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Analysis Of The Errors Made By Preservice Mathematics Teachers When Learning Abstract Algebra

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Abstract

The aim of this study was to identify the types of error made by preservice mathematics teachers when learning abstract algebra concepts and analyzing. The study used a descriptive analytical approach and observation was used as a data collection tool. The study sample consisted of (27) preservice teachers enrolled in the College of Education at Sana'a University. The study found ten errors that students make when learning abstract algebra concepts, the most important of which are: misunderstanding basic concepts, failure to understand proofs, faulty reasoning, and misunderstanding the relationship between abstract algebra and other branches. Based on the results of the study, the researchers recommended focusing on students' conceptual and procedural understanding when teaching the course, as it requires a deeper level of insight and development.

Keywords: *Error analysis, preservice mathematics teachers, conceptual understanding, abstract algebra.*

Introduction to the Study:

Abstract algebra is one of the most important branches in mathematics due to its cornerstone role in advanced mathematical studies, as well as its various applications in other fields such as computer science, physics, chemistry, data communications, and others ((Fatmiyati & et al., 2020).

Abstract algebra requires more logical thinking than the arithmetic skills that students are accustomed to in other courses, and the ability to think logically involves more mental activity, justifying, and verifying mathematical ideas (Putra & Kristanto, 2017). In order to understand abstract algebra, student[1]s must be able to comprehend each definition, term, and theory, and be prepared to use their prior knowledge and apply it in different new situations.

Abstract algebra provides pre-service teachers with a logical foundation for the mechanisms that many mathematical operations in school mathematics are based on and enhances the expected cognitive development of mathematics teachers. Despite its great importance, it is the only course in the School of Education that does not provide an overview of it, even for basic concepts, to students in general education.

This has led many pre-service mathematics teachers to believe that there is no connection between abstract algebra and the school mathematics they will eventually teach. This means that the purpose of learning abstract algebra is unknown to the majority of university

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students, which makes it extremely challenging, especially for students who do not intend to pursue further studies (Fitzmaurice & Greene, 2018).

In addition to the abstract nature of abstract algebra concepts and theories, and their inability to be memorized and repeated, this has led students to consider the abstract algebra course to be a serious educational problem. They have encountered many difficulties learning to read this. Through the review of the educational literature by researchers and previous studies related to abstract algebra and its learning difficulties, they noticed the differences in these difficulties from one study to another.

Titova (2007) indicated in his study that difficulties in learning abstract algebra manifest in thinking about abstract concepts, and that students prefer dealing with tangible objects, and they feel confused when the problem is stated in more general terms. While (Risnanoanti & Destania, 2018) mentioned in their study that there are several difficulties in learning abstract algebra, the most important of which is the inability to grasp concepts. Less proficient concepts make students unable to think abstractly and do not enable them to create different mathematical formulas.

Subedi (2020) explained that difficulties in learning abstract algebra manifest in students' weak ability to envision algebraic truths, construct examples and counterexamples, and prove theories. Through teaching the researchers of the abstract algebra course for fourth-level students at the College of Education, University of Sana'a, they noticed the existence of difficulties that prevent students from reaching a higher level of understanding, which makes it difficult to achieve the goals of the course. Therefore, the researchers applied a questionnaire to identify the nature of these difficulties and found that there are several difficulties that have been classified into three domains: (difficulties related to the student, difficulties related to the educational content, and difficulties related to assessment and evaluation). The most difficult for the students was the surface understanding of the abstract algebra vocabulary, which falls within the third domain.

Baldingre (2014) pointed out in his study that developing a deep mathematical understanding among teachers prior to serving in the abstract algebra course enables them to effectively engage in mathematical practices and handle their students' questions and inquiries with high flexibility.

Veith & et al. (2022) confirmed that understanding abstract algebra has brought about a significant transformation in teachers' beliefs (from kindergarten to twelfth grade) and their practices in the classroom, as it has developed their understanding and perspectives on the nature of mathematics in general. In light of the aforementioned, researchers came up with the idea of conducting this study to analyze the errors made by preservice mathematics teachers in order to identify the reasons that hinder them from achieving a deep understanding of abstract algebra concepts.

Problem of the study

The problem of the study can be identified in the following questions:

- What are the errors made by preservice mathematics teachers when learning abstract algebra?
- What types of error are made by preservice mathematics teachers when learning abstract algebra?

The importance of studying:

1- Identifying the mistakes that students and teachers make when learning abstract algebra.

2- Knowing the mistakes of student teachers allows a faculty member to design teaching methods better suited to address those errors and problems, leading to more effective educational outcomes.
3- Enabling student-teachers to identify and correct their mistakes, thereby improving their understanding and application of abstract algebra.

Study Objective:

The current study aims to analyze the errors made by preservice mathematics teachers when learning abstract algebra, and to classify them.

Study limitation:

1- Spatial limitation:

Students of the Mathematics Department at the College of Education, Sana'a University.

2- Human limitation:

Student teachers enrolled in the fourth level of the Mathematics Department at the College of Education, Sana'a University.

3- Subject limitation:

Abstract algebra terms (group theory and ring theory).

4- Temporal limitation:

This study was conducted in the academic year 2023-2024.

Theoretical Framework and Previous Studies:

This chapter deals with the theoretical framework for the subjects of the study and reviews previous relevant studies, as this part of the study comes as a review of the previous educational literature.

The Theoretical Framework

Abstract algebra is a branch of mathematics, also known as modern algebra, that deals with the study of algebraic structures and mathematical structures, such as groups, rings, fields, and vector spaces. These structures are defined through sets of elements and operations on them. However, unlike other branches of mathematics, the nature of abstract algebra requires its students to understand each concept and theory to organize the necessary concepts to prove theorems and distinguish between them.

The importance of abstract algebra:

Jafer & et al (2018) and Cetin & Dikci (2021) have pointed out the importance of abstract algebra, which can be summarized as follows:

Abstract algebra has the ability to provide a unified language and framework for studying mathematical structures. It also provides the foundations for various branches of mathematics, such as number theory, algebraic geometry, and cryptography. A deep understanding of group theory is necessary to explore physical and algebraic structures in various scientific fields. In addition, it provides a mathematical tool for analyzing symmetries and physical and chemical patterns. Abstract algebra also plays an active role in cryptography, where it is used to develop encryption algorithms and ensure secure communication. The abundance of concepts and theories in the subject provides students with an opportunity to develop their skills in logical thinking, mathematical proof, problem-solving skills, and analytical skills. It is also an important course for students who wish to pursue advanced studies in pure mathematics.

The objectives of studying abstract algebra are:

1- To provide students with a solid foundation in understanding and comprehending algebraic structures and how to work with them, as well as understanding the relationships between them.

2- Develop students' ability to abstract and deal with evidence.

3- To equip students with the ability to think logically.

4- The ability to distinguish between premises and conclusions.

5- Find solutions to abstract exercises using theories and results.

6- Deepen the understanding of group symmetry by introducing how to classify and enumerate abstract groups.

7- To provide examples for each structure of algebraic structures covered in the curriculum.

Previous studies:

The researchers reviewed studies related to the topic of the current study, and the following is a presentation of those studies in chronological order from the most recent to the oldest as follows:

Subroto & et al (2023):

This study aimed to exploring misconceptions in group theory lectures among prospective teachers, To achieve this the study employed a descriptive approach(case study), The study sample consisted of six preservice mathematics teachers, The study utilized interview and observation as research tools, The findings revealed two misconceptions: the emergence of misunderstandings due to the proximity of a general concept with realistic knowledge and the abundance of concepts with their interweaving, and the misunderstanding arising from incomplete understanding of the concept.

Fatmiyati & et al (2020):

This study aimed to describe of the types of errors among university students in solving problems related to ring theory, to achieve this, the study employed a descriptive approach (case study), The study sample consisted of 46 male and female students from the College of Teacher Training at Sebelas Maret University, the research tools included interviews and test. The study identified errors made by students, which could be categorized as expression errors, conceptual errors, strategic errors, and encoding errors.

Yerizon & et al (2019):

This study aimed to analysis of the errors students made by students when attempting to write proofs in group theory, to achieve this, the study employed a descriptive approach, The study sample consisted of 18 male and female students at the Andalusia University. The research tools included a proof test and interviews, The study found that students face several difficulties in learning abstract algebra, with the most significant ones being: 1- Students' achievement in proofs remains a problem. 2- Most students face difficulties in verifying the existence of identity and inverse elements.

Lewis (2013):

This study aimed to analyze the difficulties faced by students when learning group theory, To achieve this, the study employed a descriptive approach, The study sample consisted of 4 undergraduate students, three of whom are bachelor's students, and one is a master's graduate student, The research tool used was interviews, The study concluded that there are two sources of difficulties: the first is related to the conceptual understanding of the nature of group theory, and the second is related to the abstract formulation of modern topics taught in the group theory course, necessitating explanation and meaningful representation in abstract algebra.

Commentary on previous studies:

Reviewing previous studies related to the topic of the study, we find that they agreed on some aspects and differed on others, and we clarify this as follows:

- In terms of objectives: All studies agreed on analyzing errors, although they differed in classifying them from one study to another. The current study distinguished itself from the mentioned studies in that it addressed the analysis of errors related to abstract algebra, specifically group theory and ring theory. On the other hand, the studies by Lewis, Yerizon et al. and Subroto et al. focused only on errors related to learning group theory, while the study by Fatmiyati et al. was limited to ring theory.
- In terms of the methodology used: the current study agreed with the study by Lewis and Yerizon et al. in using the descriptive method, while the studies by Fatmiyati et al. and Subroto et al. used the descriptive method (case study).
- In terms of research tools: the study by Lewis used interviews, while the study by Subroto et al. used observation and interviews. On the other hand, the studies by Yerizon et al. and Fatmiyati et al. used tests and interviews. The current study used observation.

Study methodology and procedures:

This study adopted a descriptive-analytical approach.

Study sample:

The total study sample consisted of 27 pre-service teachers who teach at the fourth level in the Mathematics Department at the College of Education, University of Sana'a. The sample was purposively selected because it represents students of the fourth level who study the curriculum.

Study Tools:

To achieve the objective of the study and answer its questions, the researchers used observation to assess the performance of the teacher students in the lectures provided to them and the effort required from them, as well as to analyze their errors when learning abstract algebra vocabulary.

Study procedures:

Observe the students' performance in the first semester regarding group theory vocabulary and then observe their performance in the second semester regarding ring theory vocabulary.

Study Results:

Presentation, Analysis, and Discussion of Errors in Abstract Algebra Learning:

Abstract algebra presents a myriad of challenges for its learners, and these difficulties frequently manifest themselves as errors committed during the process of learning abstract algebra. The following section provides an overview of specific errors identified within the study sample, accompanied by a detailed analysis:

- A set of examples was presented to the students, comprising symmetry groups associated with regular geometric shapes, namely the equilateral triangle symmetry group (D_3), square symmetry group(D_4), regular pentagon symmetry group(D_5), and regular hexagon symmetry group (D_6), Students were instructed to determine the center of each group and develop a systematic approach to its determination, avoiding the need to exhaustively test each element. This method was designed to facilitate a more efficient and expedited process, particularly when faced with geometric shapes characterized by a substantial number of sides.

The students faced challenges in presenting a coherent rule, with all their attempts exhibiting significant discrepancies. They encountered difficulties in formulating explicit and rigorous proofs, relying on unsupported assumptions. This can be attributed to a deficiency in their conceptual foundation. Consequently, the students concluded that the establishment of a rule was unfeasible and that the determination of the center of the group could only be achieved through a trial-and-error approach.

Upon presentation of the rule to the students and subsequent request for proof, the students opted to rely solely on the examples provided to them. This tendency can be traced back to a misconception regarding mathematical proof and its various forms. It is crucial to emphasize that furnishing examples alone does not constitute a comprehensive proof, unless the objective is specifically to establish the falsity of a theory. This observation aligns with the research of (Ioannou, 2019), who, in his study, found that students often grapple with the initiation of mathematical proofs and the systematic progression through proof steps when faced with such tasks.

- Students were presented with the following question: "Is $\mathbb{Z}_3$ a subgroup of $\mathbb{Z}_9$?"

The majority of students answered in the affirmative, stating that both represent subgroups, as they are groups. However, some students indicated that $\mathbb{Z}_3$ represents a cyclic subgroup, since both are permutation groups. When asked to construct the Cayley table for both groups, the students observed differences in the elements. This misconception represents an incorrect application of the concept of a subgroup. The students' error can be attributed to an incomplete understanding of the concept of a subgroup. For a group to be a subgroup of another, it must contain elements from that group. The students' confusion, assuming that the elements of $\mathbb{Z}_3$ are subgroups of $\mathbb{Z}_9$ and that the remainder when divided by 3 is the same as the remainder when divided by 9, stems from a foundational weakness, particularly in the Number Theory course. Additionally, the discrepancy in the binary operation further complicates matters, since it is not feasible for one group to be a subgroup of another when the binary operations differ. This observation aligns with the findings of (Subroto & et al., 2023), who noted in their study that students often fall into two misconceptions: misunderstanding and misinterpretation.

- The students were presented with the following questions:

- If $2\mathbb{Z}$ and $4\mathbb{Z}$ are subgroups of $\mathbb{Z}$, does their union, $2\mathbb{Z} \cup 4\mathbb{Z}$ also represent a subgroup of $\mathbb{Z}$? Clarify your answer.

The students responded affirmatively, asserting that both $2\mathbb{Z}$ and $4\mathbb{Z}$ share the same binary operation, meeting the necessary conditions to be groups.

- If $2\mathbb{Z}$ and $3\mathbb{Z}$ are subgroups of Z, does their union, $2\mathbb{Z} \cup 3\mathbb{Z}$, represent a subgroup of $\mathbb{Z}$? Explain your answer.

The students responded negatively, citing the lack of closure property as the reason for their assertion.

- Provide a suitable generalization to determine when the union of subgroups constitutes a subgroup and prove it.

The students struggled to present an appropriate generalization, indicating difficulty in grasping the examples and linking them to understand the specific cases. Their tendency to generalize and overlook counterexamples was noted. (Subedi, 2020) highlighted in his study that students face challenges in comprehending logical and inclusive relationships, as well as providing evidence and arguments when learning abstract algebra.

- The students were presented with the following question: Provide an example of a group G where $A, B \preccurlyeq G$ such that $A \lhd B \lhd G$, but A is not a normal subgroup of G.

The predominant response from the students was an assertion that they could not furnish an example due to the imperative requirement of ensuring the transitivity property, as typically demanded in broader algebraic inquiries. This misconception can be ascribed to a deficiency in the application of previously acquired knowledge when faced with novel situations. (Agustyaningrum & et al., 2021) underscored in their study that students often grapple with the challenge of being unable to effectively utilize their existing knowledge in diverse contexts.

- Students were presented with the question: "Is ${}^{4\mathbb{Z}}/_{12\mathbb{Z}} \approx \mathbb{Z}_3$? Explain your answer.

In their response, the students proceeded to enumerate the elements of ${}^{4\mathbb{Z}}/_{12\mathbb{Z}}$ as a quotient group, denoting it as S, Subsequently, they endeavored to establish a mapping $f: S \to \mathbb{Z}_3$, with the aim of confirming its status as an isomorphism. However, the students failed to formulate a specific rule for the application of f, leading them to the conclusion that it could not represent an isomorphism. A more careful examination of the problem could have enabled the students to leverage the first isomorphism theorem, streamlining the solution process and conserving time and effort. This particular error falls under the umbrella of algebraic mishandling during the simplification of symbolic expressions, indicative of an incomplete grasp of the theory and its practical application to facilitate diverse problem-solving scenarios.

This misinterpretation may be ascribed to the students' tendency to memorize theoretical content without achieving a deeper comprehension. In addition, their struggle to effectively apply previously acquired knowledge in different contexts played a role in this instance. (Agustyaningrum & et al., 2021) emphasized in their study that students often confront substantial difficulties in deploying their existing knowledge across various situations.

- The students were presented with the following question: "In the ring $(\mathbb{Z}_4, +, .)$, find the invertible elements.

The students encountered two difficulties. Firstly, they faced challenges in grasping the meaning of the term "invertible elements" due to their inability to express terms with more than one meaning. After clarifying the meaning, the students managed to identify these elements using the Cayley table. However, when asked to determine the relationship between invertible elements and the identity element for easier and quicker solutions, especially when n is large, the students struggled to establish this relationship. This difficulty stemmed from their unfamiliarity with independently presenting conclusions, as they tended to receive knowledge directly. Furthermore, even after deducing the relationship, the students found it challenging to symbolize it, indicating a weak conceptual foundation in fundamental concepts. This hinders their understanding of higher-level topics. (Yerizon & et al., 2019) noted in their study that most students face difficulties when identifying invertible elements.

- The students were presented with the following question: "Find the neutral element in $(Z_{20}, \oplus, \otimes)$".

Students responded that there is no neutral element, stating that Z_{20} is not considered a ring with the multiplication operation because 20 is not a prime number. This stems from the students' lack of understanding of the concept of a ring with a neutral element, as they tend to limit themselves to memorizing only fundamental concepts. Their difficulty aligns with findings from the study by (Yerizon & et al., 2019), which indicated that most students struggle when tasked with finding the neutral element.

- The question posed to the students was: Determine the characteristic of the ring $Z_3 \times Z_9$

The majority of students responded stating that the product $3 \times 9 = 27$ and consequently, the characteristic is considered 27.

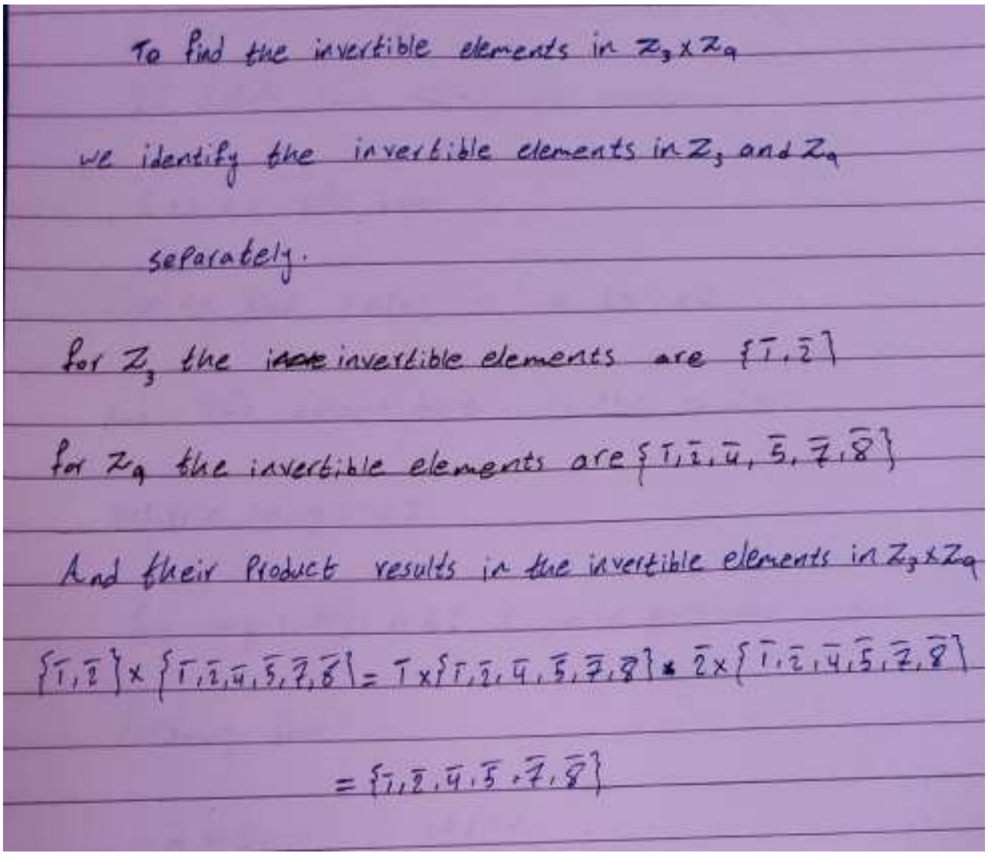
Conversely, some students provided an answer based on the least common multiple, identifying it as 18, and hence, indicating a characteristic of 18.

These responses can be attributed to the deficiencies of the students in fundamental concepts and their struggles to appropriately integrate these concepts. The difficulty observed reflects an incapacity to differentiate between the notions of greatest common divisor and least common multiple. Additionally, the students demonstrated an inability to apply their prior knowledge effectively. In a study conducted by (Mairing, 2021), it was highlighted that $(8{,}73\%)$ of students encounter arithmetic errors when delving into abstract algebra, predominantly stemming from a weak grasp of fundamental concepts and an inability to establish effective connections and applications of information.

- The students were presented with the following question: "Find the invertible elements in $Z_3 \times Z_9$".

Some of the students provided the following answers:

Figure (1): Student answers



To find the invertible elements in $Z_3 \times Z_9$

we identify the invertible elements in Z_3 and Z_9

separately.

for Z_3 the ~~inve~~ invertible elements are $\{\bar{1},\bar{2}\}$

for Z_9 the invertible elements are $\{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\}$

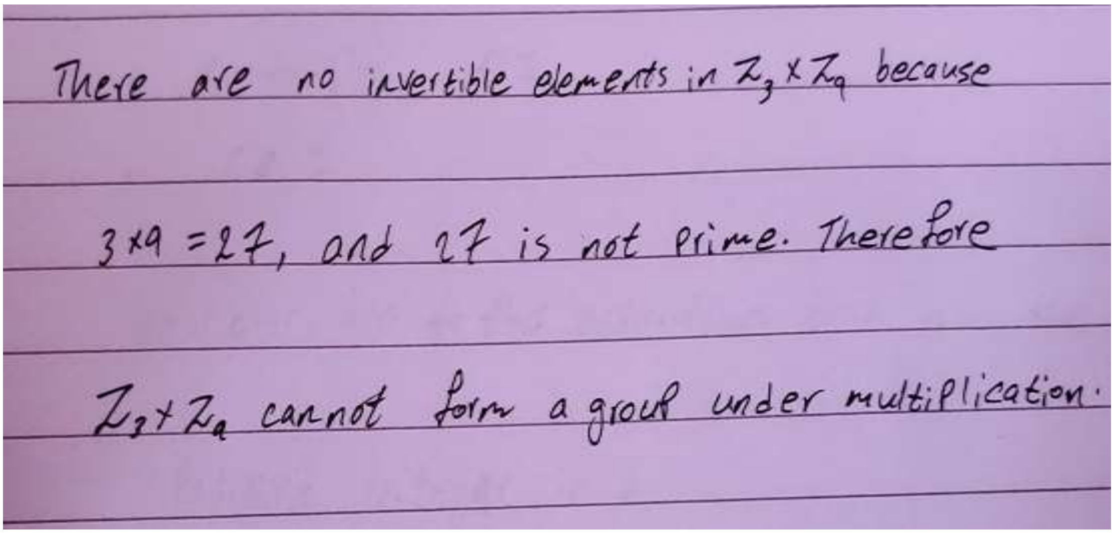
And their Product results in the invertible elements in $Z_3 \times Z_9$

$\{\bar{1},\bar{2}\} \times \{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\} = \bar{1} \times \{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\} \ast \bar{2} \times \{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\}$

$= \{\bar{1},\bar{2},\bar{4},\bar{5},\bar{7},\bar{8}\}$

The rest of the students presented their answers as follows:

Figure (2): Student answers



There are no invertible elements in $Z_3 \times Z_9$ because

$3 \times 9 = 27$, and 27 is not Prime. Therefore

$Z_3 \times Z_9$ cannot form a group under multiplication.

Both responses are incorrect, highlighting the students' struggle to solve exercises that deviate from the familiar. This difficulty arises from their tendency to memorize information without grasping its essence and to apply solution steps in a routine manner, neglecting alternative approaches. (Yerizon & et al., 2019) emphasized in their study that a majority of students face challenges when tasked with identifying invertible elements.

- The students were presented with the following question: "If a, b are two nil potent elements in a commutative ring R, prove that $(a + b)$ is also a nil potent element in R".

The students faced challenges at the beginning of the proof, struggling to establish the appropriate strategy for the demonstration. Moreover, they expressed skepticism regarding the application of the theory of idempotents in the proof. These difficulties stemmed from their limited ability to effectively employ their prior knowledge during the proof, a misconception regarding the relationship between abstract algebra and other branches of mathematics, and an erroneous belief that abstract algebra is a distinct and isolated branch within the realm of mathematics. Consequently, this lack of comprehension hindered their ability to apply their knowledge effectively in the context of abstract algebra. (Ioannou, 2019) highlighted in his study that students encounter difficulties in utilizing their prior theoretical knowledge when required to prove abstract algebraic concepts.

- Students were presented with the question: "Find the subrings and maximal ideals of the ring $(Z_{27}, \oplus, \otimes)$" .

The students responded that there are no maximal ideals in the ring $(Z_{27}, \oplus, \otimes)$. As for the subrings, they identified Z_3‘ Z_5‘ Z_9‘ Z_{11}‘This error can be attributed to an incomplete understanding of the concept of subrings among the students. They may not be aware that one ring cannot be a subring of another if its binary operations differ. This misunderstanding reflects a lack of understanding of the fundamental concepts presented to them, leading to inaccurate applications.

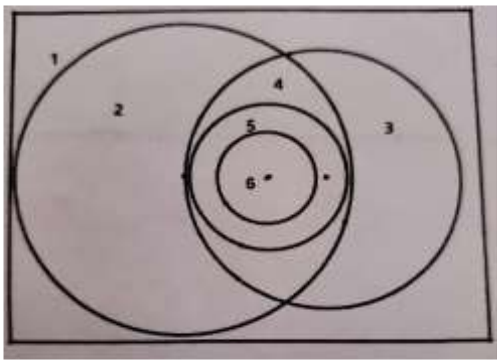
- Students were presented with the question: prove that the union of two subrings does not necessarily form a subring.

The students provided a direct proof that was time consuming to construct and contained numerous errors. They could have alternatively presented a counterexample, saved time and effort, and demonstrated the skills of the 21st century to reach solutions through faster and more efficient methods.

- Another question was presented to the students: 'If R is a ring in which every element is idempotent, then R is a commutative ring.

The students gave an example of two idempotent elements, thinking that this would suffice to fulfill the requirement. However, this approach is incorrect as the example neither proves nor disproves the statement. The error in both examples can be attributed to the students' limited ability to identify the appropriate strategy for solving the problem. In the first example, a counterexample would have been more suitable, while a direct proof was the appropriate strategy for the second example. Studies (Fatmiyati & et al., 2020) indicate that students often struggle with determining the optimal strategy to reach a mathematically correct solution.

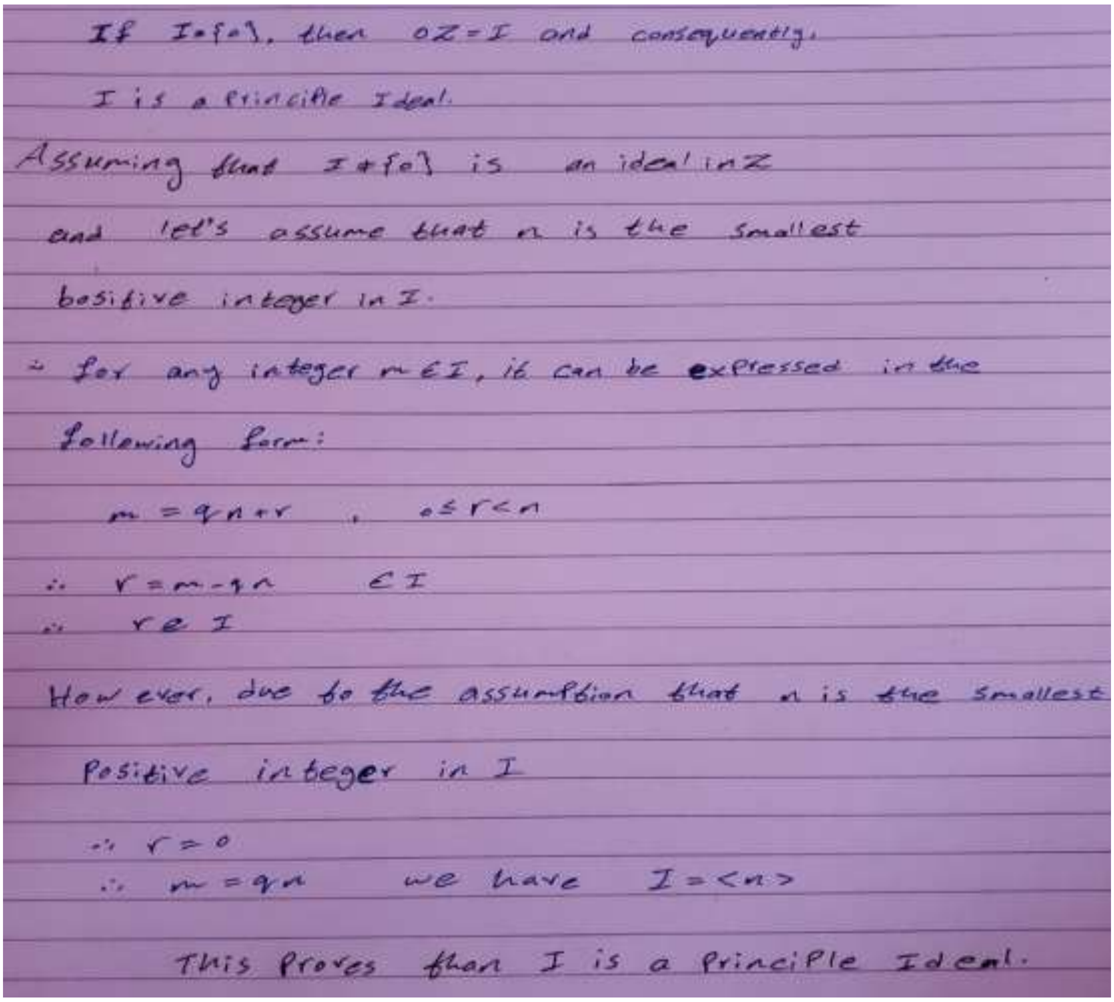
Figure (3): Question presented to Student



- presenting the question, Identify the concepts indicated by the numbers in the adjacent figure. The concepts include 'Integral domains, Commutative rings, All rings, Rings with unity, Fields, Commutative rings with unity.' It should be noted that an integral domain is a unity commutative ring that does not contain zero divisors. The students were unable to correctly arrange the concepts, attributing their difficulty to an inability to comprehend the relationships between the concepts due to an incomplete understanding. Their approach involves memorization, presenting the information directly when requested, without the ability to apply it differently. According to

(Subroto & et al., 2023), students tend to fall into the error of misinterpretation resulting from an incomplete understanding of the concept.
- Students were tasked with providing an example of an ideal that is not a subring.
Most of the students presented examples such as $(\mathbb{Z},+,.)$, $(\mathbb{N},+,.)$, $(\mathbb{Q},+,.)$, $(M_n(R),+,.)$ This can be attributed to their misunderstanding of the nature of ideals, believing that an ideal cannot exist without being a subring. As highlighted by (Subedi, 2020) in his study, students often face challenges in presenting examples and counterexamples.
- The students were presented with the following question: $(\mathbb{Z},+,\ .)$ let be the ring of integers, and let I be an ideal in $\mathbb{Z}$. Prove that I is a principle ideal.
The students presented the proof correctly as follows:

Figure (4): Student answers


If $I=\{0\}$, then $0\mathbb{Z}=I$ and consequently,
I is a principle Ideal.
Assuming that $I\neq\{0\}$ is an ideal in $\mathbb{Z}$
and let's assume that n is the smallest
bositive integer in I.
∴ for any integer $m\in I$, it can be expressed in the
following form:
$m=qn+r \quad , \quad 0\le r<n$
∴ $r=m-qn \in I$
∴ $r\in I$
However, due to the assumption that n is the smallest
Positive integer in I
∴ $r=0$
∴ $m=qn$ we have $I=\langle n\rangle$
This proves than I is a principle Ideal.

The students encountered difficulties in providing a correct and coherent proof for the given question. When discussing certain steps they followed, they were unable to present valid reasoning. For example, when asked why they assumed that n is the smallest positive integer in I, one student mentioned using the division algorithm in the subsequent step, another referred to $\mathbb{Z}^+$ being bounded below by zero. Unfortunately, most of them collectively agreed that it was an ideal, and all of these responses were incorrect and far from the correct answer. This reveals a limited and constrained thinking pattern among the students. When asked why they concluded that $r \in I$, their responses indicated a superficial understanding, as they simply mentioned that r is an integer. These errors stem from the students memorizing the proof without truly understanding and articulating its steps correctly.

- The students were presented with the following question: If R is a ring in which both P_1, P_2 are prime ideals in R, is$P_1 \cap P_2$ also a prime ideal?

Most of the students answered affirmatively, based on the theory of intersecting subrings. However, this error can be attributed to their tendency to overgeneralize the concept.

- The students were tasked with providing a definition of a local ring.

The definition they presented is as follows:

For every non-empty set equipped with the operations$(+,.)$, if we have the triple system $(R, +, \ .)$then it is a ring if it satisfies the following conditions.

1- $(R, +)$ abelian group.

2- $(R, .)$semi group.

3- The operation $(\ .)$is distributive over $(\ +)$.

It is evident that the students encountered challenges in comprehending various definitions associated with abstract algebra. Their tendency to concentrate on fundamental concepts while neglecting finer details aligns with the observations made by (Mairing, 2021), who found that (57.94%) of students commit conceptual errors in learning abstract algebra due to misunderstandings of definitions and intuitions.

- The students were asked the following question: "Consider the ring $(Z_6, +, \ .)$and let $I = \{\ \bar{0}, \bar{2}, \bar{4}\}$. Determine the ring $(^{Z_6}/_{I}, +, .)$ and generate its Cayley table for addition and multiplication.

Regrettably, the students only produced a Cayley table for addition, showcasing a challenge in distinguishing between group properties and their corresponding symbols, as well as differentiating between ring properties and symbols. This difficulty in discernment resulted in inaccuracies in their applications.

Classification of the types of error that students make when learning abstract algebra:

1- Error in misinterpreting basic concepts, which is the most dangerous error. Students' error in understanding basic concepts leads to incorrect application, and as a result of falling into this error, many other errors occur. (Risnanoanti & Destania, 2018) clarified in their study that students' less mastery of concepts makes them unable to think abstractly and unable to create different mathematical formulas.

2- Error in algebraic processing and simplification of expressions when presenting solutions to examples and exercises, or when presenting proofs.

3- Error in weak ability to link previous knowledge with current knowledge.

4- Error in understanding proofs, often the proofs in abstract algebra are complex and require a deep understanding of the basic structures. Students often face difficulty in understanding the mechanism of presenting proofs. (Arnawa & et al, 2019) pointed out in their study that there are four concepts that must be known in order to present proofs correctly: understanding proofs, constructing proofs, verifying the validity of proofs, and evaluating proofs.

It is necessary to focus on presenting proofs in order to improve students' understanding of the vocabulary of abstract algebra. (Yerizon & et al., 2019) indicated that one of the best methods to develop abstract thinking skills and improve students' understanding is through meaningful participation in building mathematical proofs.

5- Error in unfamiliarity with examples and determining their correct place, overlooking counterexamples, and making generalizations without considering exceptions, which leads to incorrect conclusions.

6- Error in merging concepts, properties, and symbols of groups and rings, which leads to errors in proofs and presenting solutions.

7- The error of the incorrect application of theories and concepts, where the abstract algebra depends heavily on deductive reasoning, and therefore the deep understanding of its theories and concepts is very important, yet students face difficulty in applying concepts and theories properly, which often leads to incorrect evidence, or provide wrong explanations for sports data.

8- The routine gradient in the solution without thinking about other methods, whether when providing proofs or when providing the given exercises solutions, and this is a mistake; Because what is appropriate to solve an exercise or provide proof of what may not be commensurate with another exercise or another proof, also there may be exercises that can be reached in easier and faster ways without applying all steps literally.

9- The conclusion error: Students face great difficulty if they are asked to conclude a concept, a relationship, or a theory, and they fall into many mistakes, and this is due to their accustomed to receiving knowledge by the faculty member without researching and thinking to reach them themselves, and their weak ability to make assumptions and provide logical evidence for it.

10- The error of misunderstanding algebra, the abstract algebra with other mathematics branches, which leads to a lack of understanding how to apply what was learned in other mathematics branches in abstract algebra and vice versa as well.

Conclusions:

A diverse set of mathematical questions related to abstract algebra was presented to the students, with a specific focus on various algebraic structures. The students faced challenges in understanding and applying fundamental algebraic concepts, such as identifying invertible elements, neutral elements, partial rings, and ideals. Furthermore, they encountered difficulties in formulating definitions and proofs accurately. The study revealed that the students relied heavily on memorization without achieving deep understanding, which led to challenges in applying their knowledge to solve unfamiliar problems. The study also indicated that a lack of familiarity with abstract concepts and weak foundational knowledge hindered their ability to express mathematical ideas symbolically.

In conclusion, the results highlight the importance of addressing conceptual gaps and strengthening foundational knowledge in the education of abstract algebra. Developing students' ability to connect abstract algebra with other mathematical branches and enhancing their deeper understanding of fundamental concepts are crucial. This requires a focus on improving problem-solving skills, encouraging critical thinking, and fostering a comprehensive understanding of abstract algebraic principles.

Study recommendations and proposals:

Recommendations:

1- Clarify the purpose of studying abstract algebra for the student as a future teacher.

2- Highlight the importance and role of abstract algebra in mathematics specifically and other sciences in general.

3- Teaching abstract algebra needs to focus on conceptual understanding, procedural knowledge, and emphasize providing examples and nonexamples as much as possible to enhance student learning, as it requires a deeper level of insight and development.

4- Encourage students to prove theories and employ them in solving mathematical problems related to the course vocabulary to improve their understanding and confidence.

5- Encourage students to turn to various sources to acquire knowledge on their own.

6- Establish a mechanism to connect the course vocabulary or some of it to other sciences and real-life situations.

Proposals:

1- Conduct studies on mechanisms that can help pre-service mathematics teachers avoid errors when learning abstract algebra.

2- Conduct studies on the extent to which deep understanding can be developed in students when learning concepts of abstract algebra.

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