

Archimedes Screw to Generate Hydroelectricity at Moderate Flow, Tucumachay-Huancavelica

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Abstract

The Archimedes screw has been used as a pump since ancient times and more recently has been used to generate hydroelectricity in power plants up to about 200 kW, thus representing an optimal way to generate hydroelectric power in places with very low altitude and moderate flow. The objective of this study is to generate hydroelectricity at moderate flow rate by using the Archimedes screw to supply lighting. An analysis of the behavior of a prototype of an optimizing mechanism of kinetic energy at low flow for electric power generation was carried out at the South American Camelids Research and Development Center "Lachocc", by physical and non-conventional mechanisms, without the intervention of other types of energy such as photovoltaic electricity and other conventional ones, based on the scientific study, carrying out field and laboratory tests taking advantage of the natural fall source located in the same area. It is highlighted among the results that the theoretical electric power was increased from 9.84 to 12 watts, a very important value because the study considered prototypes of 10 cm of blade diameter in the test prototype. It is concluded that, for the development of an optimal Archimedes hydro-generator peak, the best combination of factors that generate greater mechanical power involves wide blades, the turbine axis in a horizontal position, and the fixed structure, because if the axis is in movement, turbulences are generated that do not allow obtaining greater power.

Keywords: Archimedean screw, moderate flow, hydroelectricity.

INTRODUCTION

The demand for energy consumption increases as the population increases, so for the overall development of a region, electricity is of paramount importance; therefore, the need for energy-to-electricity conversion is of paramount importance for sustainable development. Hydroelectric projects are divided into two categories according to their size: large and small; small hydroelectric projects with capacities ranging from 10 MW to 50 MW are classified differently in different countries since a technological change for repowering and conservation is the most suitable way for optimum productivity (Singh et

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al., 2023; Hinojosa et al., 2021). The community of Lachocc in Huancavelica has enormous water potential, and the Archimedes Screw Turbine (AST) is a potential tool for generating energy from the flow of a river (Zamani et al., 2023), The AST rotates as the water flows through it, turning the primary motor of the generator connected to it, and they operate with a low head of 0.8 to 10 m and a relatively lower flow rate, compared to other turbines, being more cost-effective and highly efficient (Gogoi et al., 2018).

AST is a new way of generating electricity that has been implemented in different countries, which allows for improving the electrification scenario in rural areas (Kuo et al., 2023), especially where there is continuous flow of river or canal, and at low cost. It is quite feasible and profitable to use existing water mills for electricity generation employing the Archimedes screw turbine (Kashyap et al., 2022) in the Peruvian Andean zone, which has grain mills. Also, the increasing demand for electricity and the scarcity of fossil fuels due to their decrease in recent years makes the use of AST an opportune solution for the generation of electricity (Maulana y Putra, 2019).

ASTs have been used since ancient times for pumping water, providing knowledge about the effects of the level deposited at the inlet and outlet on pump efficiency; however, current research has implemented this technology for electric power generation (Lyons et al., 2020).

Currently, ASTs are used in multiple projects, due to their low operating cost, providing solutions to the problems of electrification in various regions with a low environmental impact (Simmons and Lubitz, 2017), in addition to being able to be implemented in rural areas that have a low flow to be considered as a hydroelectric power plant on a small scale, generating a good economic advantage (Yoosef & Lubitz, 2020).

The development of an AST with an outer diameter (142mm), flow rate (1.2 l/s), and height (0.25 m), produces 1.4W of peak power thanks to the analysis of the varied data of the experiment and the performance tests of the prototype, show potential to be installed in places with low water load (Nuramal et al., 2017), and is of great help in obtaining small-scale hydroelectric power, exploiting resources such as creeks and streams with support from previous studies about its optimal design (Rohmer et al., 2015).

Large-scale electricity production can cause environmental pollution and influence climate change, in addition to being very costly. However, if properly implemented, sufficient electricity can be produced to light a small household at night (Raza et al., 2013). In the dry season, the country faces serious inconveniences due to lack of electricity, resorting to energy mainly from hydrocarbons as a primary energy source, so to avoid the use of electricity from hydrocarbons, several projects are being proposed for the implementation of mini power plants throughout the country, which is why the objective of this study is to generate hydroelectricity in moderate flow through the use of the Archimedes screw to reduce costs.

MATERIALS AND METHODS

Location

This research was carried out at the South American Camelid Research and Development Center “Lachocc” located at km 32 of the Huancavelica-Pisco highway, belonging to the National University of Huancavelica, in a humid subalpine-subtropical zone, between 4,225 and 4,850 m.a.s.l., with water sources that are not being used very much.

2n factorial design method

The main blocks for the construction of an electric generator employing an Archimedes screw are the turbine, the connection shaft, the speed multiplier box, and the electric generator. The aim was to obtain measurement parameters for the electrical power

generated by the ASTs to identify the most efficient one.

For this purpose, 3 study parameters were taken as a reference:

- A: Number of inputs (A1: 1 input; A2: 2 inputs)
- B: Shaft diameter (B1: Thick shaft; B2: Thin shaft)
- C: Shaft position (C1: Shaft at 30°; C2: Shaft at 20°) $2^3 = 8$

Table 1 shows the 8 possible models, making the respective combinations.

Table 1 Obtaining 8 possible models by making the respective combinations

N°	Combination	Treatment levels		
	Treatments	A	B	C
1	A1B1C1	1 input	Thick shaft	Axis at 30°.
2	A2B1C1	2 inputs	Thick shaft	Axis at 30°.
3	A1B2C1	1 input	Thin shaft	Axis at 30°.
4	A2B2C1	2 inputs	Thin shaft	Axis at 30°.
5	A1B1C2	1 input	Thick shaft	Axis at 20°.
6	A2B1C2	2 inputs	Thick shaft	Axis at 20°.
7	A1B2C2	1 input	Thin shaft	Axis at 20°.
8	A2B2C2	2 inputs	Thin shaft	Axis at 20°.

The 8 combinations are modeled in Solid Works software (Fig. 1).

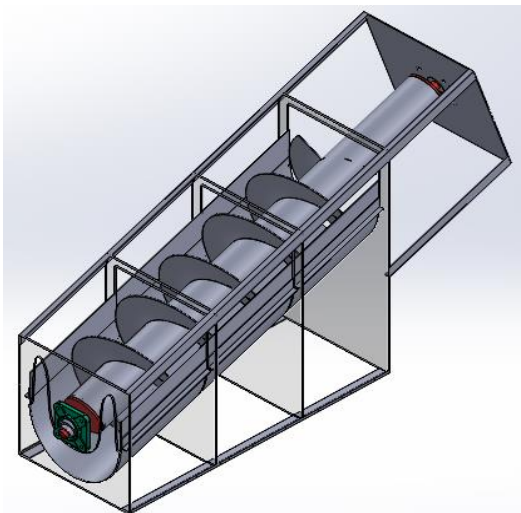


Fig. 1. Assembly of Model 5

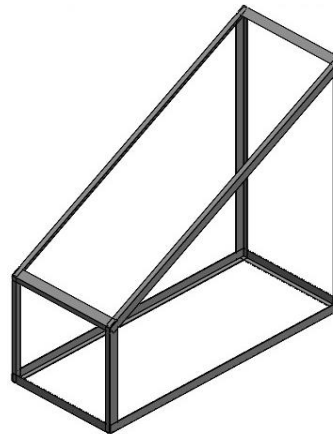


Fig. 2. Design of aluminum profiles

Materials

Aluminum profile: Aluminum profiles (Fig. 2) are used for the AST support structure due to their high corrosion resistance since they will be in contact with water. Its ease of molding and high toughness allow the reinforcement to be compact and easy to assemble.

Aluminum tube: The aluminum tube (Fig. 3) is implemented as the central axis of the structure, which supports the centripetal forces originated by the propellers or blades, in addition to transferring the torque to the generator.

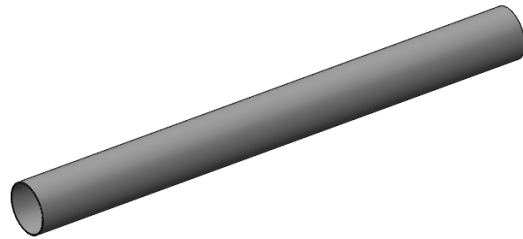


Fig. 3. Aluminum tube design

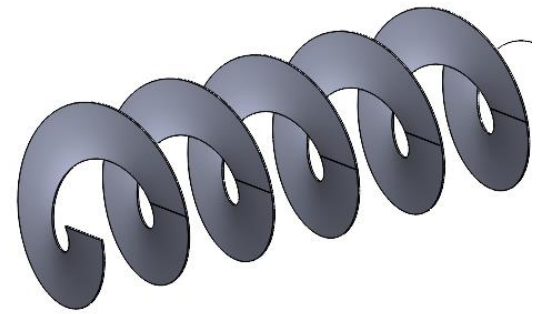


Fig. 4. Blade design

Tin plate: Used for the design of the blades, which has a high flexibility property, which facilitates its elaboration, since the blades have a complex structure (Fig. 4).

Acrylic: It is implemented in the separations of the structure; as support of the channel (Fig. 5).

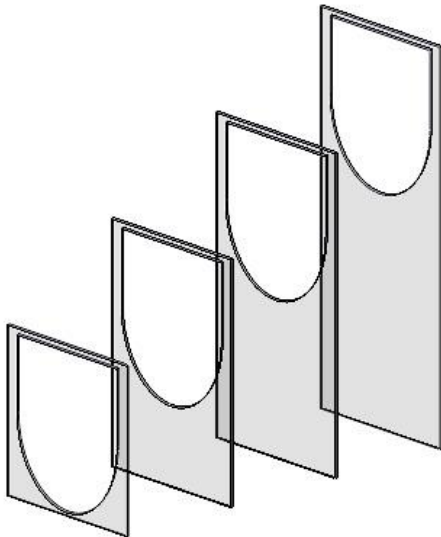


Fig. 5. Aluminum bracket design

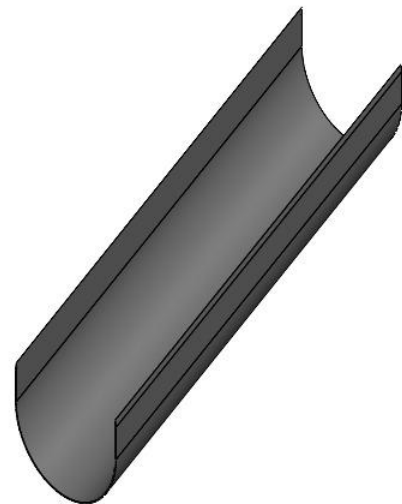


Fig. 6. Design of PVC pipe as a channel

PVC pipe: 40 cm long, used for the channel of the structure, which will allow the passage of water (Fig. 6).

Chumacera: To adapt the central axis to different degrees of inclination, it is advisable to use the bearing (Figure 7), due to its ease of positioning.

Bearing: The bearing (Figure 8) allows the rotation of the central shaft, avoiding high friction and preventing slippage of external components.

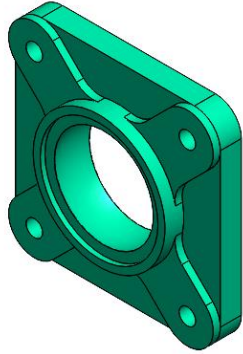


Fig. 7. Design of the chumacera

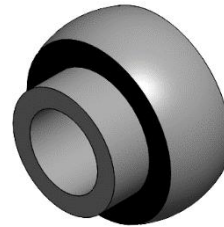


Fig. 8. Bearing design

Calculations:

Blades

For the three-dimensional implementation of the blade design for a real environment, 2D modeling is required to match the laminar measurements required for the construction, using the following equations:

$$A = \frac{D-d}{2} \quad \square \square \square$$

$$Ext = \sqrt{(D\pi)^2 + E^2} \quad \square 2 \square$$

$$Int = \sqrt{(d\pi)^2 + E^2} \quad \square 3 \square$$

Where “A” is the width of the loop, “D” is the external diameter of the loop, “d” is the internal diameter of the loops, “Ext” is the external perimeter, “Int” is the internal perimeter, and “E” is the distance between loops, as shown in Figure 9.

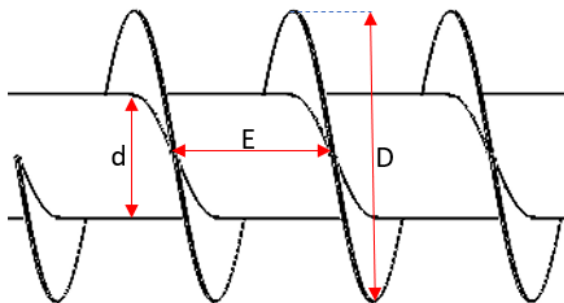


Fig. 9. Design of initial blade parameters

For the calculation of the previously mentioned parameters, an external diameter of 10 cm, an internal diameter of 3 cm, and a distance between the turns of .2 cm are taken as a reference. With the aforementioned data, it is obtained a spiral width of, an external perimeter of, and an internal perimeter of.

Equations 1, 2, and 3 are used to determine the largest radius “R” and the smallest radius “r” of the 2d design, which is used to make the respective cuts for each blade.

$$r = \frac{A \cdot \text{Int}}{\text{Ext} - \text{Int}} \quad \square 4 \square$$

$$R = r + A \quad \square 5 \square$$

Equation 4 gives a smaller radius, while equation 5 gives a larger radius.

The following equation is used to find the perimeter of the larger radius “P”:

$$P = 2\pi R \quad \square 6 \square$$

Finally, to determine the cut to be made in the perimeter of the larger radius, the difference between equation 6 and equation 2 is made.

$$\text{Cut} = P - \text{Ext} \quad \square 7 \square$$

By performing equation 7, a cut perimeter is obtained, giving as a final result the design shown in Figure 10.

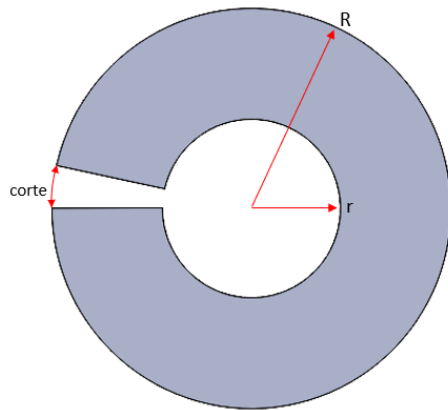


Fig. 10. 2D blade cutting design for a real-world environment

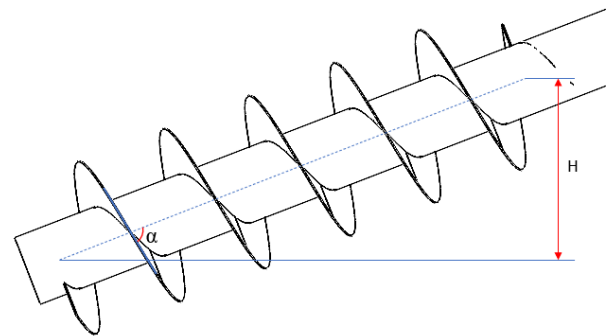


Fig. 11. Design of the parameters for power calculation

Maximum power

In previous studies, a reduced formula was determined to find the theoretical power in generators using the AST (Santa Cruz, 2018).

$$P_{teo} = \rho * g * Q * H * (\tan(\alpha))^2 \quad \square 7 \square$$

Where “ρ” is the density of water at kg/m³, “g” is the acceleration of gravity in m/s², “Q” is the water flow rate in m³/s, “H” is the water jump (height) in m, and “α” propeller outer angle in rads, as shown in Figure 11.

The power calculation is based on a density of 997 kg/m³, an acceleration of gravity of 9.81 m/s², a flow rate of 2m/s, a height of 24.6 cm, and an angle of 38°. Obtaining a theoretical power of 12 Watts.

RESULTS

In the study, it was possible to increase the theoretical electrical power from 9.84 to 12 watts, a very important value because prototypes with a blade diameter of 10 cm have been used in the test prototype. The final module was able to generate a minimum electrical power of 1 kW at very low water flow.

N°	Combinación	NIVELES TRATAMIENTO			REPLICAS POTENCIA ELÉCTRICA (Watts)			
		Tratamientos	A	B	C	I	II	III
1	A1B1C1	2 entradas	Eje delgado	Eje a 38°	12	12.01	11.96	11.9975
2	A2B1C1	3 entradas	Eje delgado	Eje a 38°	10.01	9.98	10.1	10.0225
3	A1B2C1	2 entradas	Eje grueso	Eje a 38°	11	11.04	11.3	11.3
4	A2B2C1	3 entradas	Eje grueso	Eje a 38°	9.08	8.98	9.03	9.0625
5	A1B1C2	2 entradas	Eje delgado	Eje a 20°	11.58	11.6	11.49	11.5425
6	A2B1C2	3 entradas	Eje delgado	Eje a 20°	10.18	10.2	10.15	10.2525
7	A1B2C2	2 entradas	Eje grueso	Eje a 20°	9.57	9.61	9.5	9.575
8	A2B2C2	3 entradas	Eje grueso	Eje a 20°	9.21	9.2	9.18	9.17

Fig. 12. Measurements of factorial experiment

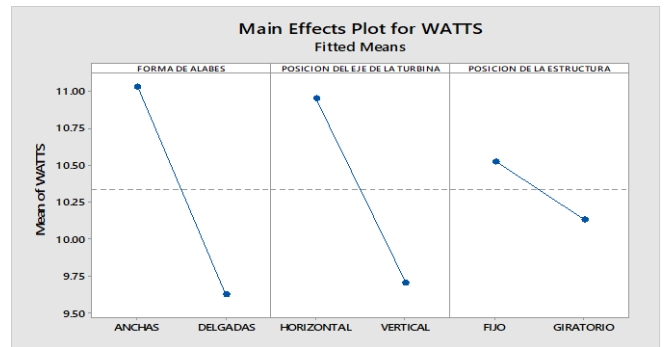


Fig. 13. Graph of main effects

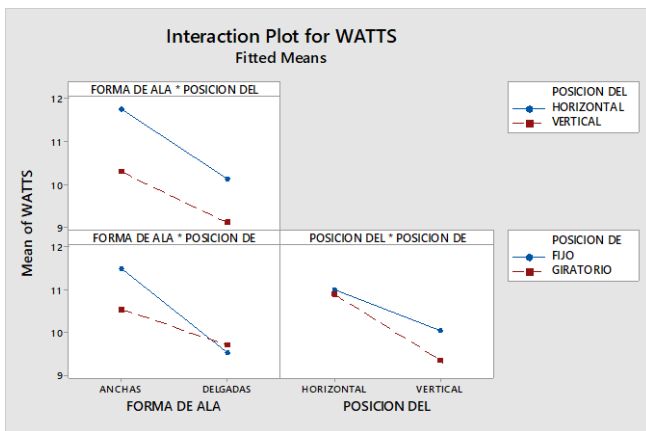


Fig. 14. Interaction graph for response

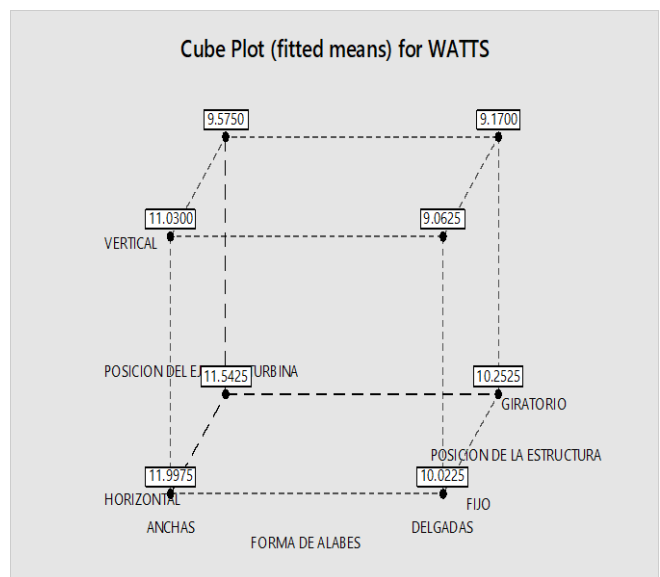


Fig. 15. Cube plot (data averages)

Factor analysis

a. For all analyses, the following factor names must be kept in mind:

A turbine blade inlet number (A1: 2 inlets, A2: 3 inlets); B turbine shaft diameter (B1: Thick shaft, B2: Thin shaft) and C hydro-generator turbine shaft position or pitch (C1: 38-degree shaft, C2: 20-degree shaft).

b. Figure 12 shows the effects in the experimental design, where it is seen that the most important effect is the one corresponding to factor A in the low level followed by B in the low level and effect C in the high level, the interrelation AC in the high level, AB in the low level, BC in the low level and the triple interrelation ABC in the high level. In the same way, it can be seen that the treatment where the highest electrical power is achieved is in the first (-1) where all the factors have a low level.

c. Figure 13 shows the main effects, it is analyzed how the average response changes according to the levels of a factor, it can be said that factor A has the highest relative strength with a downward trend, followed by B also with a downward trend, factor C has a very low strength and a downward trend.

d. Figure 14 shows how an interaction effect affects the response variable, the double interactions of the factors can be evaluated by assessing the response in its change from

low to high level; the interaction AC together with the interaction BC have a high level of interaction, while AB has a low level of interaction.

e. Figure 15 of the cube plot shows the relationships between the three factors, applying the adjusted means after analyzing the design of each combination of factor levels, it can be seen that the BC treatment is where the highest electrical power is obtained from the generator.

Analysis of variance

- The Anova of one factor in Figure 16 clearly shows that Fo for B with a value of 2339.61 is the highest value followed by factor A with a value of 3001.2 and C in the last place with a value of 223.74 corroborating the results of the factorial analysis.
- The normalized standardized effects graph in Figure 17 clearly shows the significant and non-significant effects; those with the highest positive significance are on the right side of the graph, the significant ones at the negative level are on the left side of the graph and the non-significant ones are close to the line; they can also be seen by their color, red for the significant ones and blue for the non-significant ones. In the present graph, it can be seen that the most significant factor is factor A followed by factor B on the negative side and factor AC on the positive side.
- In Figure 18, a Pareto plot of standardized effects where the effects are evaluated according to their magnitude and importance also shows the absolute value of the effects and draws a reference line where you can see the effects that are to the right of the graph line are potentially important; it can be noted that all factors of double and one triple interaction are important since none are on the left side of the red line.

Residue analysis

- All the data obtained by the prototype are normal and there is not much dispersion among them as shown in Figure 19 of the residuals in time sequence.
- From Figure 20 in the histogram of the residuals graph, it can be seen that the central tail with the value zero makes intuit that the tests have been carried out without outliers or a skewed distribution.
- From Figure 21 of the plot of the residuals against the fitted values, it can be concluded that the model is correct since there is no pattern of growth beyond 3.
- From Figure 22 of the normal probability plot it can be deduced that there are no anomalies because there are no residuals that are very different from others or much larger, therefore, there are no outlier residuals.

Fuente	GL	SC	CM	F
		Ajust.	Ajust.	
1-Efectos principales	3	29.5148	9.8383	1858.1
Número de entradas de los ababes	1	15.8907	15.8907	3001.2
Diámetro del eje	1	12.3878	12.3878	2339.61
Posición del eje	1	1.2364	1.2364	233.51
2-Interacciones de (No.) factores	3	3.5539	1.1846	223.74
Número de entradas de los ababes	1	0.3983	0.3983	75.22
Diámetro del eje	1	2.5256	2.5256	477
Posición del eje	1	0.63	0.63	118.99
3-Interacciones de (No.) factores	1	0.385	0.385	72.71
Número de entradas de los ababes, Diámetro del eje, Posición del eje	1	0.385	0.385	72.71
Error	24	0.1271	0.0053	
Total	24	33.5808		

Fig. 16. Analysis of variance

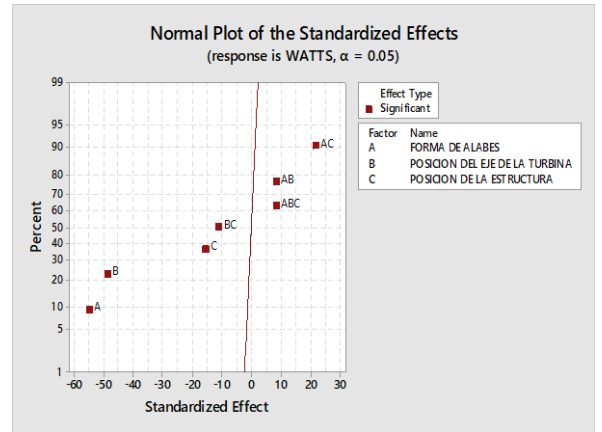


Fig. 17. Normal plot of standardized effects

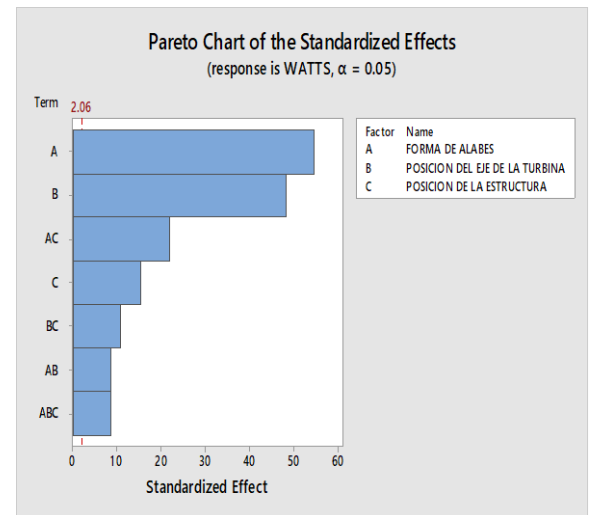


Fig. 18. Pareto chart of standardized effects

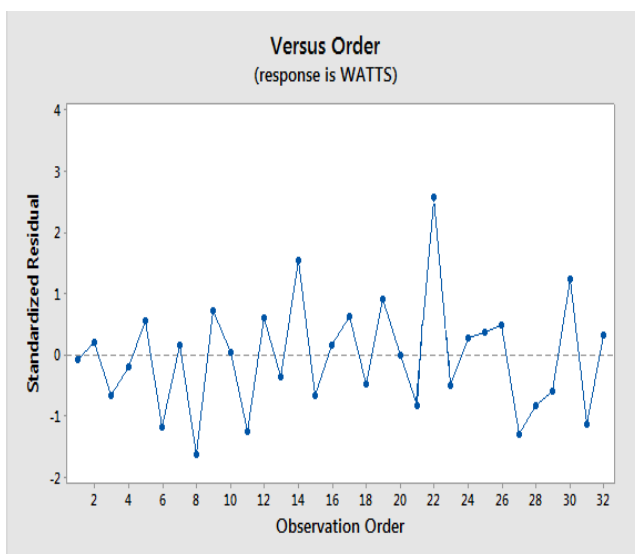


Fig. 19. Graph of standardized waste by order of observation

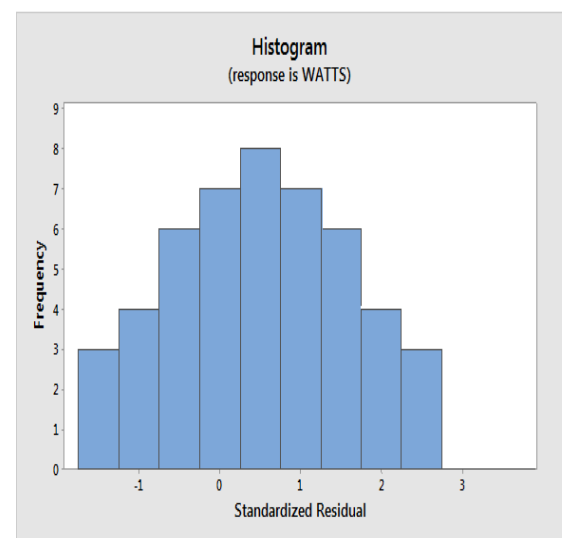


Fig. 20. Graph Histogram of standardized residuals

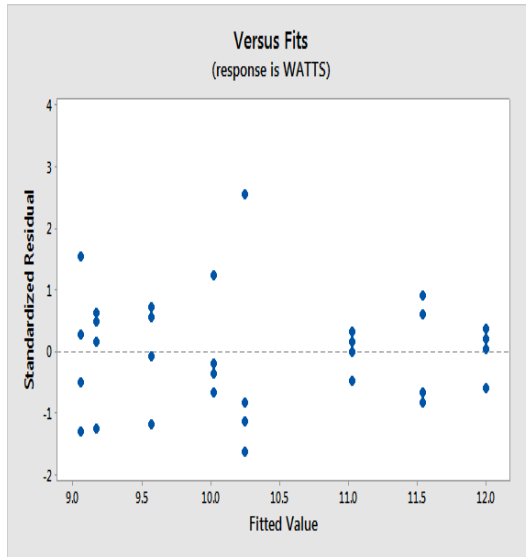


Fig. 21. Graph Residuals versus adjusted values

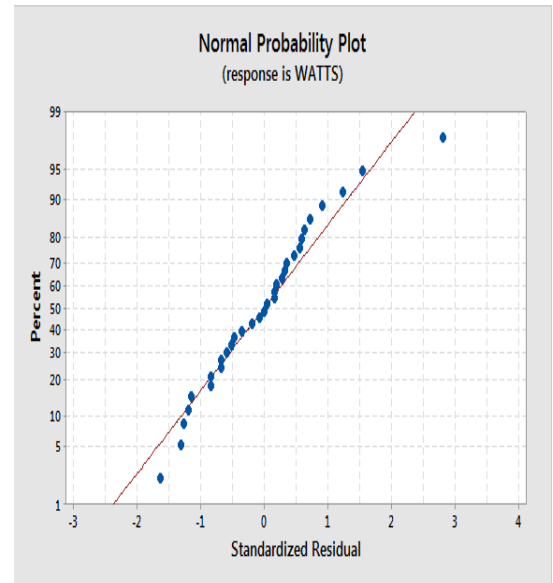


Fig. 22. Normal Probability Plot of standardized residuals

DISCUSSION

In the present investigation, the shape of the blades, the position of the shaft and the position of the turbine of the generator have been considered as having a great positive influence on the increase of the electric power of the hydro peak Archimedes generator in places with low flow intensity, coinciding with Shahverdi (2021) who developed a model to predict the optimum efficiency of the AST in different conditions, using the response surface methodology, so that, among the different geometrical and flow parameters of an AST, the outer diameter of the screw, the number of blades, the angle of inclination, the rotation speed, the available head, and the flow were used to define the efficiency of ASTs; In contrast to Reinaldo and Sebastian (2023), who only propose as an important factor the turbine blade, concerning the hydrodynamic characteristics of the spiral blade turbine, the following factors were used to define the efficiency of the ASTs.

Regarding small-scale power generation using the so-called Archimedes blade, the present research work studies the position of the turbine axis, as this factor is considered of utmost importance for the orientation of the turbine when the wind changes direction, unlike Moreno (2023) who focuses only on the properties of the turbine material for flowing water.

Concerning the residuals in time sequence, in the present investigation it can be corroborated that there is no reason to suspect any violation of the independence assumptions and that there is not a very wide dispersion since the values are between 3 and -2, according to Montgomery (2005) who indicates that about 68% of the residuals should be close to ± 1 , about 95% of them should be close to ± 2 and virtually all should be included between ± 3 , the residuals greater than 3 are potential outliers.

CONCLUSIONS

When optimizing the mechanical power generation factors, the blade shape and the position of the turbine must always be considered because of the importance of their effects on the increase of the electrical power of the hydroelectric peak Archimedes generator, so the best combination of factors that generate greater mechanical power involves wide blades, the turbine shaft in horizontal position and the fixed structure

because if the shaft is moving, turbulence is generated that does not allow obtaining greater power.

The design and elaboration of the Archimedes turbine are not very complicated, only the calculations indicated in the theoretical framework must be used correctly, since the use of the Archimedes turbine for the generation of electric energy is a good alternative for renewable energies, being necessary to carry out more studies since it is a new technology worldwide.

ACKNOWLEDGEMENT

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