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Hierarchization of Educational Elements based on Morganov-Heredia Matrices using Hypergraphs

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Abstract

For educational institutions in general and teachers in particular, it is important to be able to prioritize thematic contents, objectives and competencies, among others, for their study plans and programs. To carry out this hierarchy, analysis techniques are responsible for obtaining an optimal sequence from the vague idea that a person (in this case managers or teachers) initially has about the sequence relationship between the elements to be studied. One of these techniques is the hypergraph (which is a generalization of a graph), and can relate any number of vertices, instead of only a maximum of two as in the particular case, to determine a logical sequence of the elements to be hierarchize based on the criteria managed by a person or group and, by virtue of mathematical formalism, allows its application to be carried out by a computer. This work describes the Hypergraph technique used to provide a solution to one of the sequencing techniques called Morganov-Heredia matrices, which is based on the relationship of rows and columns to prioritize the elements, using specific competencies as a particular case.

Keywords: *Graphs, Morganov-Heredia Matrix, hierarchy, competencies.*

Introduction

In the work of teaching, it implies that some efforts must be made in the updating and training of teachers, beyond the knowledge of their own discipline. This situation is more complicated for the area of science (understood as such for this work to be physics, mathematics, biology and chemistry) given the resistance of teachers to the incorporation of elements beyond traditional teaching (Barolli et al, 2019). While it is true that a teaching sequence can be decided intuitively, this implies a high risk of making operational errors that result in poor teaching.

The term content analysis is often used to refer to the systematic work of determining an optimal sequence of teaching (Flores, 1983). The elements that content analysis works with are called content elements. This versatility allows the analysis of content elements to be applied in activities as diverse as educational planning, course design, class preparation, programming of teaching material, objectives to be met, tasks to be carried out, topics to be taught, concepts to be established, subjects to be taken, etc., depending on the interests pursued. Content analysis is a resource to obtain a logical sequence of elements, according to the criteria and premises that are handled in a given situation. When initially faced with the problem of sequencing a set of content elements in the best possible hierarchical order,

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a person actually has a vague idea of the sequence to be followed. You may well know the beginning and final elements of the sequence, but you will be unable to describe the sequence of intermediate elements step by step. Moreover, he will not be certain that the idea he has about the sequence between the elements is free of cycles (contradictions consisting of circular relations of sequence between the elements), for example, such as the one that takes place when the topic "Evolution" of a biology course requires the previous study of the topic "Natural Selection". at the same time that "Natural Selection" requires the prior study of "Evolution".

Therefore, content analysis can be understood as a task that allows:

- Systematically obtain information from the interested party in order to formalize the initial idea they have about the sequence in which they think the content elements should be arranged;
- To uncover the presence of cycles between the elements, for their correction;
- Transform the initial information of the relationships between the elements, in order to know which of these relationships are direct; and
- Determine the optimal sequence from the direct sequence relationships between the elements.

Based on these points, this paper shows the use of Morganov-Heredia matrices to order content elements hierarchically for their development.

Development

In mathematics and computer science, a hypergraph is a generalization of a graph, whose edges here are called hyperedges, and can relate to any number of vertices, rather than just a maximum of two as in the particular case (López, 2022).

In 1970 Claude Berge defined the term hypergraph as a set of vertices and a set of hyperedges. A hyperedge is a subset if (Pardal, 2015). Similarly, the hyperedges of a hypergraph can have weight where the weight of the hyperedge is. A hyperedge is said to be incident with when .$HG(V,\mathcal{E}) V \mathcal{E} e V \cup_{e\in\mathcal{E}} e = V HG(V,\varepsilon,\mathcal{w}) \mathcal{w}: \mathcal{E} \to \mathbb{R}^{+} e v v \in e$

A hypergraph has an incidence matrix where:$H \in \mathbb{R}^{|V|\times|E|}$

$$h(v,e) = \begin{cases} 1 & \text{si } v \in e \\ 0 & \text{si } v \notin e \end{cases} \quad (1)$$

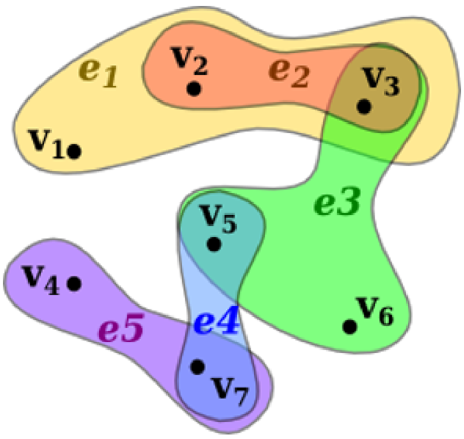
The vertex and degree of the hyperedge are defined as follows:

$$d(v) = \sum_{e\in E} \mathcal{w}(e) h(v,e)$$
$$\delta(e) = \sum_{v\in V} h(v,e) = |e| \quad (2)$$

The simplest way to represent a hypergraph is by means of a matrix, this matrix is called an incidence matrix (Figure 1). This matrix consists of a two-dimensional array of size "n", where "n" is the maximum number of nodes in the graph. Each box in the matrix is filled with values of true "1" or false "0".

The main advantage is its simplicity, since it facilitates the operations that can be carried out on the network. The disadvantage is that it is limited to a maximum number of nodes in the graph, which makes it impossible to store more information in the array.

Unlike adjacency matrices representing graphs, which can only have two ones in each column, incidence matrices representing hypergraphs can have any number of ones per column.



	v1	v2	v3	v4	v5	v6	v7
e1	1	1	1	0	0	0	0
e2	0	1	1	0	0	0	0
e3	0	0	1	0	1	1	0
e4	0	0	0	0	1	0	1
e5	0	0	0	1	0	0	1

Figure 1: Representation of a hypergraph.

Constructing the Matrix

The columns of the matrix represent the different nodes, the rows represent the hyperedges of the graph.

For each node joined by a hyperedge, we put a "one" (1) in the corresponding place, and fill the rest of the locations with "zeros" (0). A list containing all those nodes that are adjacent to it is associated with each node of the hypergraph, this list will also store the length of the edge to the adjacent vertex. A hypergraph cross-sectional is a set of vertices that meets all the hyperedges of the hypergraph and that do not have empty intersections (Sánchez, Lazo, & Almanza, 2023).$HG(V, \mathcal{E})T \subseteq V$

A family of cross-sections of a hypergraph is called a transverse hypergraph Tr(HG). An example is shown in Figure 2.

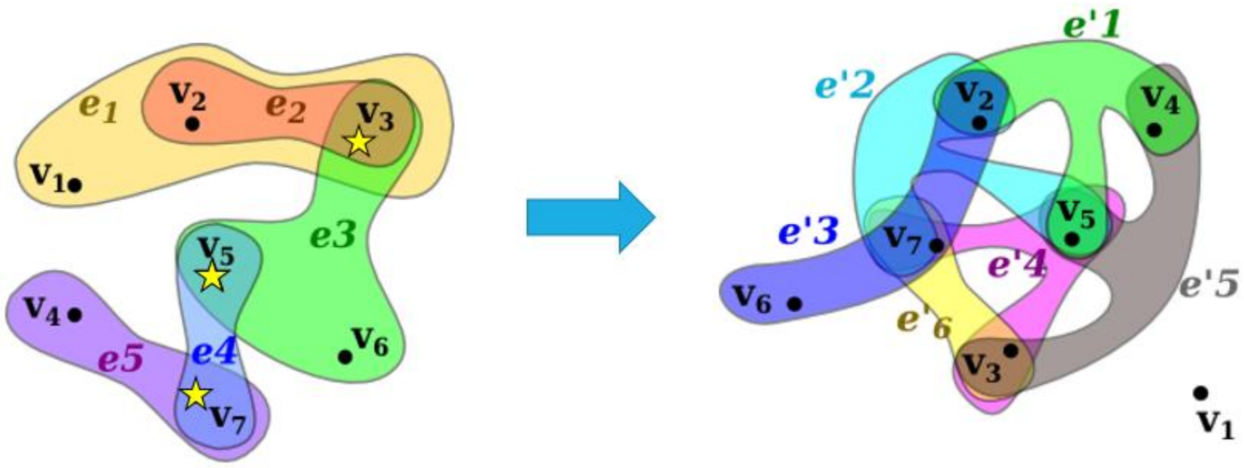


Figure 2: Transverse hypergraph set.

On the other hand, a minimal cross-sectional of a hypergraph is a set of vertices that meets all the hyperedges of the hypergraph, that they do not have empty intersections and that the cardinality of each of their hyperedges is minimal (Elbassioni, 2008).$HG(V, \mathcal{E})V' \subseteq V$

Algorithms for obtaining cross-sectional and other characteristics

Here we will explain the steps followed by the algorithms to find the minimum cross-sections from the incidence matrix. Below, we will describe some of the concepts needed to understand how algorithms work.

A set is said to be compatible if, under a reordering of columns or rows, an identity matrix is formed.

Example

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

YYC Algorithm

The fundamental idea of the YYC algorithm is that, instead of analyzing the entire basic matrix in order to determine the set Ψ* of minimal cross-sectionals, that process can be decomposed into an incremental process, which, during each iteration, calculates the set of cross-sections that lie within the first rows of the matrix (Cabrera et al, 2014). The YYC algorithm has an incidence matrix as input, unlike other algorithms the input matrix does not necessarily have to be in its basic form; however, it is advisable to reduce the search space. As an output, the algorithm delivers a set of transversals.

Steps of the YYC algorithm

The pseudocode of the algorithm is shown below (code in Spanish, Figure 3):

```
1. Inicializar la lista Ψ* = []
2. Añadir a Ψ* las columnas que contengan 1 en el primer renglón
3. Para cada renglón ri, i = 2…n
     a. Inicializar la lista Ψaux = []
     b. Para cada elemento Tj de Ψ*
          i. Si ∃Xp ∈ Tj tal que ri[Xp]=1 entonces añadir Tj a la
             lista Ψaux
         ii. De lo contrario, para cada Xp∈ ri tal que ri[Xp]=1
               1. Si Tj + Xp es un conjunto compatible, añadir
                  Tj ∪ Xp a la lista Ψaux
        iii. Ψ* = Ψaux
4. Regresar Ψ*
```

Figure 3. Pseudocode YYC algorithm (Authors' own elaboration).

The representation of the process followed by the algorithm can be seen in the flowchart in Figure 3. The degree of the vertex is the number of hyperedges contained in the vertex.d(v)$\mathcal{E}$v

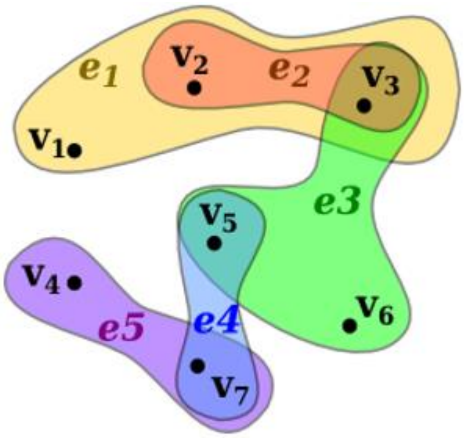


Figure 4: Representation hypergraph with 7 nodes and 5 hyperedges.

In the image above (Figure 4) you can see that node v1 only belongs to one hyperedge, while node v3 belongs to 3, so the incidence count is 1 and 3 respectively. Below is the hyperedge incidence count for the previous figure (Figure 5).

```
1. V3 = 3
2. V2 = 2
3. V5 = 2
4. V7 = 2
5. V1 = 1
6. V4 = 1
```

Figure 5. The algorithm ranks the nodes from highest to lowest according to their incidence value and delivers the n nodes with the highest incidence as a response (Own elaboration).

In the following section, the tools described in the hierarchy of competencies for the area of physics are used.

Implementation

To exemplify the Morganov-Heredia matrix method (Ramírez et al, 2013, Rillo et al, 2022) we will use the competencies proposed in the Latin America Tuning Project (Beneitone, 2007). The Tuning Latin America Project, in its section on physics programs, considers within the 22 specific competencies proposed a division of these that it calls "cognitive competencies", which in turn allowed them to be complemented by those that were considered important to develop for a professional who would dedicate himself to the teaching of physics (Ramírez et al, 2016). To choose the competencies to be developed, we worked with a group of 46 professors from the Physics and Mathematics career of the National Polytechnic Institute of Mexico, all of them with master's and doctorate degrees in the area of physics willing to contribute by choosing which competencies already mentioned, based on the 22 proposed by Tuning. They considered the most important ones to develop for the physics teacher. A consensus was reached among the most recurrent ones. From this exercise, 10 specific competencies were obtained, which are shown in the following list:

1. To propose, analyze and solve physical problems, both theoretical and experimental, through the use of analytical, experimental or numerical methods.

2. Construct simplified models that describe a complex situation, identifying its essential elements and making the necessary approximations.

3. Demonstrate a thorough understanding of the fundamental concepts and principles of both classical and modern physics.

4. Describe and explain natural phenomena and technological processes in terms of physical concepts, principles, and theories.

5. Estimate the order of magnitude of measurable quantities to interpret diverse phenomena.

6. Demonstrate work habits necessary for the development of the profession such as teamwork, scientific rigor, self-learning and persistence.

7. Seek, interpret, and use scientific information.

8. Communicate scientific concepts and results in oral and written language to their peers, and in teaching and dissemination situations.

9. Know and understand the conceptual development of physics in historical and epistemological terms.

10. Know the relevant aspects of the teaching-learning process of physics, demonstrating willingness to collaborate in the training of scientists.

These competitions were numbered from 1 to 10 in no specific order.

On the other hand, in order to give a didactic order to the specific competencies and to be able to link them with the topics and subtopics of learning in a specific course, it is recommended that the following question be asked in pairs: To develop competency X, is it necessary to develop competency Y first? (Ramírez et al, 2013).

Based on the previous question and in order to order the content of the competencies, a Morganov-Heredia matrix is made, in which each competency will be considered in a line and a column of the matrix (Ramírez & Olvera, 2012). In each box of the matrix, the answer to the question asked above will be considered, taking a positive answer as "1" and the opposite case as "0". In the case of the same competence in both the row and the column, no comparison is made since it cannot be a requirement of itself.

Thus, using the competencies already numbered and applying consensus among the group of physics professors who supported the aforementioned research, the following Morganov-Heredia matrix was obtained (Figure 6):

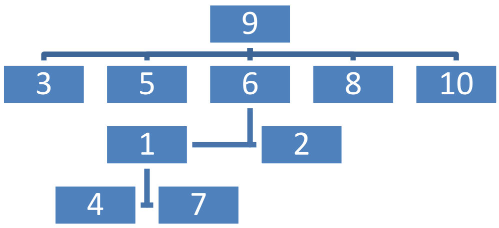
	1	2	3	4	5	6	7	8	9	10
1	X	0	1	1	1	0	1	1	1	0
2	1	X	1	1	0	1	1	1	1	0
3	0	0	X	1	1	0	1	1	1	0
4	1	0	1	X	1	1	0	1	1	1
5	0	0	1	0	X	0	1	1	1	1
6	0	0	1	0	0	X	1	1	1	1
7	0	1	1	1	1	1	X	1	1	1
8	1	1	0	0	1	1	0	X	0	1
9	0	0	1	0	1	0	0	1	X	1

10	1	0	1	1	0	1	0	1	0	X

Figure 6. Morganov-Heredia matrix obtained for the proposed competencies (Authors' own elaboration)

The next step in the procedure is to count the "ones" in each line for the competencies and then order from the competency that had the fewest to the largest to obtain the didactic order of development of the competencies. In our case, the order from the matrix is as follows (Figure 7):

Figure 7. Hierarchical order obtained from the Morganov-Heredia matrix (Authors' own



elaboration).

To illustrate how the hypergraph technique works, we will use the results shown in Figures 6 and 7:

```
[guillermosipe@gsp-linux algoritmo_YYC]$ python yyc.py -f olvera.json
Matriz de entrada:
          [0, 0, 1, 1, 1, 0, 1, 1, 1, 0]
          [1, 0, 1, 1, 0, 1, 1, 1, 1, 0]
          [0, 0, 0, 1, 1, 0, 1, 1, 1, 0]
          [1, 0, 1, 0, 1, 1, 0, 1, 1, 1]
          [0, 0, 1, 0, 0, 0, 1, 1, 1, 1]
          [0, 0, 1, 0, 0, 0, 1, 1, 1, 1]
          [0, 1, 1, 1, 1, 1, 0, 1, 1, 1]
          [1, 1, 0, 0, 1, 1, 0, 0, 0, 1]
          [0, 0, 1, 0, 1, 0, 0, 1, 0, 1]
          [1, 0, 1, 1, 0, 1, 0, 1, 0, 0]
psiaux 1 6
```

Resulting hypergraph (Figure 8):

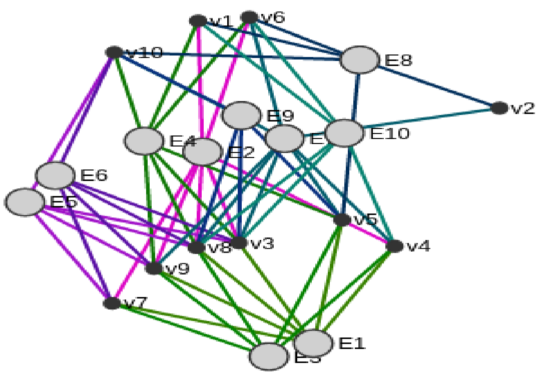


Figure 8. Hypergraph resulting from Morganov-Heredia matrix (Authors' own elaboration).

The results obtained were as follows:

Hyperedge incidence count (ordering from lowest to highest):

```
Conteo de incidencia en hiperaristas:
Counter({8: 9, 3: 8, 9: 7, 5: 6, 10: 6, 4: 5, 6: 5, 7: 5, 1: 4, 2: 2})
[2, 1, 4, 6, 7, 5, 10, 9, 3, 8]
```

Count by membership of transversals (ordering from lowest to highest):

```
=== Testores típicos ===

[3, 5]
[1, 3, 4]
[2, 3, 4]
[3, 4, 6]
[4, 10]
[1, 5, 10]
[5, 6, 10]
[1, 3, 7]
[2, 3, 7]
[1, 5, 7]
[4, 5, 7]
[5, 6, 7]
[3, 6, 7]
[1, 7, 10]
[3, 7, 10]
[6, 7, 10]
[1, 8]
[2, 8]
[5, 8]
[6, 8]
[8, 10]
[1, 3, 9]
[2, 3, 9]
[1, 5, 9]
[4, 5, 9]
[5, 6, 9]
[3, 6, 9]
[1, 9, 10]
[3, 9, 10]
[6, 9, 10]

Longitud psi: 30
Hits: 217
max_len: 30

Conteo de transverales:
Counter({3: 12, 5: 10, 10: 10, 1: 9, 6: 9, 7: 9, 9: 9, 4: 6, 8: 5, 2: 4})
[2, 8, 4, 1, 6, 7, 9, 5, 10, 3]
```

It can be seen that in order to match the method originally described by the Morganov-Heredia technique (Ramírez et al, 2013) using the above results, the matrix in its transposed form of the following form and the corresponding hypergraph (Figure 9) are necessary:

```
[guillermosipe@gsp-linux algoritmo_YYC]$ python yyc.py -f olvera.json
        [0, 0, 1, 1, 1, 0, 1, 1, 1, 0]
        [1, 0, 1, 1, 0, 1, 1, 1, 1, 0]
        [0, 0, 0, 1, 1, 0, 1, 1, 1, 0]
        [1, 0, 1, 0, 1, 1, 0, 1, 1, 1]
        [0, 0, 1, 0, 0, 0, 1, 1, 1, 1]
        [0, 0, 1, 0, 0, 0, 1, 1, 1, 1]
        [0, 1, 1, 1, 1, 1, 0, 1, 1, 1]
        [1, 1, 0, 0, 1, 1, 0, 0, 0, 1]
        [0, 0, 1, 0, 1, 0, 0, 1, 0, 1]
        [1, 0, 1, 1, 0, 1, 0, 1, 0, 0]


Matriz de entrada:
        [0, 1, 0, 1, 0, 0, 0, 1, 0, 1]
        [0, 0, 0, 0, 0, 0, 1, 1, 0, 0]
        [1, 1, 0, 1, 1, 1, 1, 0, 1, 1]
        [1, 1, 1, 0, 0, 0, 1, 0, 0, 1]
        [1, 0, 1, 1, 0, 0, 1, 1, 1, 0]
        [0, 1, 0, 1, 0, 0, 1, 1, 0, 1]
        [1, 1, 1, 0, 1, 1, 0, 0, 0, 0]
        [1, 1, 1, 1, 1, 1, 1, 0, 1, 1]
        [1, 1, 1, 1, 1, 1, 1, 0, 0, 0]
        [0, 0, 0, 1, 1, 1, 1, 1, 1, 0]
```

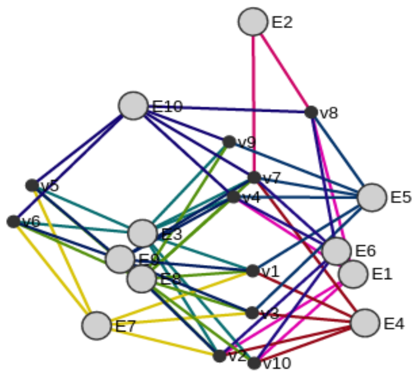


Figure 8. Transposed matrix and corresponding hypergraph (Authors' own elaboration).

From Figure 8 we get the following results:

Hyperedge incidence count (ordering from lowest to highest)

```
Conteo de incidencia en hiperaristas:
Counter({7: 8, 2: 7, 4: 7, 1: 6, 3: 5, 5: 5, 6: 5, 8: 5, 10: 5, 9: 4})
[9, 3, 5, 6, 8, 10, 1, 2, 4, 7]
```

Count by Transversal Membership (Ordering from Lowest to Highest)

```
=== Testores típicos ===

[2, 7]
[1, 4, 7]
[3, 4, 7]
[4, 5, 7]
[4, 6, 7]
[1, 8]
[2, 8]
[3, 4, 8]
[3, 5, 8]
[3, 6, 8]
[3, 7, 8]
[5, 7, 8]
[6, 7, 8]
[3, 8, 9]
[3, 8, 10]
[5, 8, 10]
[6, 8, 10]
[1, 7, 10]
[3, 7, 10]
[5, 7, 10]
[6, 7, 10]

Longitud psi: 21
Hits: 157
max_len: 21

Conteo de transverales:
Counter({7: 12, 8: 12, 3: 8, 10: 7, 4: 5, 5: 5, 6: 5, 1: 3, 2: 2, 9: 1})
[9, 2, 1, 4, 5, 6, 10, 3, 7, 8]
```

What we were able to see is that in the first case, Hyperedge Incidence Count shows the same results as Ramírez et al (2013), while in the case of Cross-sectional Count, only the first and last position is preserved (Figure 9).

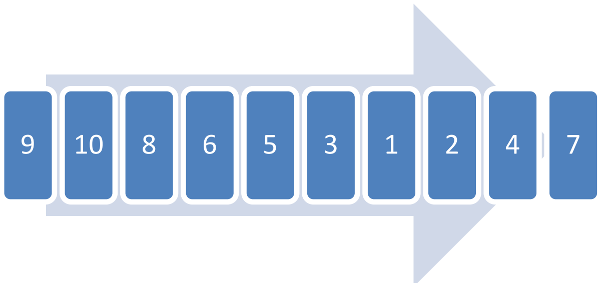


Figure 9. Hierarchy of competencies obtained by counting transversal competencies

Conclusions

From the Morganov-Heredia matrices, it can be seen that the algebraic technique of content analysis has an advantage that techniques with similar purposes do not have in mathematical formality to solve the problem of sequencing and hierarchization in a fixed, predetermined way. Such formality makes it possible to reduce operational work to a minimum compared to when operations are carried out manually, and even eliminate it completely when the data is processed automatically. In this way, the person or group of people in charge of organizing a course, a program, a curriculum, etc., can have the time they would waste doing work not directly related to discussion and decision-making. Experience with the algebraic technique allows us to estimate that it reduces the work by 20 percent when the operations are performed manually and up to 5 percent and even less when the program is available. This is of particular interest in the field of education in general, where it is common to find projects of great diversity and with many elements to be properly organized.

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