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Design and Implementation of Prepared Learning Environments to Develop Algebraic Thinking Skills

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Abstract

Within the framework of the doctorate in educational sciences of the Universidad de San Buenaventura, Medellín-Colombia, attached to the research line: critical studies on education and curriculum, the present research was developed where some problems were identified in the teaching of mathematics, specifically in the learning environments used in the classrooms that do not favor the development of interesting classes, where students solve exercises in a fictitious way, making it impossible to apply knowledge outside the classroom, specifically within the social context in which students and their families develop. The thesis aimed to determine the incidence that prepared learning environments have on the development of algebraic thinking skills through a didactic proposal that involves the process of mathematization and mathematical modeling supported by the use of technology such as embedded systems and sensors, measuring the impact through an instrument that was designed to measure the first four levels of algebraization categorized in the ontosemiotic approach. A quasi-experiment is implemented by analyzing the data obtained by the control and experimental groups in the pretest and post-test, obtaining as a statistical result, the positive influence of the didactic proposal in levels 1, 2, and 4 of algebrization.

Keywords: *Algebraic thinking, levels of algebraization, prepared learning environments.*

Introduction

During the background search, some problems have been identified in the teaching of mathematics such as the mathematical formulation to a literal and symbolic language (Cuili, 2008), the mechanical reproduction of concepts the, memorization of formulas and methods (Valverde & Näslund-Hadley, 2010), how complicated it is for students to understand what varies (Parada et al., 2016), the absence of measurement instruments (Arrieta, 2003), which in the end, has only generated disinterest in learning (Cantoral, 2019).

According to the Organization for Economic Cooperation and Development (OECD) in the report generated by the (Colombian Institute for the Evaluation of Education - [ICFES, (2020)], the results of PISA locate Colombian students in the lowest level of the six categorized levels highlighting that students who develop the ability to model situations mathematically represent only 1%.

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From the characterization of the students, it is known that many of them live on farms that they own or that their guardians manage; in addition, the students help with farm chores. Also, following what is stated by the OECD (2006), to generate functional mathematical knowledge for life itself and what Bressan et al. (2016), the relevance of taking up topics of daily life, generating in the student, value judgments; in addition, authors such as (Somasundran, 2018) in Malaysia, (Chimoni, 2015) in Republic of Cyprus and Chimoni & Pitta-Pantazi, (2016) in Czech Republic, recognize the need to develop algebraic thinking even from elementary school, as indicated by the National Council of Teachers of Mathematics [NTCM, (2000)].

It is evident the need to generate a didactic proposal that has as a primary basis the social context of the students of Cumaral and the productive sector of the region, involving some elements such as the modeling of processes and phenomena of reality defined in the basic standards of competence in mathematics (MEN, 2006). With all the above, the idea of recreating a prepared learning environment that helps and contributes to the development of the socioeconomic context of the municipality of Cumaral in the department of Meta, specifically in the agricultural sector through automated irrigation systems, with the acquisition of environmental data but simultaneously, allows the development of mathematical skills especially in algebraic thinking - levels of algebrization Aké (2013).

In summary:

Problems in the teaching of mathematics
- Mathematical formulation of a literal and symbolic language (Cuili, 2008).
- Mechanical reproduction of concepts and memorization of formulas and methods (Valverde & Näslund-Hadley, 2010).
- Difficulty for students to understand what varies (Parada et al., 2016).
- Absence of measurement instruments (Arrieta, 2003).
- Generation of disinterest in learning (Cantoral, 2019).
PISA results in Colombia according to the OECD
- Colombian students are at the lowest level of the six levels categorized in PISA.
- Only 1% of students develop the ability to model situations mathematically.
Characterization of Cumaral students
- Many of the students live on farms and help with farm chores.

- The need to generate functional mathematical knowledge for daily life (OECD, 2006).
- Importance of relating mathematics to everyday life (Bressan et al., 2016).
- Recognition of the need to develop algebraic thinking from primary education (Somasundran, Chimoni, Chimoni & Pitta-Pantazi) and following the basic standards in mathematics competence (MEN, 2006).
Didactic proposal
- Creation of a prepared learning environment that relates to the socioeconomic context of Cumaral's students.
- Focus on the agricultural sector and automated irrigation systems.
- Environmental data acquisition.
- Development of mathematical skills, especially in algebraic thinking and algebrization levels Aké (2013).
- Objective to contribute to the development of the municipality of Cumaral in the department of Meta.

Methodology

The study will be conducted from the positivist paradigm, understanding the paradigm as a system of beliefs about reality (Flores, 2004), understanding positivism according to (Ricoy, 2006) of quantitative, empirical - analytical, and scientific character, sustaining the studies through the testing of hypotheses through statistics; likewise, experimentation is constituted in the essential way to generate theory (Hernández et al., 2010), with a quantitative approach, since it focuses its study, analyzing the data with statistical instruments (Pimienta & De La Orden, 2017), characterized by being rational, objective, observable and verifiable (Cuenya & Ruetti, 2010), under pre-established scientific parameters and criteria.

Two of the three specific objectives stated in the doctoral thesis, referred to in the writing of this article briefly described: To design a didactic proposal from the prepared learning environments, which favors the development of algebraic thinking skills and to evaluate the impact of the use of the prepared environments in the development of algebraic thinking. Although this article does not seek to specify the methodology used for the validation of the measurement instrument of the four levels of algebraization categorized in the ontosemiotic approach (Godino, 2012; Godino et al., 2007), it is worth clarifying that a pilot test was conducted, with multiple-choice questions with a single answer (dichotomous data), performing an external consistency analysis of the initial test through expert judgment and internal consistency, making selection filters per item with the calculation of discrimination and difficulty per item, and to calculate the reliability index of the instrument, according to (Reidl-Martínez, 2013), the most widely used test to estimate the internal consistency of an instrument with dichotomous responses (correct and incorrect) is the one developed by Kuder and Richardson (KR-20) (Kuder & Richardson, 1937).

A study with a quasi-experimental design will be conducted, being the independent variable the didactic proposal based on the Prepared Learning Environments, accepting seven (7) principles of the potentially significant teaching units (Moreira, 2011), observing its effect on the variable Algebraic Thinking skills that will be discriminated in reagents that are strongly linked to the first four (4) levels of algebraization.

Pre-tests and post-tests will be performed on the experimental and control groups. The analysis of the stored data will be performed through RStudio. The analysis of each of the data begins by performing a normality test through the Shapiro-Wilk test according to Novales (2010), this statistic is used to contrast normality when the sample is less than 50 observations and has greater test power for a given significance. Some of the data analyzed showed that they were not normally distributed, so parametric tests such as Welc Two Samplt T-Test were used in the data analysis to compare independent groups. In addition, the Mann-Whitney test was used for the analysis of the data that did not show normality.

Population and sample: The intervention is carried out with students of the educational institution Teniente Cruz Paredes, specifically in the two tenth grades, being a non-probabilistic sample chosen at the discretion of the researcher, since the courses are pre-established, being the experimental group the 10-1 course and the control group the 10-2 course. It is assumed that both groups have similar conditions in different situations such as economy, social context, and knowledge, among others.

The didactic proposal is initially implemented in PowerPoint using macros (Visual Basic) as shown in Figure 1 and videos designed in the Vyond platform are added; however, the proposal of the environment transcends to be supported by technology kits that are made up of Arduino, soil moisture sensors, air temperature and relative humidity, actuators such as small electric pumps for irrigation, allowing access to pleasant and interactive information, where students are the programmers and scientists of the moment.

The research identifies with Montessori (1988) when he says that the child has an absorbent mind that allows him to absorb information directly from the surrounding environment through the senses, experience, and interaction; therefore, it is essential to prepare and maximize the conditions of the environment, so that the child absorbs information that enriches him. Therefore, education is not acquired by listening to words, but under experiences carried out in the environment" (Montessori, 2006, p. 19).

In the creation of the environment, seven (7) principles of the Potentially Significant Teaching Units (PSHE) proposed by Moreira (2011) are taken into account: previous knowledge; thoughts, feelings, and actions; problem situations that give meaning to new knowledge; the creation of a functional mental model; the role of the teacher as a provider of problem situations. The proposal raises a global crisis problem announced by the FAO (Food and Agriculture Organization of the United Nations) and the need to increase food production; therefore, the importance of encouraging automated irrigation systems to improve agricultural processes in the region and in the country.

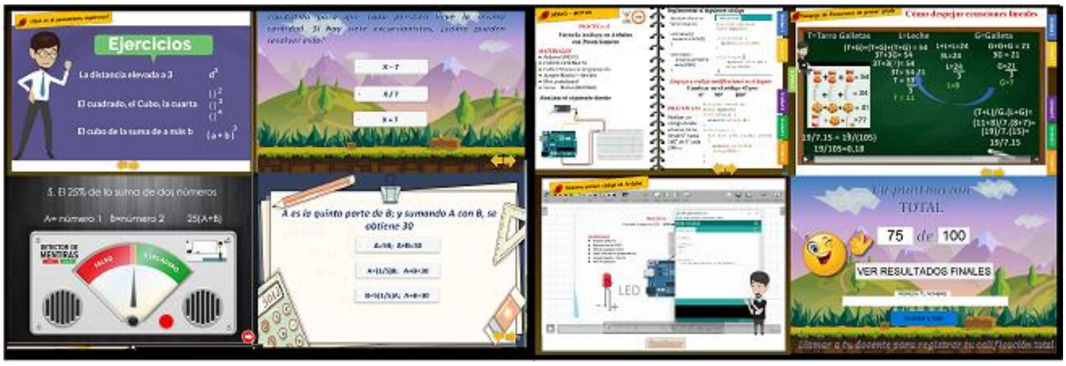

Figure 1 Visualization of the didactic proposal

Figure 2 shows the interaction that students have, as defined by Montessori, through the senses and experimentation within the environment. The students embrace the proposal as a group and with fieldwork to collect data through the sensors, which enriches the practice within the prepared learning environment.

Figure 2 Intervention with students of the didactic proposal

Results

The analysis begins with the recording of the data obtained by the control and experimental groups in the pretest (Figures 3 and 4)

0:H 1:M	APELLIDOS Y NOMBRES DEL ESTUDIANTE	P1	P2	P3	P4	P5	P6	P7	P8	P9	P1	P2	P3	P4	P5	P6	P7	P8	P9	V	F	PORCENTAJE	DESEMPEÑO
0	GCACRUL1F16	C	D	D	A	C	B	B	D	C	1	0	0	0	1	0	0	0	1	3	6	33,33333	BAJO
1	GCBAAMJ2F16	C	B	D	A	C	D	C	D	A	1	1	0	0	1	1	0	0	0	4	5	44,44444	BAJO
0	GCBALED3F16	C	C	D	D	D	A	D	D	B	1	0	0	1	0	0	1	0	0	3	6	33,33333	BAJO
1	GCCAGAD4F15	C	D	B	D	C	B	A	D	A	1	0	0	1	1	0	0	0	0	3	6	33,33333	BAJO
1	GCCARUL5F16	C	A	C	C	D	A	D	C	D	1	0	1	0	0	0	1	0	0	3	6	33,33333	BAJO
0	GCCAMAC6F17	C	C	D	A	C	B	A	D	C	1	0	0	0	1	0	0	0	1	3	6	33,33333	BAJO
0	GCCHCHC7M15										0	0	0	0	0	0	0	0	0				
0	GCCRRIJ8F15	C	D	D	A	C	C	NR	C	B	1	0	0	0	1	0	0	0	0	2	7	22,22222	BAJO
1	GCDICOY9M16	C	C	A	B	C	A	NR	C	D	1	0	0	0	1	0	0	0	0	2	7	22,22222	BAJO
1	GCGAPEJ10M15										0	0	0	0	0	0	0	0	0				
0	GCGAHEK11M16	A	C	D	C	A	A	D	C	D	0	0	0	0	0	0	1	0	0	1	8	11,11111	BAJO
1	GCGUSÁM12F15	C	B	D	A	C	A	C	D	A	1	1	0	0	1	0	0	0	0	3	6	33,33333	BAJO
1	GCHEMAJ13M15										0	0	0	0	0	0	0	0	0				
0	GCMEORM14M16	C	C	B	D	D	B	C	C	D	1	0	0	1	0	0	0	0	0	2	7	22,22222	BAJO
0	GCMOTOA15M15	C	B	A	B	C	A	B	A	A	1	1	0	0	1	0	0	0	0	3	6	33,33333	BAJO
0	GCMOPIL16F16	C	A	D	B	C	A	D	C	D	1	0	0	0	1	0	1	0	0	3	6	33,33333	BAJO
1	GCMOQUM17F15	C	D	D	C	C	D	A	D	D	1	0	0	0	1	1	0	0	0	3	6	33,33333	BAJO
0	GCMOGOA18M18	A	D	C	C	D	B	C	C	A	0	0	1	0	0	0	0	0	0	1	8	11,11111	BAJO
0	GCMOFEJ19M15	B	B	A	B	C	B	B	B	A	0	1	0	0	1	0	0	1	0	3	6	33,33333	BAJO
1	GCORORG20F15										0	0	0	0	0	0	0	0	0				
1	GCREVEK21M16	A	B	A	D	C	B	A	C	A	0	1	0	1	1	0	0	0	0	3	6	33,33333	BAJO
0	GCREACA22M15	C	B	D	C	A	B	A	D	B	1	1	0	0	0	0	0	0	0	2	7	22,22222	BAJO
0	GCRÍORO23M16	C	B	D	B	C	A	B	C	D	1	1	0	0	1	0	0	0	0	3	6	33,33333	BAJO
1	GCROGOJ24M17	C	B	D	C	D	B	A	D	C	1	1	0	0	0	0	0	0	1	3	6	33,33333	BAJO
1	GCSÁGOJ25F15										0	0	0	0	0	0	0	0	0				
1	GCURACK26F15	A	B	D	D	C	B	A	D	D	0	1	0	1	1	0	0	0	0	3	6	33,33333	BAJO

Figure 3 Record of data obtained by the control group in the pretest

0:H 1:M	APELLIDOS Y NOMBRES DEL ESTUDIANTE	P1	P2	P3	P4	P5	P6	P7	P8	P9	P1	P2	P3	P4	P5	P6	P7	P8	P9	V	F	PORCENTAJE	DESEMPEÑO
0	GEACRAA1M15	C	C	D	A	B	A	C	D	D	1	0	0	0	0	0	0	0	0	1	8	11,11111	BAJO
1	GEAGMOH2M16	C	A	B	B	C	B	NR	NR	B	1	0	0	0	1	0	0	0	0	2	7	22,22222	BAJO
0	GEALGÓD3F16	C	D	B	D	D	D	C	B	B	1	0	0	1	0	1	0	1	0	4	5	44,44444	BAJO
1	GEBEGAD4M16	C	B	C	D	B	C	B	C	D	1	1	1	1	0	0	0	0	0	4	5	44,44444	BAJO
1	GECOMAS5M15	C	A	D	B	C	D	D	A	D	1	0	0	0	1	1	1	0	0	4	5	44,44444	BAJO
0	GECRMOY6F15	C	C	B	NR	B	D	D	C	A	1	0	0	0	0	1	1	0	0	3	6	33,33333	BAJO
0	GEESAGM7F16	C	C	D	B	D	C	C	B	NR	1	0	0	0	0	0	0	1	0	2	7	22,22222	BAJO
0	GEGASAK8M16	C	D	D	D	B	D	D	B	D	1	0	0	1	0	1	1	1	0	5	4	55,55556	BAJO
1	GEGUOSL9M17	C	B	B	C	B	A	NR	D	A	1	1	0	0	0	0	0	0	0	2	7	22,22222	BAJO
1	GEMEPAD10M16	C	B	D	B	D	D	C	D	C	1	1	0	0	0	1	0	0	1	4	5	44,44444	BAJO
0	GEMOROR11M15	C	B	D	B	B	D	A	B	D	1	1	0	0	0	1	0	1	0	4	5	44,44444	BAJO
1	GEMOSAS12F16										0	0	0	0	0	0	0	0	0				
1	GENIXXL13F16	C	C	A	D	D	B	A	D	D	1	0	0	1	0	0	0	0	0	2	7	22,22222	BAJO
0	GENOPEC14M15	C	C	D	A	C	A	C	B	B	1	0	0	0	1	0	0	1	0	3	6	33,33333	BAJO
0	GENÚMAS15F15	C	C	D	B	B	D	B	D	D	1	0	0	0	0	1	0	0	0	2	7	22,22222	BAJO
0	GEOLROS16M15	C	C	D	D	C	B	B	D	D	1	0	0	1	1	0	0	0	0	3	6	33,33333	BAJO
1	GEPEPUJ17M16	C	C	D	D	C	D	NR	D	NR	1	0	0	1	1	1	0	0	0	4	5	44,44444	BAJO
0	GEPEBOJ18M15	C	C	D	D	A	B	C	D	B	1	0	0	1	0	0	0	0	0	2	7	22,22222	BAJO
0	GEPOCAM19F16	A	D	A	A	A	D	C	B		0	0	0	0	0	1	0	1	0	2	7	22,22222	BAJO
1	GERIGRL20F16	C	C	D	A	B	B	C	D	D	1	0	0	0	0	0	0	0	0	1	8	11,11111	BAJO
1	GEROAGS21M16	C	C	D	B	C	A	C	D	D	1	0	0	0	1	0	0	0	0	2	7	22,22222	BAJO
0	GEROPÉV22F15	C	C	D	C	C	A	C	D	D	1	0	0	0	1	0	0	0	0	2	7	22,22222	BAJO
0	GEROBAM23F15	A	C	D	B	B	C	B	A	D	0	0	0	0	0	0	0	0	0	0	9	0	BAJO
1	GESAVAI24M15	C	A	C	B	C	D	D	A	D	1	0	1	0	1	1	1	0	0	5	4	55,55556	BAJO
1	GEVEARC25M15										0	0	0	0	0	0	0	0	0				
1	GEVIMEM26M15	C	A	D	D	C	A	D	D	D	1	0	0	1	1	0	1	0	0	4	5	44,44444	BAJO

Figure 4 Record of data obtained by the experimental group in the pretest

Subsequently, a separation is made only with dichotomous data, taking into account that items 1 and 2 belong to algebrization level 1, items 3 and 4 to algebrization level 2, items 5 and 6 to algebrization level 3, and items 7, 8, and 9 to algebrization level 4. The pretest data of the control group are recorded in Table XX and the data of the experimental group are recorded in Table XXI.

Table 1 Dichotomous data - Pre-test control group

P1	P2	N1	P3	P4	N2	P5	P6	N3	P7	P8	P9	N4	TOTAL
1	0	1	0	0	0	1	0	1	0	0	1	1	3
1	1	2	0	0	0	1	1	2	0	0	0	0	4

1	0	1	0	1	1	0	0	0	1	0	0	1	3
1	0	1	0	1	1	1	0	1	0	0	0	0	3
1	0	1	1	0	1	0	0	0	1	0	0	1	3
1	0	1	0	0	0	1	0	1	0	0	1	1	3
1	0	1	0	0	0	1	0	1	0	0	0	0	2
1	0	1	0	0	0	1	0	1	0	0	0	0	2
0	0	0	0	0	0	0	0	0	1	0	0	1	1
1	1	2	0	0	0	1	0	1	0	0	0	0	3
1	0	1	0	1	1	0	0	0	0	0	0	0	2
1	1	2	0	0	0	1	0	1	0	0	0	0	3
1	0	1	0	0	0	1	0	1	1	0	0	1	3
1	0	1	0	0	0	1	1	2	0	0	0	0	3
0	0	0	1	0	1	0	0	0	0	0	0	0	1
0	1	1	0	0	0	1	0	1	0	1	0	1	3
0	1	1	0	1	1	1	0	1	0	0	0	0	3
1	1	2	0	0	0	0	0	0	0	0	0	0	2
1	1	2	0	0	0	1	0	1	0	0	0	0	3
1	1	2	0	0	0	0	0	0	0	0	1	1	3
0	1	1	0	1	1	1	0	1	0	0	0	0	3
MATCHES		25			7			16				8	56

Tabla 2 Datos dicotómicos – Pre test grupo experimental

P1	P2	N1	P3	P4	N2	P5	P6	N3	P7	P8	P9	N4	T
1	0	1	0	0	0	0	0	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	0	0	0	0	2
1	0	1	0	1	1	0	1	1	0	1	0	1	4
1	1	2	1	1	2	0	0	0	0	0	0	0	4
1	0	1	0	0	0	1	1	2	1	0	0	1	4
1	0	1	0	0	0	0	1	1	1	0	0	1	3
1	0	1	0	0	0	0	0	0	0	1	0	1	2
1	0	1	0	1	1	0	1	1	1	1	0	2	5
1	1	2	0	0	0	0	0	0	0	0	0	0	2
1	1	2	0	0	0	0	1	1	0	0	1	1	4
1	1	2	0	0	0	0	1	1	0	1	0	1	4
1	0	1	0	1	1	0	0	0	0	0	0	0	2
1	0	1	0	0	0	1	0	1	0	1	0	1	3

1	0	1	0	0	0	0	1	1	0	0	0	0	2
1	0	1	0	1	1	1	0	1	0	0	0	0	3
1	0	1	0	1	1	1	1	2	0	0	0	0	4
1	0	1	0	1	1	0	0	0	0	0	0	0	2
0	0	0	0	0	0	0	1	1	0	1	0	1	2
1	0	1	0	0	0	0	0	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	0	0	0	0	2
1	0	1	0	0	0	1	0	1	0	0	0	0	2
1	0	1	1	0	1	1	1	2	1	0	0	1	5
1	0	1	0	1	1	1	0	1	1	0	0	1	4
MATCHES		26	MATCHES		10	MATCHES		19	MATCHES			12	67
		54			21			40				17	31

Continuing with the data recording, the same procedure is carried out with the data obtained in the post-test for the control and experimental groups.

The results obtained in the pre-test and post-test (dichotomous data) are recorded and the normality analysis is performed through QQplot graphs and the Shapiro-Wilk test utilizing the R-Studio software. It is found that some data do not present a normal distribution; therefore, parametric and non-parametric tests must be performed according to each case.

For the present article, one (4) of the eight hypotheses raised is highlighted.

Did the experimental group have a significant improvement over the control group?

H0: Udif_e – Udif_c <= 0 H1: Udif_e – Udif_c > 0

```
        Wilcoxon rank sum test with continuity correction

data:  DATOS$DIF_EXP and DATOS$DIF_CTRL
W = 365, p-value = 9.259e-05
alternative hypothesis: true location shift is greater than 0

        Welch Two Sample t-test

data:  DATOS$DIF_EXP and DATOS$DIF_CTRL
t = 4.4622, df = 32.038, p-value = 4.695e-05
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 1.329445      Inf
sample estimates:
mean of x mean of y
2.8571429 0.7142857
```

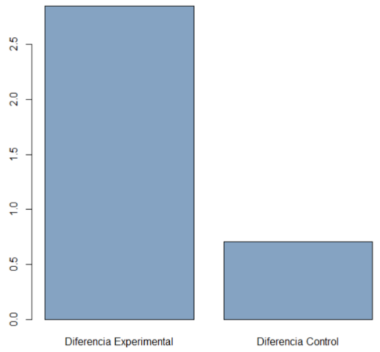

Figure 5. Learning difference plot for experimental and control

As the p-value is less than 0.05 then H0 is rejected, i.e., the mean score of the difference of correct points between the experimental and control groups is greater than zero, i.e., there was a significantly higher improvement in the test score in the experimental group.

In the control group was there a significant improvement in the number of hits?

H0: Upos_c – Upre_c <= 0

H1: Upos_c – Upre_c > 0

```
        Welch Two Sample t-test

data:  DATOS$POSTEST_CTRL and DATOS$PRETEST_CTRL
t = 2.4906, df = 35.245, p-value = 0.008805
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 0.2388791       Inf
sample estimates:
mean of x mean of y
 3.409091  2.666667
```

```
        Wilcoxon rank sum test with continuity correction

data:  DATOS$POSTEST_CTRL and DATOS$PRETEST_CTRL
W = 295, p-value = 0.02388
alternative hypothesis: true location shift is greater than 0
```

Conclusion: As the p-value is less than 0.05 then H0 is rejected, i.e. the mean score of the posttest is higher than that of the pretest.

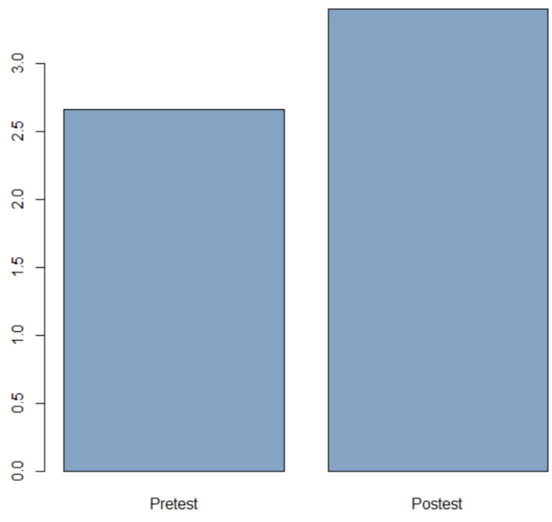

Figure 6. Learning difference graph for the control group.

In the experimental group was there a significant improvement in the hit percentage?

H0: Upos_e – Upre_e <= 0

H1: Upos_e – Upre_e > 0

```
        Welch Two Sample t-test

data:  DATOS$POSTEST_EXP and DATOS$PRETEST_EXP
t = 7.1771, df = 45.447, p-value = 2.64e-09
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 2.119627      Inf
sample estimates:
mean of x mean of y 
 5.680000  2.913043 
```

```
        Wilcoxon rank sum test with continuity correction

data:  DATOS$POSTEST_EXP and DATOS$PRETEST_EXP
W = 413, p-value = 4.946e-07
alternative hypothesis: true location shift is greater than 0
```

Conclusion: As the p-value is less than 0.05 then H0 is rejected, i.e. the mean score of the posttest is higher than that of the pretest.

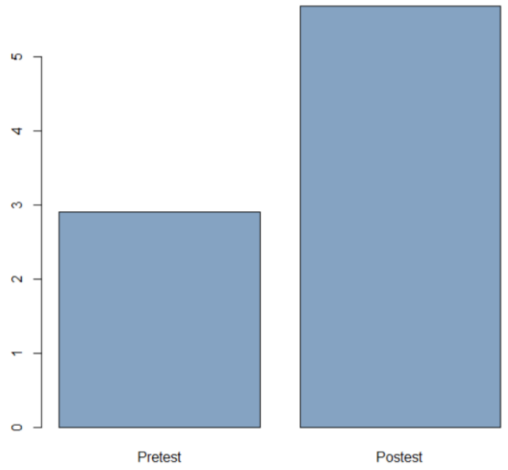

Figure 71 Learning difference graph for experimental group

Did the experimental group have a significant improvement over the control group?

H0: Udif_e – Udif_c <= 0

H1: Udif_e – Udif_c > 0

```
        Welch Two Sample t-test

data:  DATOS$DIF_EXP and DATOS$DIF_CTRL
t = 4.4622, df = 32.038, p-value = 4.695e-05
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 1.329445      Inf
sample estimates:
mean of x mean of y
2.8571429 0.7142857
```

```
        Wilcoxon rank sum test with continuity correction

data:  DATOS$DIF_EXP and DATOS$DIF_CTRL
W = 365, p-value = 9.259e-05
alternative hypothesis: true location shift is greater than 0
```

Data that could not be related due to student absences were omitted.

Conclusion: As the p-value is less than 0.05 then H0 is rejected, i.e., the mean score of the difference of correct points between the experimental and control groups is greater than zero, i.e., there was a significantly higher improvement in the test score in the experimental group.

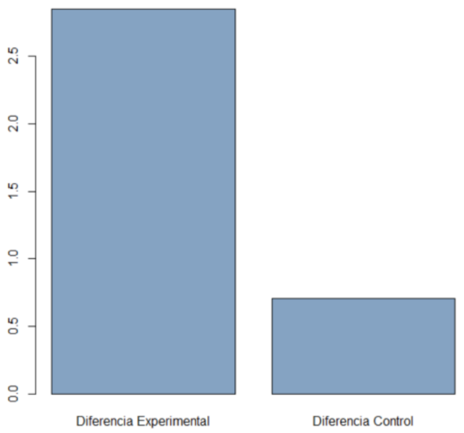

Figure 8 Difference graph of learning difference between the experimental and control groups

In addition, it was decided to analyze the incidence of the didactic proposal on each level of algebrization by taking the data corresponding to each level of algebrization symbolized in tables N1, N2, N3, and N4 for the control and experimental groups.

Discussion

From the prepared learning environments, a space is identified that should be harmonious and that fosters autonomy, confidence, and self-discipline. Within the research, it was observed how students who usually do not participate in the mathematics class, were very active during the intervention. In the tasks (challenges) assigned, they became expert programmers for the activities posed with Arduino. At no time are roles attributed to the members of the group; however, they distributed the tasks themselves as the one in charge of taking the data, who performs the programming, who performs the electronic connections of the circuits. The way the work was done allowed the students themselves to make decisions, create new codes, and test their knowledge during the activities.

The results obtained from the methodology and data analysis in their study are fundamental to understanding the impact of the didactic proposal based on Prepared Learning Environments on the development of algebraic thinking. The most important findings are summarized here:

1. Initial conditions: The analysis showed that in the pretest, the control and experimental groups started in similar conditions in terms of hit scores, which is fundamental for the subsequent comparison.

2. Improvement in the Control Group: After the implementation of the didactic proposal, a significant improvement was observed in the control group in terms of the percentage of correct answers between the pretest and the post-test. This suggests that even without the specific intervention, there was an increase in performance in algebraic thinking.

3. Improvement in the Experimental Group: In the experimental group, which was subjected to the didactic proposal based on Prepared Learning Environments, a significant improvement was observed in the percentage of correct answers between the pretest and the post-test. This indicates that the intervention had a positive impact on the development of algebraic thinking.

4. Difference between Groups: When comparing the improvement in the percentage of correct answers between the experimental group and the control group, it was found that the experimental group experienced a significantly greater improvement in the test score. This suggests that the didactic proposal based on Prepared Learning Environments had a

positive impact on the development of algebraic thinking compared to traditional teaching.

5. Analysis by Algebrization Levels: A more detailed analysis by algebrization levels was performed, which can provide valuable information on which specific aspects of algebraic ability improved. This can help identify areas of strength and weakness in the didactic proposal.

In summary, the results of the study support the effectiveness of the didactic proposal based on Prepared Learning Environments in the development of algebraic thinking. The methodology used, which included parametric and non-parametric statistical tests, as well as detailed analysis by levels of algebraization, provides a complete picture of the effects of the intervention. These findings have important implications for the teaching and learning of algebraic thinking and may contribute to the design of more effective pedagogical strategies in this field.

Conclusions

In both groups (experimental and control) there was apparent equality in the initial conditions considering the statistic evaluated (mean of the student's grades). The average grade increase obtained for the experimental group was much higher than that obtained by the students in the control group (2.85 VS 0.71), i.e., a difference of more than two points, which implies that the methodology applied was more effective.

The experimental methodology demonstrated high efficiency at algebrization levels 1, 2, and 4, while at level 3 there was no significant difference in the scores because of the application of the methodology.

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