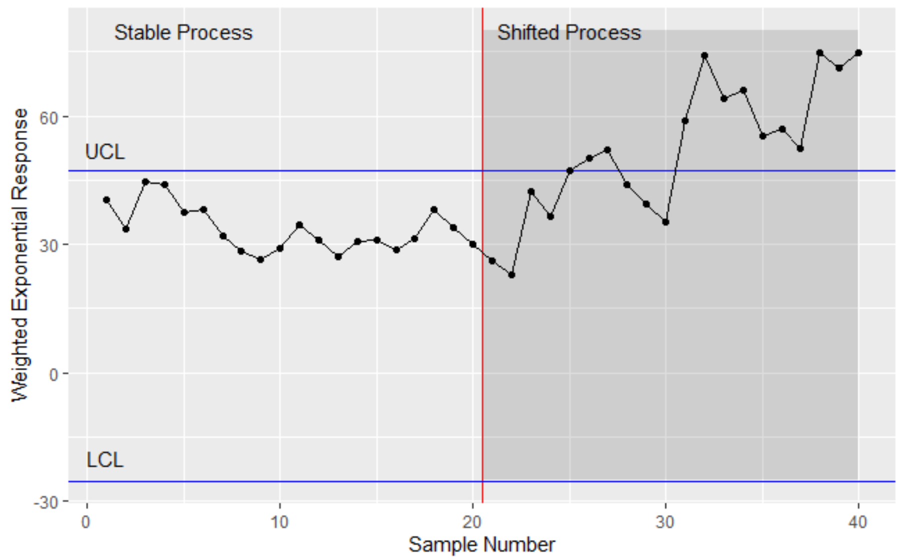

Figure 7: The proposed control chart applied to breast cancer data.

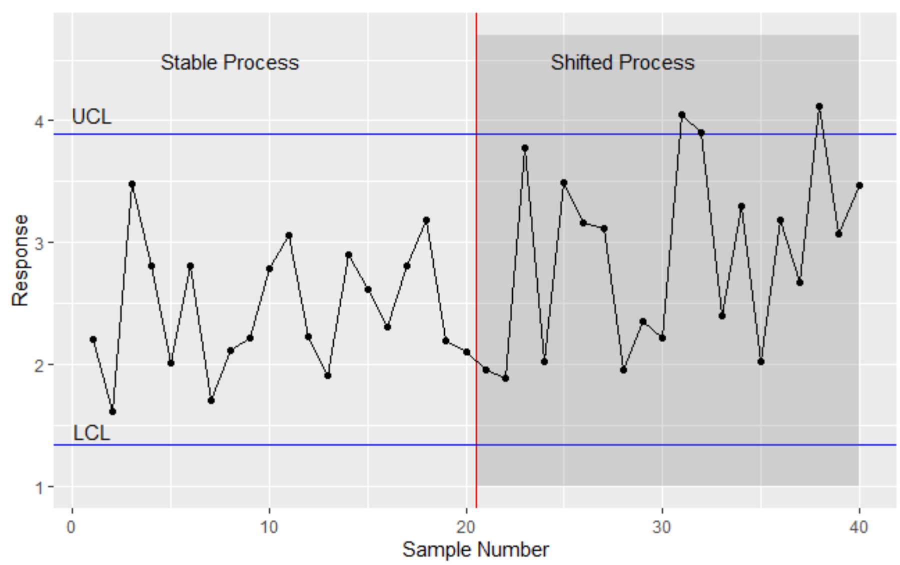

Figure 8: The existing control chart applied to breast cancer data.

4.3. Application to Textile Experiment Data

The third example is related to textile experiment. The data set represents time to failure of a polyester/viscose yarn in a textile experiment for testing the tensile fatigue characteristics of yarn. It consists of a sample of 100 cm yarn at 2.3% strain level. This data set is recently used by [18].

For the comparison purposes, we have randomly selected a sample of 40 observations. For stable process, the first 20 observations of random sample have been selected. To assess the detection ability of both the proposed and existing charts, a shift of size $\delta = 1.5$ was introduced in the scale parameter for the last 20 observations, where the initial estimated scale parameter was $\hat{\theta}_o = 110.98$ and $\hat{\theta}_1 = \delta\hat{\theta}_o$. Subsequently, the charting statistics were plotted against control limits, allowing for a comparison between the proposed and existing charts.

From the Figures 9-10, it can be concluded that the proposed control chart performs much better than the existing chart in detection the shift in the process. the advantage of the proposed control chart is not only to detect the shift earlier but also it has identified the shift in the process overall on 11 out of shifted 20 sample points at 25^{th}, 26^{th},27^{th}, 32^{nd}. 33^{rd}, 34^{th}, 35^{th}, 36^{th}, 37^{th}, 38^{th} and 39^{th} samples. Whereas the unweighted-based chart detected the shift only at 32^{nd} sample.

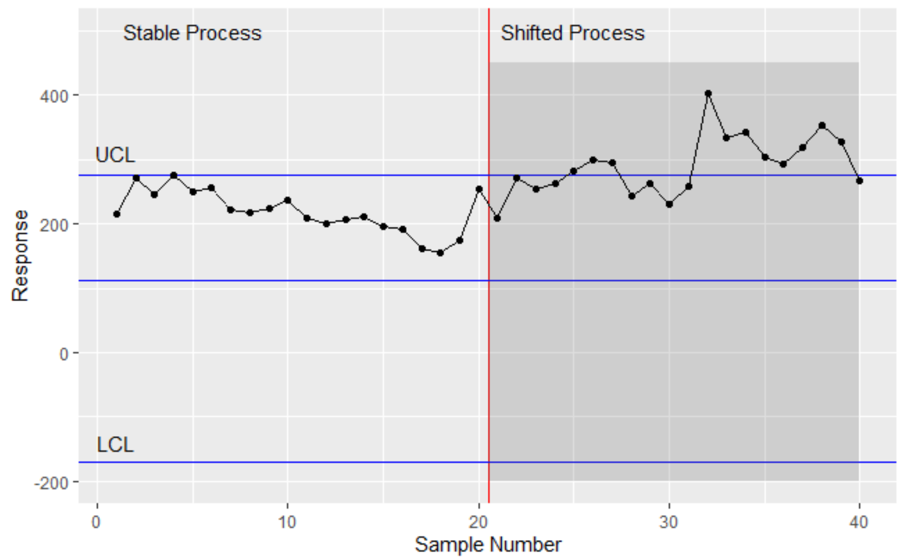

Figure 9: The proposed control chart applied to textile experiment data.

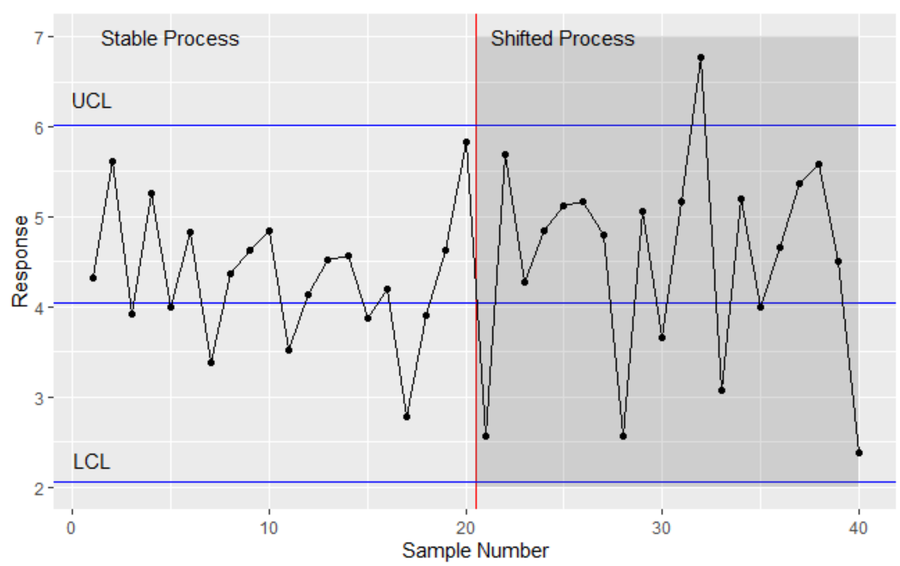

Figure 10: The existing control chart applied to textile experiment data.

This observation suggests that employing the weighted distribution instead of the unweighted distribution could potentially improve the detection probability of control charts when monitoring Time Between Events (TBE) data. Additionally, utilizing the weighted distribution may help prevent the production of faulty items resulting from undesirable variations, thereby safeguarding the production lot.

5. Results and discussions

The objective of the study presented in this article was to develop and compare a new control chart for the weighted exponential distribution against the conventional control chart relying on the standard exponential distribution. Traditional and existing control charts have been criticized for overlooking the possibility that sampling units in a production process may not have equal probabilities of selection.

The new control chart for the weighted exponential distribution takes this into account and provides a more accurate representation of the process. Through extensive simulations to compare the performance of the new control chart with the existing one. The results of the simulations demonstrate that the new control chart yields smaller values of Average Run Length (ARL), indicating that it detects process shifts earlier than the existing control chart.

In addition to the simulations, real-life examples are presented to demonstrate the effectiveness of the new control chart. The authors of the article recommend extending the proposed control chart to other probability distributions, such as Weighted-Weibull, Weighted-Gamma, and Weighted-Erlang distributions.

Overall, the results of the study suggest that the new control chart for the weighted exponential distribution is a superior alternative to the existing control chart based on the standard exponential distribution and can provide more accurate and reliable monitoring of the production process.

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