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Improving Methods of Finding the Least Common Multiple and the Greatest Common Divisor Using Conceptual Map

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ABSTRACT

In this research methods of teaching and finding of the least common multiple (LCM) and the greatest common divisor (GCD) were visited hoping to improve both using conceptual maps. The subject matter was introduced to the 22 math education students of Hebron university in 4 lectures where we divided them into 6 groups. Seven activities each of 3 pre and post introducing conceptual maps questions were included, presentations, discussions and brain storming were used notes were taken and they include problems from the real life. The questions before introducing conceptual maps showed that the majority of students used the standard methods for finding factors, multiples, common factors, and common multiples of a number. Four groups used the tables and the remaining two used the trees. After introducing conceptual maps strategy, the students concentrated on using it through trees. Through meetings and discussions some methods of teaching were presented and finding GCD and LCM using conceptual maps was observed.

Keywords: *Conceptual maps, greatest common divisor, least common multiple, methods, strategy.*

1. Introduction

Mathematics is very well known as an inter-related subjects science and that for one to succeed in one area he must master its pre-requisites (abu-aqeel, 2016). One of the most important topics in the preparatory stage at school is the LCM and GCD and the students must master finding both (Lee and Boyadzhiev,2020) as these concepts are heavily used in the study of other concepts such as fractions and operations on fractions. (Meric and Yakin,2021) from among the difficulties students face in solving real life problems concerning LCM and GCD is the understanding of the prerequisites, not understanding the problem and the difficulty in solving the problem (Kolitsch and Kolitsch,2011; Triyani et al,2012).

According to Mohyuddin and Khalil (2016) many students don’t understand the meaning of LCM and GCD, where the concepts "multiple of" and "factor of" are needed in addition to the concept of prime numbers and confusion between these concepts (Unaenah et al, 2020), finally confusion mistaking the above mentioned concepts (Tugba, 2021).

Students also face problems in understanding LCM and GCD because of weakness in the concept of "multiplication", finding a prime factor and not being able to distinguish between multiple and factor (Sutarto et al,. 2021). Suffolk (2007) said that a few students found the GCD (12, 24) and did not realize that the answer is 12 as it is a factor of 24. In

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addition, he found that many schools did not teach the basic meaning of the LCM and GCD. In fact when the students were asked to find GCD (12,16) it was crystal clear that they were taught only how to solve the question but not the meaning of the concept.

In the school math book by the Palestinian ministry of education students were taught how to find the GCD using factorization or by repeated division in tables and that the LCM can be found by listing multiples of numbers and choosing the smallest common factor (ministry of education,2022).

The real life applications are very important in helping students gain logical thinking and clarify to them the importance of such mathematical concepts side by side with educational aids, graphs and plans that illustrate the concepts in a clear way.

In addition, storytelling, acting and drama are used to achieve the goals in an entertaining method (Triyani et al,2012).

The conceptual map considered to be an educational method through sketching flexible plan that governs information using relations. Conceptual maps aim at simplifying solving problems.

Fauzan et al (2020) studied the use of realistic math education to teach LCM and GCD in addition to videos, work sheets, writing notes, interviews and analysis of students work-sheets. He concluded that using realistic math teaching helped offering better education to the students and that students can repeat applying LCM and GCD by themselves in addition to solving real life problems and improving their ability in logic.

Yuliant and Fauzan (2018) developed the local theorem on math education, so that it can be used efficiently in teaching LCM and GCD. Khatoon and Akhter (2010) designed an education strategy using creative cooperation together with group education to help educate slow learners. There was significant improvement in their performance.

The researcher noticed during his many years of teaching university math education courses weakness in creating and developing teaching methods of mathematical concepts. The weakness may have resulted because of lack of cooperation, sharing and generating ideas by students or is due to the lack of ability to work in teams and use of constructive discussions that lead to new and easier teaching methods of mathematical concepts through organizing the concepts via relations between them such as conceptual maps. The question is how can we improve teaching mathematical concepts if we don't get involved in teams work and exchange the general ideas? In looking for answer for the question above the researcher decided to experiment teaching LCM and GCD.

This study aims at knowing the methods and strategies of finding LCM and GCD which are preferable to the students and to introduce the use of conceptual maps in finding LCM and GCD and hoping for new strategies and creative methods for finding LCM and GCD.

2. Methodology:

The qualitative methodology was used through interviews, discussions, learning via groups, comparison between a section situation before and after introducing conceptual maps. We chose randomly a section from among many math and math education sections composed of 22 students (males and females). The researcher lectured the sample 4 times presenting the experiment and the most important strategies and methods for finding LCM and GCD through presenting the interviews questions. Then, through the discussions, groups working, solving these activities using conceptual maps and then hoping to arrive at improving the methods for finding LCM and GCD (using a conceptual map). Finally, comparison between the students answers pre and post discussions was done.

Collection of data was done by interview in the form of 7 activities where each activity contains 3 questions on the same subject of one goal and after presenting questions, notes about solution methods were taken. The researcher then (with participant students) through the discussion groups solved another question using conceptual maps and leaves the solution of the third question to the students, then notes were taken and conceptual maps was noticed.

In the first activity, the students were asked to find some multiples of a number and explain the methods students used for that in order to determine their understanding of multiples and the concept of multiplication. The second activity leads to finding the factors of a number and indicate which of them is prime and

noting the method used and their understanding of prime numbers and prime factors was determined. The third activity requires the students to find the common multiples for two numbers (or more) indicating the used solution method. The fourth activity asks the students to find the common factors for two numbers (or more) with concentration on the method used in the solution. The fifth activity asks the students to find LCM of two numbers (or more) with emphasis on the method used. The sixth activity asks students to find the GCD. Finally, the seventh activity directs participants towards the relation between the LCM and GCD where the students were also asked to indicate methods and strategies used to find the two concepts and suggests improved and creative methods and strategies through the previous discussions of the 6 groups.

Validity of the activities was agreed upon by a group of experts in math education and very experienced and distinguished math teachers at schools. Data on the activities was collected in writing

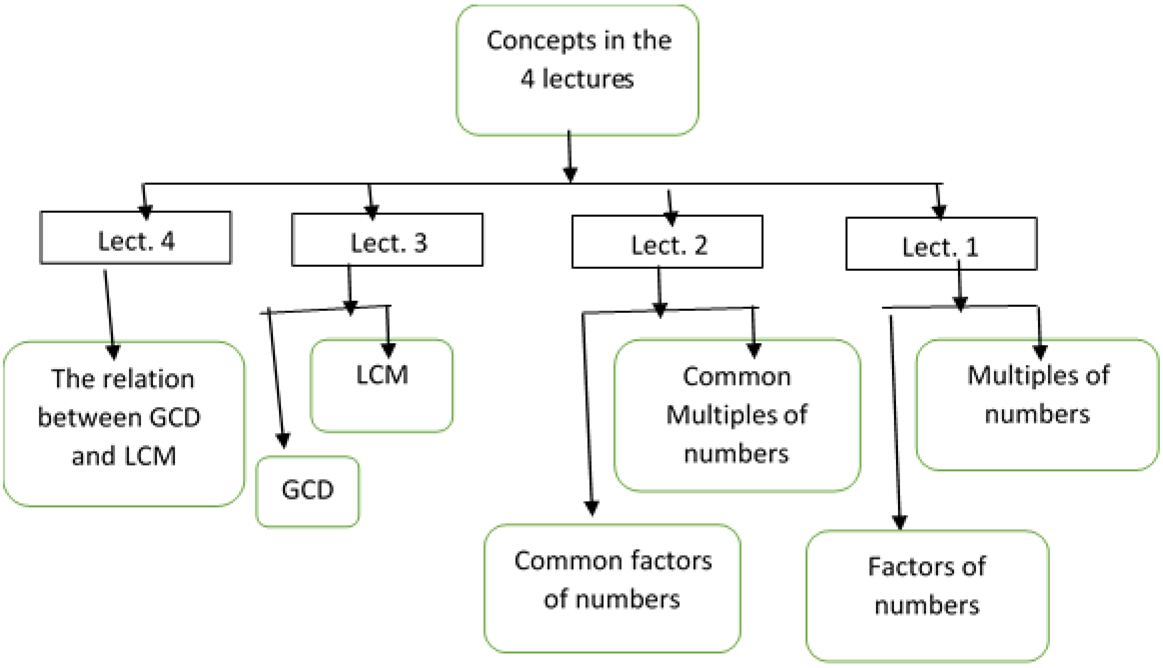


Fig.1 Concepts in the 4 lectures

The process of teaching a single concept using conceptual maps includes 3

stages:

The first aims at knowing the methods of finding the concept and what mechanism was used in the solution which is considered to be the pre comparison while the second concentrates at the use of conceptual maps in understanding and studying the different aspects guided by the researcher and finally the third stage aims at generating new ideas to improve the methods of teaching the concept using the conceptual maps and this is considered to be the after comparison.

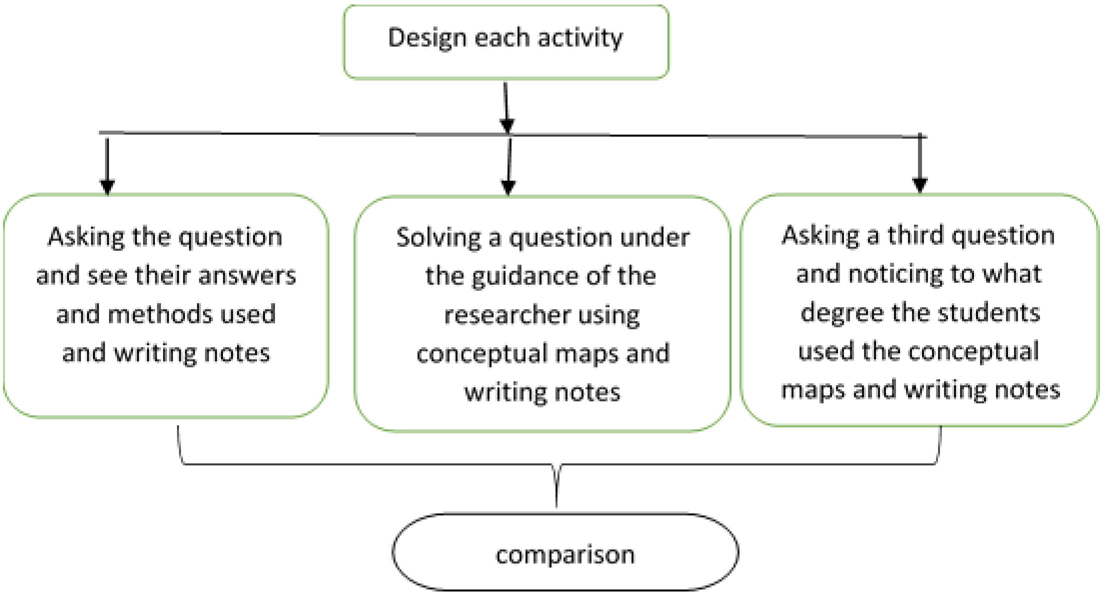


Fig.2 Activity Design

3. Data collection and analysis:

The researcher gave 4 lectures, each about 75 minutes, the content of the

lectures was as follows:

The first concentrates at presenting the first and second activities where we presented the activity and the solution methods used by the students were noted. The students had an idea about the concepts under consideration from school. Common multiples and common factors (divisors) are considered previous experiences.

Students at the beginning used the usual methods to find CM and CD. Through learning activities such as the use of team work, learning through solving problems, allowing students to share their solutions within the team discussions of their solutions on white boards, allowing students to ask questions and concentration on recalling definitions of related concepts (mean while the researcher took notes). The students when asked: what are the prime factors of 24? all groups used the tree method to factorize 24 into its prime factors as in the figure below.

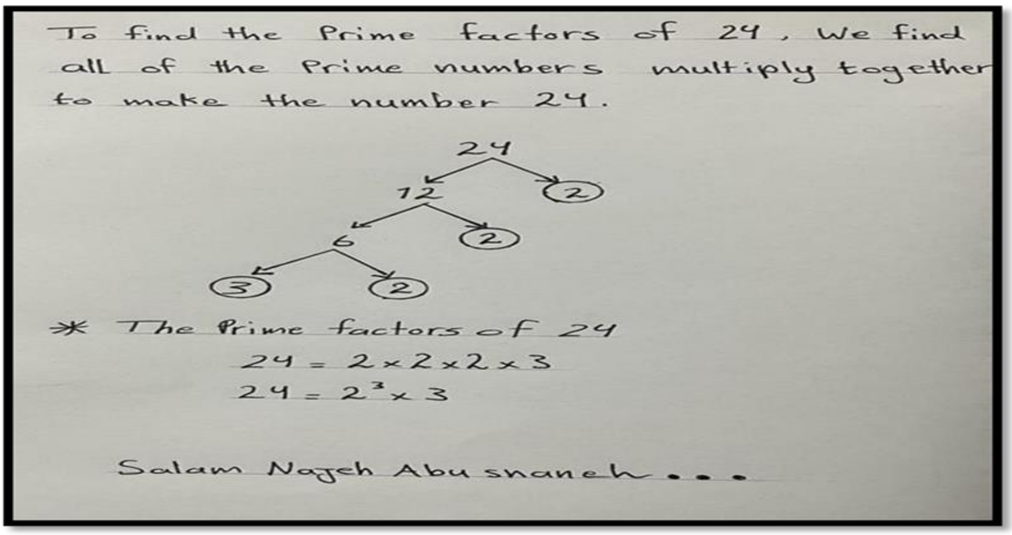
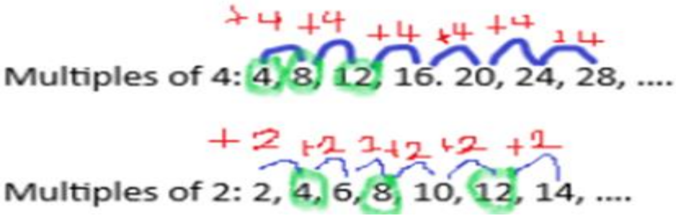


Fig.3 prime factors of 24

The second lecture contains the concepts of common factors (CF)and common multiples (CM), applying them on real life problems and finding each concept through the third and fourth discussions activities within the teams. The students found the CM of 2 and 4 where thy used multiplication and finding the first three CM as follows:



So the first 3 CM (2,4) are {4,8,12}.

When asked about CM (24,48,56), 4 teams found the factors through hand division and divisibility by 2 then 3 then 4 then 5 which is definitely not convenient to say the least. In fact, they faced some problems especially when the numbers get larger. The other two teams used the tree method for each number and then found the CD as follows:

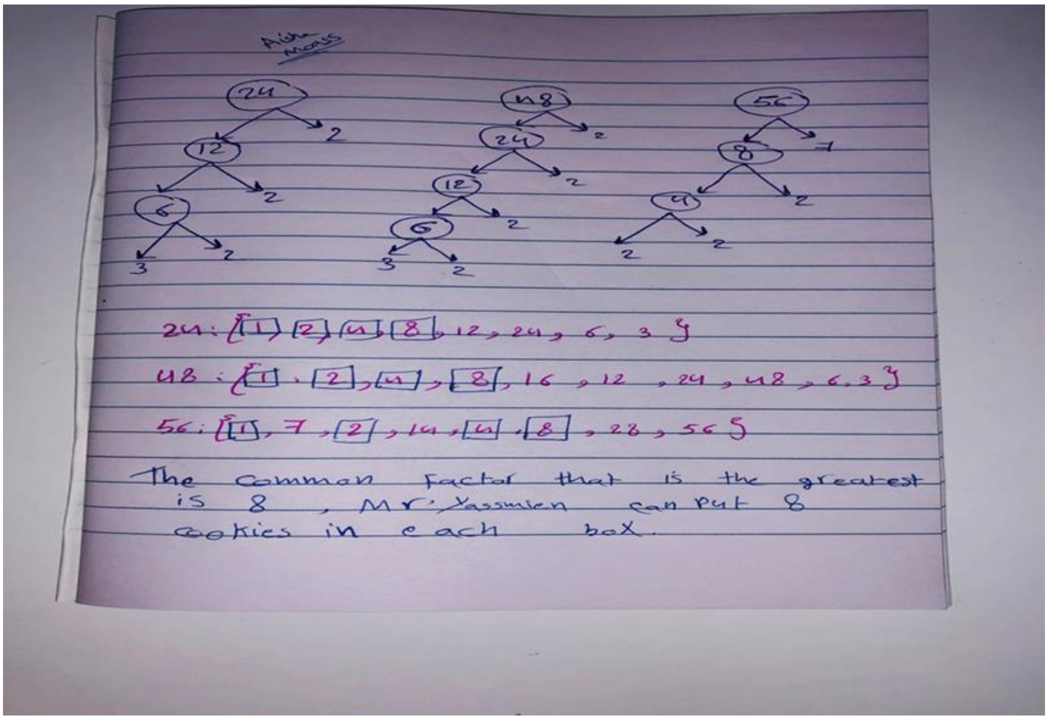


Fig.4 finding CD

The third lecture was centered at finding LCM and GCD and determines methods for finding each of them for the same numbers through discussing activities number five and six. All teams were asked to present all methods they used to find LCM and GCD.

Fieger 5 below shows the work of the 4th team to answer the following activity: Palestinian custom (apiece of Palestinian heritage) of length 350 cm, width 300 cm is to be divided into equal squares. Find the length of the largest square that can be used. The team used tables to find GCD and used sketching a rectangle indicating its length and width

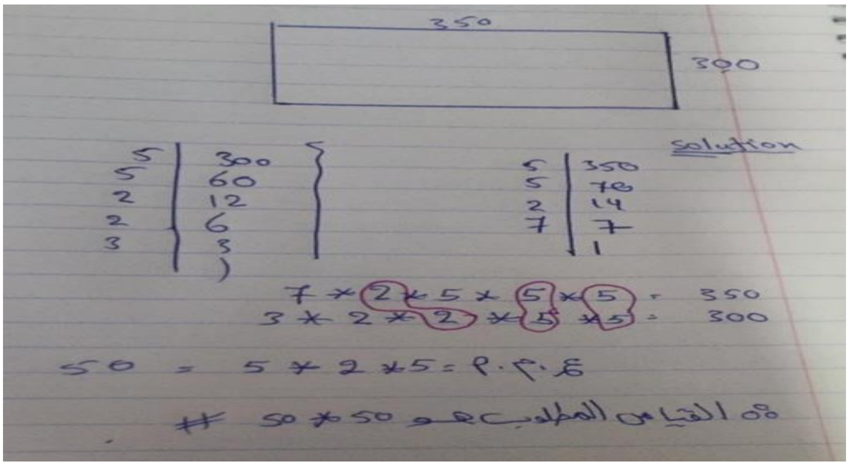


Fig.5 finding GCD using factorization into its prime factors

When the following real life problem was asked, the third team used solving problems using a plan and factorization of the numbers into their prime factors to find GCD as in the figure below. The problem is:

Sara wanted to distribute 36 kg of rice and 48 kg of sugar to a number of needy families. Find the largest amount of rice and sugar to be distributed where the amount of sugar equals the amount of rice?

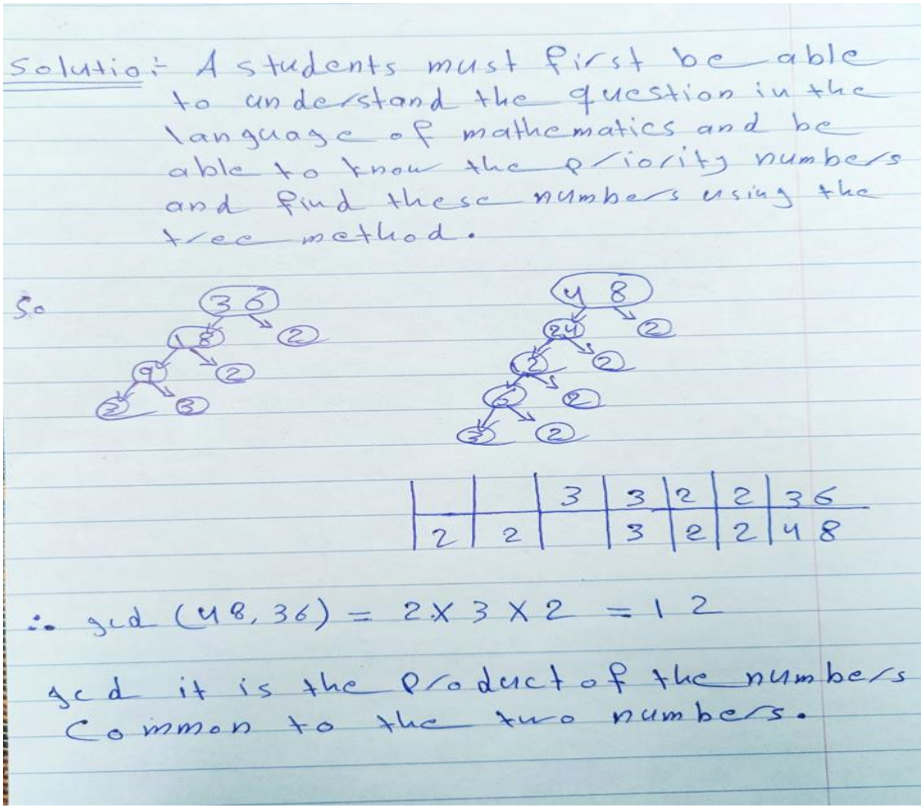


Fig.6 finding GCD by using factorization into product of prime factors using a tree

Finally, the forth lecture included the seventh interview activity in which we wanted the students to find GCD and LCM and present improved creative methods using conceptual maps as discussed earlier in the groups during the previous 3 lectures as shown below.

Sameh has 4 Wooden pieces. the lengths of which are 160cm, 0.8m, 40 cm and 2.4 m in a rows, and he wants to cut them so that they are of equal length without any of them remaining. Find the largest possible length for each piece.

Sol: ⇒ 4 →

160cm, 0.8m, 40cm and 2.4m.

1m → 100 cm.

1m → 100 cm.
0.8m → ?? ⇒ $\boxed{80\ cm}$

1m → 100cm.
2.4m → ?? ⇒ $\boxed{240\ cm}$

All measurements are in Centimeters.
{160 cm, 80cm, 40cm, 240cm}.

We use the analgesia method:-

⇒

2	160
2	80
2	40
2	20
2	10
5	5
	1

2	80
2	40
2	20
2	10
5	5
	1

2	40
2	20
2	10
5	5
	1

2	240
2	120
2	60
2	30
3	15
5	5
	1

		5	3	2	2	2	2	240
2	2	5		2	2	2	2	160
		5		2	2	2	2	80
		5		x	2	2	2	40

gcd(240, 160, 80, 40) = 2 * 2 * 2 * 5 = 8 * 5 = 40.

so, the maximum possible length of each piece that Sameh should cut the wood with is 40cm.

4. Results and discussion:

In the discussions concerning the first, third and fifth activities of the interview, students answers on finding multiples of a number, 5 teams choose the regular increasing jump or the repeated addition and only one team chooses multiplication by 1,2,3,4,..

Multiples of 8: 8, 16, 24, 32, 40, …….

Multiples of 4: 4, 8, 12, 16, 20, 24, 28,

And concluded that LCM (8.4) =8

When asked about the method used to find LCM (20,15) all teams choose tables and factorization into product of prime factors.

Number (15)	Factors	Number(20)	Factors
15	3	20	2
5	5	10	2
1		5	5
		1	

15= $3^1 \times 5^1$ 20= $2^2 \times 5^1$

LCM (15,20)= $3^1 \times 5^1 \times 2^2 = 3 \times 5 \times 4 = 60$

We observe from the above that student answers did not use conceptual maps to help finding LCM and GCD which can be referred to their previous method of learning at schools. After discussing conceptual maps and whether they help in clarifying the concepts and when students were asked to solve question number 3 in each activity, all teams used conceptual maps using the tree method to find LCM and GCD of (15,20) via factorization of numbers into their prime factors. Team work played an important role in sharing ideas also.

Interview discussions concerning the second, forth, and sixth activities showed that students used division to find the prime factors of numbers and all teams used the following method:

Factors of 8: 1, 2, 4, 8.

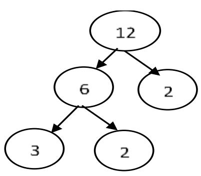
Factors of 12: 1, 2, 3, 4, 6, 12.

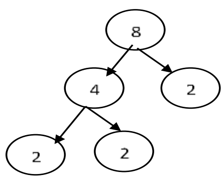
Then students were asked what method they used to find GCD (8,12), 4 teams used tables as below:

Number (8)	Factors	Number(12)	Factors
8	2	12	2
4	2	6	2
2	2	3	3
1		1	

The common prime factors between 8 and 12 are 2 and 2, the GCD (12,8)= $2 \times 2 = 4$

The remaining two teams used the tree method as follows:





12=2×2×3 and 8=2×2×2, and GCD = 2×2= 4

Team working, activities, presentations, brain storming and with the guidance of the researcher groups suggested the study of natural numbers and find possible relations between them in order to help finding LCM and GCD which lead to subdividing the natural numbers into the following 3 subsets:

1. Strange numbers: that is all relatively prime numbers such as (4,5),(7,3),… in such a case where the only common factor (divisor) is 1, we have GCD=1 and LCM= their product. For example GCD (12,7) = 1, and LCM (12,7)= 12×7 =84.

2. Containment numbers: that is when one number divides (is a factor of) the other number. In this case the GCD= the smaller number and the LCM= the bigger number. For example GCD (4,12) =4 (note:4 divides 12) and LCM (4,12)= 12.

3. Relative numbers: that is numbers that has common devisers other than 1 and they are not Containment numbers such as (6,4), (9,15),… in this case we use other known methods.

This was presented with a conceptual map as shown in the figure:

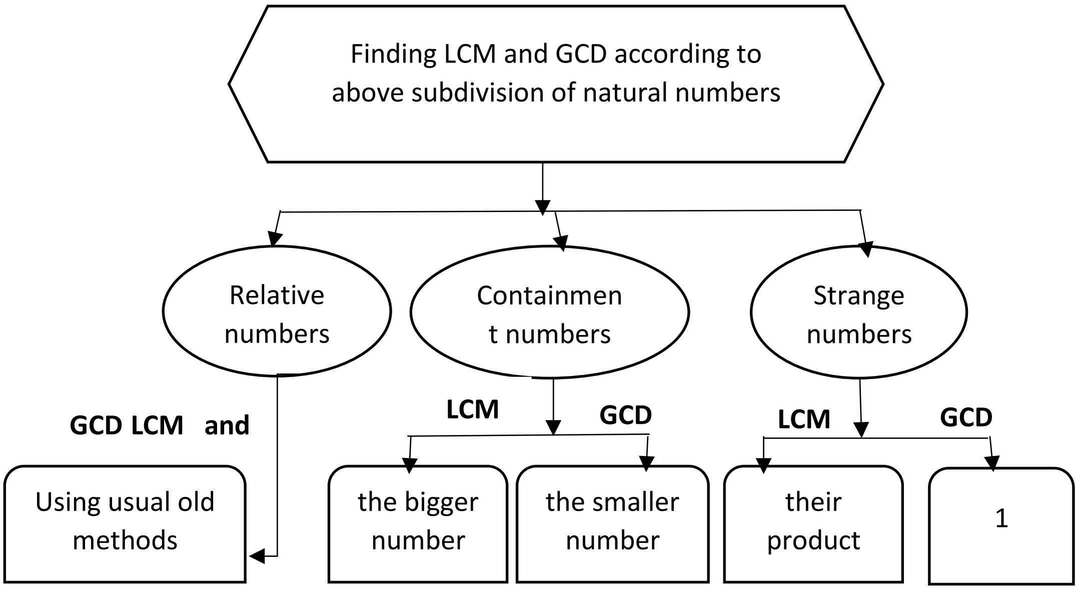


Fig.7 Finding LCM and GCD according to above subdivision of natural numbers

The groups offered a solution strategy using drama called accommodation as follows:

1. The student factorizes the two numbers (or more) into their prime factors using a tree or a table.

2. He then forms a table where he places the prime factors of each number where the first row (building) is designated for the first number prime factors and where repeated factors "lives in" different floors and the second row (building) is designated for the second number prime factors where he must place each prime factor alone in the floor on the condition that equal prime factors of both numbers lives below each other.

3. The LCM of the numbers = the product of all prime factors in all floors on the condition that we take only one prime factor from among the equal numbers living in the same floor, while the GCD = the product of one factor from equal factors and if there is no equal factors the GCD =1.

The following clarifies the drama:

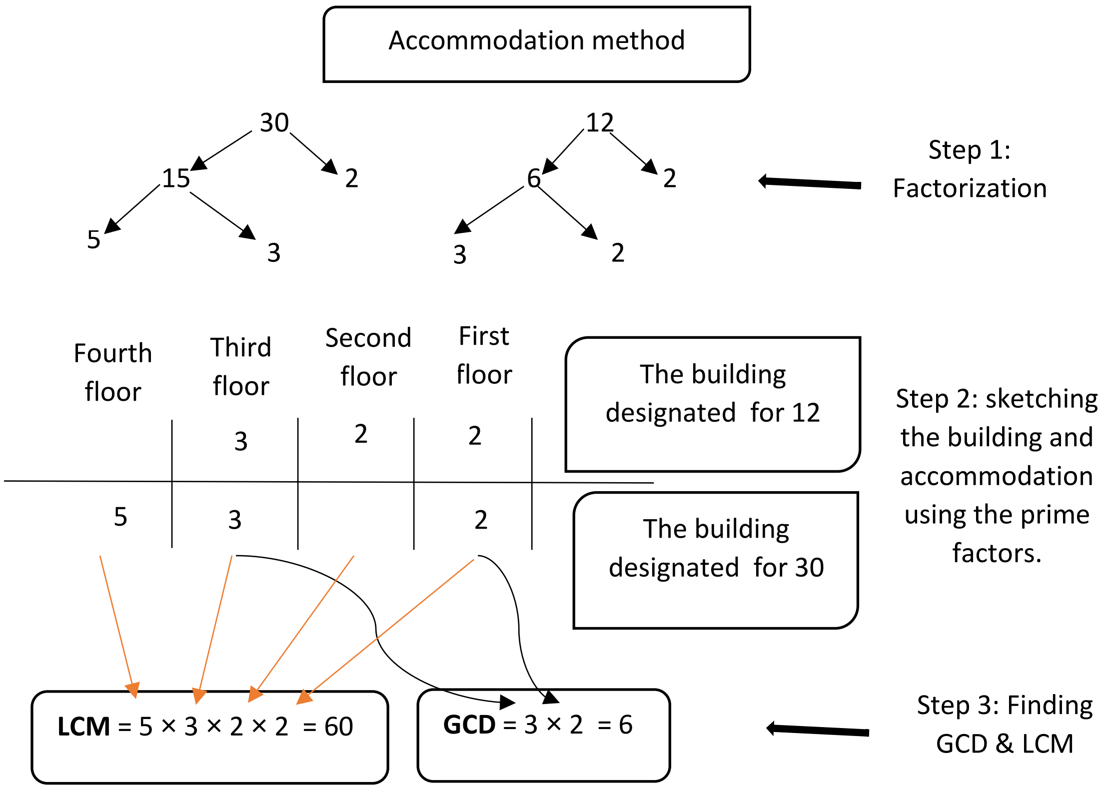


Fig 8: Accommodation method

5. Conclusions:

Focused discussion groups and interviews with each team alone and after presenting the questions in the 4 lectures students are required to present the methods they know to solve the questions and notes were taken. It was noted that some students (8 students) needed to be reminded of LCM and GCD.

In the beginning of the research the students preferable methods for finding the multiples and factors of the numbers and LCM and GCD are the usual methods. Usual division was used for finding the GCD (as it is easy for them). After introducing and discussing the conceptual maps students preferred the tree method in finding LCM and GCD and they were able to solve real life problems without support from the instructor.

When activities were presented with enhancing higher thinking skills, students were able to gain knowledge and better understanding, in addition to recalling related concepts, forming relations or bonds as conceptual maps and finally generating creative ideas especially when working in teams.

Among the important conclusions during the activities and discussions students suggested many related activities.

When conceptual maps were introduced by the instructor (researcher) to the different groups and the discussion groups, he noted a high range of concentration, deep understanding of the concepts in question, their ability to work within teams, discussing happily the different scientific aspects related to the concepts in question and being involved in deep thinking using analyzation and comparison. The groups then proposed subdividing the natural numbers into the three mentioned subsets to make learning and educating easier, in fact some students mentioned that the idea of subdividing helped them in studying some other concepts such as finding common denominator in fractions.

Using simple understandable strategies makes it much easier to learn and understand mathematical concepts, some students also insist on the importance of using drama, acting and story telling to help students understand the concept.

A sample of the participants (students) opinions is listed below:

A student from the sixth group said "dividing concepts and things according to common properties makes it easier to learn and teach the concepts" another student from the same team said " this method helped me in correlating other concepts and applying the above mentioned subdivision of the natural numbers to them as the conceptual maps method is built upon finding relations between things". A member of the fourth team emphasized the above and said "I benefited from subdividing the natural numbers in finding the common denominator in dealing with fractions". Finally a member of the third team said "I was able to use the accommodation method and I am going to teach it to my future students".

The researcher recommends that one should try to do a similar research with school students (pre university) to see whether they also will be happy to learn and use the above mentioned strategy and conceptual maps.

Conflict of Interests Statement I certify that I have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; membership, employment, consultancies, stock ownership, or other equity interest), or nonfinancial interest in the subject matter or materials discussed in this manuscript.

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