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"The Relationship Between Geometric Reasoning and the Learning of the Derivative in the Subject of Differential Calculus in University Students in the Period 2023-II"

Marisol Paola Delgado Baltazar[1], Ruben Dario Mendoza Arenas[2], Mónica Beatriz La Chira Loli[3], Josefina Arimatea García Cruz[4], Jenny María Ruiz Salazar[5], Ana Maria Holgado Quispe[6], Raquel Leonor Atoche Wong[7], Jesselle Roxana Rodas García[8], Ana María Yamunaqué Morales[9]

Abstract

The research aimed to determine the relationship between geometric reasoning and the learning of the derivative in Differential Calculus. It employed a quantitative approach, specifically a basic type of research with a non-experimental design and a cross-sectional correlational method. The sample consisted of 80 students selected through a census approach. The survey technique was employed, and the instruments used were a questionnaire and a questionnaire test.

The results indicated that 73.7% of the participants were at a medium level of the geometric algorithm, while 47.5% were at a regular level of learning the derivative. A statistical analysis using Spearman's rho revealed a correlation coefficient of 0.769, with a significance level of 0.000 (p<0.05). This suggests a direct, high, and significant relationship between the geometric algorithm and the learning of the derivative in the context of Differential Calculus in university students in the period 2023-II.

In conclusion, the research establishes a strong connection between the proficiency in geometric reasoning, as measured by the geometric algorithm, and the level of learning of the derivative in the specified calculus course. These findings contribute to understanding the interplay between geometric reasoning and mathematical learning in the context of Differential Calculus.

Keywords: *Geometric reasoning, learning, derivative, calculus.*

INTRODUCTION

In this article, has been meticulously crafted, emphasizing the significance of employing theoretical approaches. These approaches facilitate the utilization of theoretical models for solving exercises and derivative problems, notably the Van Hiele model. Additionally, the study seeks to establish a correlation between geometric algorithms and the acquisition of derivative knowledge.

[1] Universidad Nacional del Callao
[2] Universidad Nacional del Callao
[3] Universidad Autónoma del Perú
[4] Universidad Nacional de Educación Enrique Guzmán y Valle
[5] Universidad Nacional Federico Villarreal
[6] Universidad Tecnológica del Perú
[7] Universidad Nacional Federico Villarreal
[8] Universidad Cesar Vallejo
[9] Universidad Nacional del Callao

This research addresses a common issue prevalent in the university setting: the inadequacy in geometric reasoning. This deficiency is rooted in the Van Hiele Model (1957), as referenced by Vásquez and Gamboa (2013), forming the contextual framework for the study. A teaching and learning model that provides the opportunity to identify forms of geometric reasoning is the Van Hiele Model. Developed by Dutch educators Pierre and Dina Van Hiele in the 1950s, this model focuses on understanding how students reason and comprehend geometry at different levels of complexity.

Description of the problematic reality

In the mid-20th century, the global mathematical community expressed concerns about finding effective teaching methods to enhance the clarity of understanding in various mathematical topics. There was an attempt to move away from using elementary geometry due to perceived difficulties in providing a solid foundation for it.

In more recent times, Labra and Vanegas (2022) highlighted the challenges students face in grasping geometric concepts and processes. A critical issue identified is the low levels of geometric reasoning among students. Yi et al. (2020) suggested that addressing the development of geometric reasoning involves designing, implementing, and evaluating didactic sequences based on the Van Hiele (1957) model. This model simplifies the representation of didactic tasks, focusing on the development and description of the geometric algorithm.

After the pandemic, university-level teachers continue to express concerns about instilling analytical understanding of mathematical ideas and concepts in students. However, not all instructors incorporate a geometric description when presenting definitions, theorems, and propositions in basic courses such as Differential Calculus in engineering and science programs. There seems to be a lack of awareness regarding the crucial impact of the geometric dimension on the comprehension of various mathematical topics.

This research endeavors to offer solutions to learning challenges faced by engineering students, a significant portion of whom, having completed their Regular Basic Education, lack a solid geometric understanding of certain mathematical subjects. The objective is to imbue learning with meaning by emphasizing the influence of the geometric aspect on their conceptualization of theoretical content. This, in turn, aims to facilitate a smoother transition to the practical application of these concepts. To address these objectives, the following problems are formulated:

General problem.

What is the relationship between geometric reasoning and learning the derivative in the subject of differential calculus in university students in the period 2023-II?

Specific problems.

1. What relationship exists between the visualization dimension of geometric reasoning and learning of the derivative in the subject of differential calculus in university students in the period 2023-II?

2. What relationship exists between the analysis dimension of geometric reasoning and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II?

General objective.

Determine the relationship between geometric reasoning and learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

Specific objectives.

1. Establish the relationship that exists between the visualization dimension of geometric reasoning and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

2. Establish the relationship that exists between the analysis dimension and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

Theoretical bases

The theoretical bases for addressing geometric reasoning are primarily grounded in the Van Hiele model. This didactic model is a result of a simplified representation of the teaching task, encompassing the development and description of geometric reasoning, along with the didactic proposal for teaching and learning geometry. According to Jaime and Gutiérrez (1990), the Van Hiele model enables the identification of different levels of reasoning in students within a specific thematic field in geometry. They emphasize that students can only comprehend concepts corresponding to their current level of geometric reasoning. Thus, a mathematical concept cannot be effectively learned if it is not taught in alignment with the student's existing geometric reasoning level. To address this, it becomes essential to develop and patiently wait for students to reach the required level of reasoning. However, Vásquez and Gamboa (2013) posit that it is possible to expedite this process through appropriate geometry teaching methods, facilitating the student's development and progression towards more adept reasoning. They characterize the Van Hiele model as a teaching and learning framework that provides the opportunity to identify various forms of geometric reasoning.

The Ontosemiotic Approach (EOS) to mathematical knowledge and instruction, as proposed by Godino, Botanero, and Font in 2007, introduces a framework that incorporates categories of objects to distinguish between mental entities (personal objects) and institutional ones (social or cultural). In this approach, mathematics is viewed from three interrelated perspectives: as a problem-solving activity (either extra or intra-mathematical), as a language, and as a socially shared conceptual system.

The EOS framework offers a complementary perspective to identify the diversity of knowledge involved in visualization and spatial reasoning tasks. These tasks often pose challenges for students, and the Ontosemiotic Approach helps explain the difficulties by considering the intricate interplay of personal and cultural elements in mathematical understanding.

Theories related to learning the derivative

The Theory of Didactic Situations and the Connectivist Theory offer distinct perspectives on learning, each with its unique approach:

Theory of Didactic Situations:

Inspiration: Rooted in the constructivist thinking of Piaget (1983), emphasizing learning through adaptations to the environment.

Brousseau's Conception: Brousseau (2007) diverges from Piaget by defining the environment as shaped by disciplinary knowledge and mathematical situations that students must learn.

Systemic Approach: This theory takes a systemic approach to teaching and learning, examining processes within the didactic triangle formed by the teacher, students, and the knowledge in play. It explores the relationships among these three components.

Analysis of Knowledge Transmission: The Theory of Didactic Situations analyzes the transmission and acquisition of mathematical knowledge, aiming to discover, interpret, and create situations adapted to both the knowledge and the students.

Connectivist Theory:

Origin and Digital Age Emphasis: Proposed by Siemens (2004), the Connectivist Theory is considered the learning theory for the digital age, highlighting the fundamental role of technology in the teaching process.

Foundation in Chaos and Complexity: Grounded in the theories of chaos, complexity, and self-organization, this approach views learning as the formation of a network of connections. Human beings are considered the starting point for this network.

Technology's Role: Connectivism underscores the importance of technology in facilitating connections and knowledge acquisition within a networked environment.

Both theories contribute valuable perspectives to understanding how students learn, with the Theory of Didactic Situations focusing on the systemic nature of teaching and learning mathematics, and the Connectivist Theory emphasizing the role of technology and the formation of networked connections in the learning process.

Conceptual framework

Definitions of geometric reasoning

The concept of spatial reasoning, as highlighted by various scholars, encompasses a set of cognitive processes related to the construction and manipulation of mental representations of space, objects within that space, their relationships, transformations, and various translations or material representations. Here are the perspectives from:

Torres (2022), Spatial reasoning involves cognitive processes that lead to the construction and manipulation of mental representations of space and objects within that space. This includes understanding relationships between objects, their transformations, and various translations or material representations.

Chávez (2019), Spatial reasoning is characterized as the ability enabling students to comprehend diverse mathematical concepts and engage in profound introspection regarding real problems. Additionally, it contributes to a better command of movement and space. This perspective highlights the integration of spatial reasoning with mathematical understanding, emphasizing its role in problem-solving and real-world application. .

Dimensions of geometric reasoning.

In the course of the study, the researchers considered the following dimensions proposed by Van Hiele, as cited by Cabello (2013):

Visualization Dimension:

Description of Objects: The students describe objects based on their physical appearance, lacking explicit differentiation of their components or mathematical properties.

Perspective of Gamboa and Vargas (2013): Students do not differentiate the parts of geometric figures; instead, they perceive shapes as wholes.

Contributions from Llanos et al. (2016): This dimension implies i) recognizing the conservation of the size and shape of figures, ii) the ability to identify and perform movements using auxiliary materials, iii) employing strongly visual properties to identify symmetry between two symmetrical figures, including changes in orientation, iv) using elementary vocabulary related to isometries such as plane, symmetry, symmetry axis, coordinate, module, among others.

In summary, the visualization dimension in the study assesses how students describe objects based on their physical appearance, emphasizing the recognition of conservation, ability to perform movements, utilization of visual properties for symmetry identification, and the use of elementary vocabulary associated with isometries.

Analisis Dimension: In the study, it was observed that the participants demonstrated the ability to recognize the components and mathematical properties of an object or concept. However, their approach to establishing relationships between objects and components was experimental in nature. The students were found to struggle in making formal descriptions, relying more on generalizations. Here are additional insights from Gamboa and Vargas (2013) and Llanois et al. (2016):

Gamboa and Vargas (2013): Students were noted to recognize shapes but struggled to establish relationships between properties of different families of figures. Their understanding seemed limited to individual shapes without a broader connection between properties.

Llanois et al. (2016): Considering movements through their elements allows for intentional and explicit use of the elements characterizing each isometry. The ability to determine the elements characterizing a specific isometry is highlighted, particularly in particular situations that do not require resorting to property relations of the third level. Students demonstrated the capacity to discover and use new properties of isometries based on verification in specific cases. Specific tasks included working with the definitions of movements in recognition tasks, applying movements directly, and applying compositions of isometries. The participants showcased an understanding of the non-commutativity of the composition of reflections. Mathematical notation and vocabulary associated with isometries, such as perpendicularity, bisector, module, direction, sense, among others, were effectively used by the students

Hypothesis

General hypothesis

There is a significant relationship between the geometric reasoning and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

Especific hypotheses

1. There is a significant relationship between the visualization dimension of geometric reasoning and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

2. There is a significant relationship between the analysis dimension of geometric reasoning and the learning of the derivative in the subject of differential calculus in university students in the period 2023-II.

Operationalization of variables

Variable 1: Geometric Reasoning

The definition provided by Torres (2022) succinctly describes spatial reasoning as a complex set of cognitive processes. It involves the construction and manipulation of mental representations related to space objects, encompassing their relationships, transformations, and translations. This cognitive activity is not limited to a single dimension but contemplates the actions of the subject across all dimensions and spatial relationships. The subject engages with objects in space in various ways, developing multiple representations. Coordination between these representations facilitates conceptual approaches, ultimately leading to the creation and manipulation of new mental representations. This comprehensive understanding highlights the dynamic and multifaceted nature of spatial reasoning.

Variable 2: Learning of the derivative

Salvatierra et al. (2021), states that it is the sequence of concepts that ranges from function limits to derivative calculations that plays a crucial role in the preparation of students, providing them with the necessary tools to address relevant problems and applications in their professional training.

Methodological design

Approach Quantitative: This approach was used to collect data numerically, allowing the testing of hypotheses through the hypothetico-deductive methodology.

Kind of investigation: Noun, according to Sánchez et al. (2018), focused on the increase in theoretical-scientific knowledge, its principles and laws, without having a practical nature.

Research design Correlational: According to Ñaupas et al. (2018), the correlation between the variables geometric reasoning and derivative learning was measured.

Non-experimental: No variables were manipulated.

Cross-sectional: Data collection was carried out at a single moment.

Research Method Hypothetical-Deductive: It allowed the formulation of hypotheses and, with the results of the instruments, the verification of the hypotheses and deduction of conclusions.

Population and Sample:

Population: 80 students enrolled in the Calculus I subject of Semester 2023-II of the Faculty of Industrial and Systems Engineering of the National University of Callao.

Sample Determination: It also consisted of 80 students enrolled in the Calculus I subject of Semester 2023-II of the Faculty of Industrial and Systems Engineering of the National University of Callao.

This methodological design allows obtaining quantitative data that is analyzed statistically to evaluate the correlation between geometric reasoning and learning of the derivative in students of the Differential Calculus subject..

Techniques and instruments for collecting information

Instrumentation:

Technique Survey: Ander-Egg (2011) suggests that a survey allows for the direct collection of data by formulating questions that are answered by members of the sample.

Instruments: Questionnaire: According to Ávila et al. (2020), a questionnaire involves the formulation of written questions related to hypotheses, variables, and indicators. Its objective is to collect, process, and analyze data to verify hypotheses.

Calculus I Test: This test was also utilized as part of the data collection.

Reliability: According to Hernández and Mendoza (2018), reliability is related to the internal consistency of the instrument and its ability to establish a replicable average as part of the evaluation process.

Table of Instrument Reliability:

[Details of the reliability of the instruments, including internal consistency and replicability, were not provided. If you have specific data or reliability coefficients, they can be included in this section.]

The utilization of a survey and a questionnaire, along with a Calculus I test, contributes to a comprehensive data collection approach. Ensuring the reliability of these instruments is essential for the validity and trustworthiness of the study's findings.

Table 1 Confiability

Nº	Instruments	Cronbach's alpha	KR20	Nº of ítems
1	Geometric Reasoning Quiz	0,844		20
2	Differential Calculus Test		0.840	20

Validation

Validation in the research process is crucial to ensure that the measurements accurately reflect the theoretical constructs. According to Hernández and Mendoza (2018), validity establishes the importance of the link between the construct and the indicator, ensuring that the theoretical idea is faithfully reflected in the measurement. The validation process involves seeking the well-informed opinions of experts in research methodology who are recognized in the field and can offer details, evidence, verdicts, and assessments. The expert opinions contribute to establishing the validity of the instruments used for data collection.

Data Analysis and Processing: The research followed a systematic set of procedures:

Identification of Classrooms: The classrooms that would be part of the population were identified.

Authorization: Authorization to carry out the research was sought.

Pilot Test: A pilot test was conducted to determine the reliability of the instruments.

Expert Evaluation: The instruments were presented to experts for evaluation to ensure their validity and reliability.

Questionnaire Application: Valid and reliable validation questionnaires were applied to the members of the sample.

Data Collection: Data were collected from the respondents.

Results Preparation: Based on the collected data, the results were prepared.

Data Processing and Analysis:

Data were processed and analyzed using both descriptive and inferential methods.

Descriptive Results: Descriptive results were presented using bars and tables to provide a clear overview of the findings.

Inferential Results: Inferential results were obtained by applying statistical tests, specifically the Kolmogorov-Smirnov normality test, to assess the normal distribution of the data.

The systematic approach to data collection, processing, and analysis, including validation through expert evaluation, contributes to the reliability and credibility of the research outcomes.

RESULTS

Descriptive results of Geometric reasoning

Table 2: Distribution of levels of the geometric reasoning variable

Levels	Frequencies	Percentage
Low	17	21.3
Medium	59	73.7
High	4	5.0

Total	80	100,0

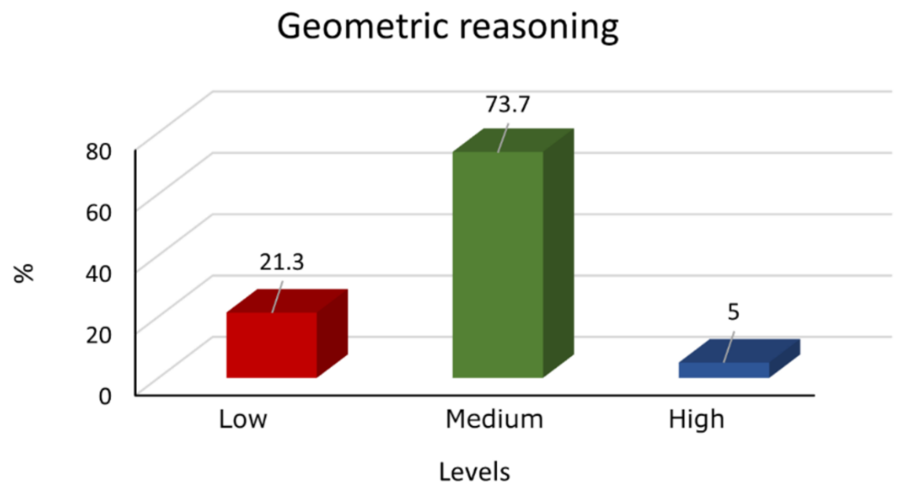

Figure 1: Levels of the geometric reasoning variable

In table 2 and figure 1, it is observed that, with respect to the geometric reasoning variable, 73.7% of students presented a medium level, 21.4% a low level and 5.0% a high level; Given this, it is stated that the majority of the students of the Calculus I Subject of Semester 2023-B of the Faculty of Industrial Engineering and Systems of the National University of Callao presented an average level.

Table 3: Distribution of levels of the dimensions of geometric reasoning

Levels	Display		Analysis	
	fi	%	fi	%
Low	18	22.5	19	23.7
Medium	58	72.5	58	72.5
High	4	5.0	3	3.8

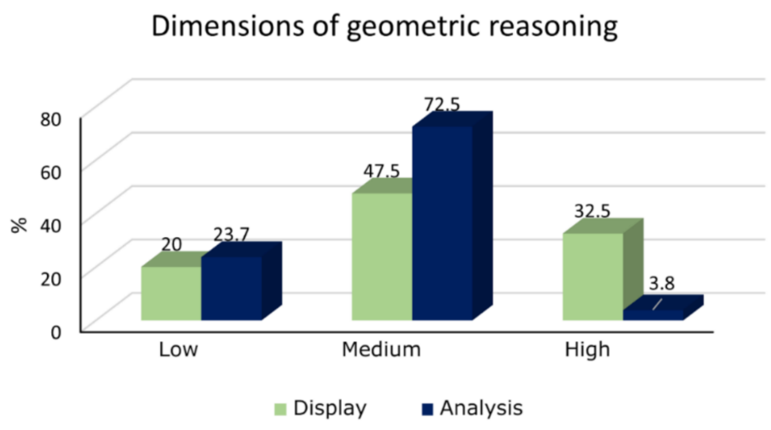

Figure 2: Levels of the dimensions of geometric reasoning

In table 3 and figure 2, it can be seen that, with respect to the dimensions of the geometric reasoning variable; In the visualization dimension, 7.5% presented a medium level, 32.5% a high level and 20.0% a low level; In the analysis dimension, 72.5% presented a medium level, 23.7 a low level and 3.8% at a high level.

Learning derivate

Table 4: Distribution of levels of the derivative learning variable

Levels	Frequencies	Percentage
Bad	16	20.0
Regular	38	47.5
Good	26	32.5
Total	80	100,0

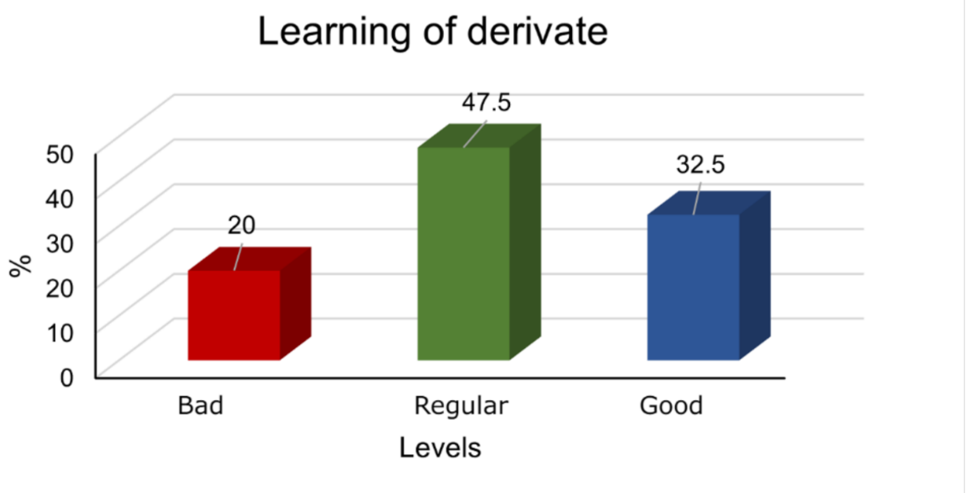

Figure 3: Levels of the derivative learning variable

The results of table 4 and figure 3 allow us to observe that, with reference to the derivative learning variable, 47.5% presented a regular level, 32.5% a good level and 20.0% a bad level. level; Given this, it can be stated that there is a prevalence of the regular level in learning the derivative in the students of the Calculus I Subject of Semester 2023-II of the Faculty of Industrial Engineering and Systems of the National University of Callao.

Table 5: Distribution of levels of the dimensions of learning the derivative

Levels	Resolution of exercises		Concept Management		Model problems	
	fi	%	fi	%	fi	%
Bad	15	18.8	19	23.8	23	28.7
Regular	49	61.2	42	52.4	57	71.3
Good	16	20.0	19	23.8	0	0

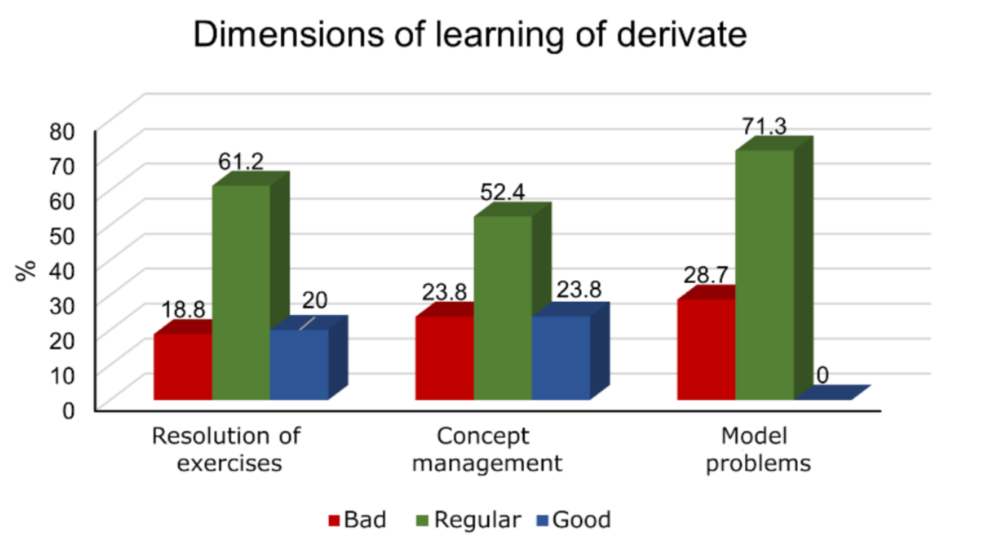

Figure 4: Levels of the dimensions of learning the derivative

Modeling problem Table 5 and Figure 4 show that, with respect to the dimensions of learning the derivative; In the exercise resolution dimension, 61.2% presented a regular level, 20.0% a good level and 18.8% a bad level; In the concept management dimension, 52.4% presented a regular level, 23.8% a good level and another 23.8% a low level; and in the dimension, modeling problems, 71.3% presented a regular level, and 28.7% a bad level.

CONCLUSIONS:

Overall Relationship:

1. Geometric Reasoning and Learning of the Derivative: The study found a significant and positive relationship between geometric reasoning and the learning of the derivative in the Calculus I Subject at FIIS-UNAC, Callao 2023. The Spearman's rho coefficient was 0.729, and the significance level was 0.000, indicating a strong correlation.

Specific Dimensions of Geometric Reasoning:

2. Visualization Dimension:

Relationship with Learning of the Derivative: There is a significant and positive relationship between the visualization dimension of geometric reasoning and the learning of the derivative in Calculus I at FIIS-UNAC, Callao 2023. The Spearman's rho coefficient was 0.610, and the significance level was 0.000, indicating a substantial correlation.

3.Analysis Dimension:

Relationship with Learning of the Derivative: A significant and positive relationship exists between the analysis dimension of geometric reasoning and the learning of the derivative in Calculus I at FIIS-UNAC, Callao 2023. The Spearman's rho coefficient was 0.669, and the significance level was 0.000, suggesting a strong correlation.

These conclusions provide valuable insights into the specific dimensions of geometric reasoning and their impact on the learning of the derivative. The findings emphasize the importance of considering visualization and analysis dimensions in understanding the relationship between geometric reasoning and mathematical learning in the context of Differential Calculus.

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