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Factors Limiting Modeling Integration in the Mathematical Thinking-Representation

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Abstract

Mathematics teaching traditionally focuses on the mechanical learning of principles, formulas, and algorithms, disconnecting from the reality students face and their mathematical interpretation. Although there is research on numerical representation in various contexts, some educational centers have not yet incorporated modeling to strengthen mathematical thinking and understand real-world situations. This study seeks to identify the factors limiting modeling integration in mathematics teaching. It is a descriptive qualitative research based on a documentary review. The analysis found that the factors that restrict the adoption of modeling are socioeconomic, pedagogical, cognitive, and psychological. Therefore, it is necessary to implement pedagogical and didactic measures that encourage the active participation of students in the construction of mathematical knowledge. At the same time, educators need to act as guides and facilitators in this process.

Keywords: *Education, Mathematics, Modeling, Mathematical thinking.*

Introduction

Since social and technological advances, mathematics teaching has undergone significant transformations. As in the past, mathematical knowledge was oriented from the operationalization and formalism of the nature of the discipline. According to Reverand (2003), this teaching method privileged the axiomatic character, leading to the interpretation of mathematics as a finished product without discussion.

Contemporary conceptions imposed by traditional teaching methods have been redefined in the modern world, making room for the linking of sociocultural factors and the student's analytical reflection based on the reality they face in their context. According to Zimmerman and Cuningham (1991), the relationship between mathematics and the environment cannot be ignored. Visualizing and representing contextualized situations allow the student to conceive mathematical thinking beyond the simple symbolic and numerical representation.

Mathematical modeling as a pedagogical strategy enables students to connect their lived experiences to the cognitive aspects of mathematics (Blomhøj, 2004; Córdoba, 2011). By representing real-world situations through mathematical models, learners can relate

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mathematical concepts to potential solutions for issues arising in problem-solving contexts. As such, Villa (2009) contends that modeling in the classroom serves the critical purpose of elucidating problems from other disciplines via mathematics. By framing mathematical objects meaningfully, modeling facilitates students' construction of mathematical knowledge. By integrating modeling approaches, teachers can support learners in connecting environmental experiences to mathematical understanding.

Despite offering many advantages, the integration of modeling into the school classroom, including the appropriation and interpretation of mathematical thinking, many teachers who guide disciplinary teaching do not link it in their pedagogical praxis. As Ortiz and Camelo (2020) point out, some different barriers or limitations hinder the didactic use of modeling, for example, managing groups with a large number of students, since each one has a uniqueness in their learning and teachers are limited in providing personalized training. Likewise, the little knowledge that educators possess has led them to continue developing the pedagogical process through traditional models.

However, Olarte (2020) emphasizes that the modeling processes implemented in school classrooms occasionally fail to align with the requisite stages for correlating the representation to the meaning of the mathematical object. In other words, modeling is utilized merely as a straightforward graphic representation. Conversely, educators prioritize the acquisition of procedural knowledge, tantamount to resolving problems utilizing theorems, formulas, and algorithms, which, to a certain degree, constrains analysis and reflection in the assimilation of mathematical knowledge.

According to Villa and Alencar (2019), while mathematical modeling is recognized as an indispensable mathematical competency for solving contextual problems, educational institutions' efforts to implement it to the depth required have been insufficient. Mathematical modeling involves navigating unfamiliar terrain for teachers. It begins with formulating modeling problems and appropriating its cycle of simplifying, mathematizing, interpreting, validating, and communicating solutions. Achieving detachment from traditional pedagogy poses an even more significant challenge.

Another factor limiting the integration of modeling for strengthening mathematical thinking is the didactics educators use in their pedagogical work. As Lingefjärd and Holmquist (2005) state, some educators have not redefined the didactic elements of the pedagogical process, preferring to utilize conventional resources like textbooks, which focus on reproducing knowledge via the educator's lecture. This situation constrains students' proactive participation in their academic training process.

Based on the references discussed, this study focuses on identifying the factors limiting the integration and pertinent use of modeling to strengthen mathematical thinking. Furthermore, it proposes guidelines and strategies that allow teachers to link mathematical modeling as a fundamental element in students' appropriation of the discipline.

Literature Review

The following section details theoretical references relevant to the object of study, particularly expanding on topics related to modeling and mathematical thinking.

Mathematical thinking in the school classroom

Mathematical thinking plays a vital role in mathematics education, as numerous scholars have argued. Pólya (1963), Stacey et al. (2010), and Hıdıroğlu and Can (2020) all contend that mathematical thinking enables students to engage in logical, numerical reasoning and reflection while also developing fundamental mathematical competencies that are critical for academic success. Moreover, incorporating mathematical thinking into mathematics pedagogy moves beyond rote numerical skills, opening space for higher-order cognitive processes, including conceptual understanding, contextualizing mathematical concepts in real-world situations, and logical relationship building through representation. By leveraging mathematical thinking in the classroom, students can thus deepen their comprehension and application of mathematics in analytic, creative, and meaningful ways.

In contrast to these approaches, several scholars have argued that mathematical thinking strengthens disciplinary knowledge and empowers educators to guide mathematics instruction through problem-solving (Borromeo, 2003; Takahashi, 2007; Newton, 2012; Capone, 2022). Furthermore, integrating mathematical thinking into the classroom can promote critical reasoning skills like inference, exemplification, abstraction, hypothesis formulation, and verification, equipping students with essential 21st-century competencies.

However, despite the purported benefits of mathematical thinking, some researchers have identified limitations in current pedagogical practices. Álvarez and Álvarez de Zayas (2004), Ron (2007), and Vargas (2020) contend that many educators' methods restrict problem-solving, favoring rote mechanical exercises and formulaic applications over analytical and reflective thinking. Additionally, critical phases of the pedagogical process sometimes occur separately from students, limiting their agency and active participation in their learning.

One of the factors limiting the development of mathematical thinking among students is their difficulty in appropriating knowledge. As Aebli (2002) indicates, students sometimes struggle to interpret the processes needed to solve problem situations. This stems from an inability to integrate conceptual frameworks, with prior knowledge failing to provide a sufficient foundation for the transition from concrete to abstract reasoning. That is, their mathematical thinking lacks consistency. Faced with these challenges, teachers and students may question the sources of errors impeding the acquisition and application of mathematical thinking. The answer lies in innovating teaching methods and didactics for mathematics education. Educators must reimagine their pedagogical praxis, restructuring mathematical activities and utilizing active methodologies that position students at the center of the learning process. Doing so can minimize problems stemming from underdeveloped mathematical thinking.

In developing school activities, Schoenfeld (2016) considers it vital that students engage in search, research, observation, and reflection. Through these actions, students can form mental representations of mathematical objects and relate them to real-world experiences. In doing so, they can also advance their mathematical thinking. Aligning with this perspective, León et al. (2016) emphasize that if mathematical thinking is to be fostered as a core educational competency, it must permeate math lessons in the classroom. However, the onus lies with teachers to integrate the necessary methods and didactics into their pedagogical practice. This will help ensure that students incorporate mathematical thinking into their cognitive structures. In summary, by intentionally designing activities centered on inquiry and reflection and approaching instruction with pedagogical techniques suited to mathematical thinking, educators can support students in developing this critical capability.

Modeling and mathematical thinking

Modeling represents an emerging field that connects mathematical thinking to real-world representations. While various researchers have attempted to define this construct, the most robust conceptualizations come from Blomhøj (2004), King et al. (2005), and Córdoba (2011). These scholars characterize modeling as manipulating, representing, and communicating real-world disciplinary objects using mathematical principles. By linking graphical and visual representations, modeling can simulate complex processes to validate assumptions and communicate modeled knowledge. At its core, modeling seeks to connect mathematical knowledge with students' lived realities, enabling them to comprehend situational and environmental phenomena. Therefore, the overarching purpose of modeling is to bridge modeled situations or phenomena with mathematical concepts.

Similarly, Blum and Borromeo (2009) argue that incorporating modeling into mathematics learning, especially for developing mathematical thinking, can motivate students to explore the world around them. By interpreting their surroundings through numbers, formulas, and mathematical principles, students can gain competencies and skills to propose, interpret, validate, and communicate problem situations involving mathematical objects.

However, modeling extends beyond the representation of mathematical knowledge. As Li and Schoenfeld (2019) explain, modeling equips students with skills to interpret reality through experimentation. Students can link data, opinions, and ideas with peers when representing relationships between variables. This process leads to conclusions framed by the application of the scientific method. Moreover, the authors argue that modeling is vital for integrating knowledge across disciplines in school environments. This integration fulfills modern educational demands that promote STEAM (Science, Technology, Engineering, Arts, and Mathematics) connections throughout the curriculum.

For those assuming the integration of modeling into mathematics classrooms is straightforward, researchers contest this notion. They argue that implementing modeling requires extensive time, complex planning, and coherent activities that genuinely incorporate the modeling cycle. As De Almeida Luna et al. (2015) and Greefrath and Vorhölter (2016) discuss, barriers exist that inhibit utilizing modeling to develop mathematical thinking. One challenge relates to educators' limited time designing modeling activities tailored to students' grade levels, as even current curricular content strains available time.

Another factor is students' diverse learning styles, as modeling activities may prove complex for some. Moreover, many educators lack sufficient training to implement modeling to support mathematical thinking, even if trained in mathematics. Additionally, insufficient resources and materials in schools hinder modeling adoption. In summary, while modeling can advance students' mathematical thinking, meaningful classroom integration demands considerable planning, resource allocation, and teacher training to address barriers posed by time constraints, student differences, and educator knowledge gaps. Overcoming these hurdles requires recognizing the complexities of curricular modeling.

Teacher training is a critical factor for the integration and utilization of modeling in the learning of mathematics. As stated by Zaldívar et al. (2018) and Méndez et al. (2020), promoting initial training in modeling for future teachers is necessary for higher education institutions where educators develop pedagogical practices. However, ongoing training processes are also imperative, as only through these can planning, design, and didactic strategies be implemented to successfully apply modeling in developing mathematical thinking. Furthermore, modeling should not just be conceived from a theoretical perspective but also from a didactic one, as teachers-in-training need clarity on how modeling can be linked and articulated with mathematics and identify activities that enable the consolidation of modeling competencies in students.

Due to the reasons above, students' use of modeling is constrained, as educators prioritize teaching mathematics from an abstract perspective, namely by focusing on formally learning formulas and algorithms in a mechanistic manner, disconnected from situating them in environmental contexts that enable establishing connections between the mathematical and real world. Furthermore, Villa et al. (2022) note that the discourse of educators guiding the teaching of mathematics centers on students appropriating mathematical ideas and procedures, neglecting students' analytical, argumentative, and reflective capacities. This results in poor student performance on standardized assessments like SABER in Colombia and PISA internationally.

Modeling processes in mathematics classrooms are often limited to the graphical representation of situations, neglecting the existing relationships between all the variables of the mathematical object. As Rendón et al. (2015) suggest, modeling refers to mathematical knowledge's visual representation. However, the modeling cycle encompasses simplification, mathematization, validation, and communication of the connections between the mathematical world and different disciplinary areas.

In summary, various factors constrain the integration and use of modeling in the mathematics classroom. Therefore, this research focuses on expanding these factors by reviewing the research tradition. However, beyond mere analysis, the aim is to formulate guidelines and actions to appropriate the modeling process so that teachers and students can relate mathematical knowledge to different real-world situations. In other words, disciplinary learning solves real-world problems by integrating modeling as a stimulating element to strengthen thinking.

Materials and Methods

Given the nature of the study, various research methods are integrated into the research process. Specifically, the approach is qualitative, through which we aim to analyze the object of study from a holistic perspective, examining fundamental aspects for the use of modeling in developing mathematical thinking. As Hammersley and Atkinson (1994) state, qualitative research enables elaboration on the situation's nuances under investigation. Additionally, the research scope is framed within the descriptive method, which, as per Ander Egg (1995), facilitates delineating the features and characteristics of the phenomenon or situation being studied, in this case, the factors constraining the integration and use of modeling as a strategy to enhance mathematical thinking in students.

Regarding the design, this study constitutes documentary review research, analyzing the perspectives of different researchers concerning aspects that impede integrating modeling as a critical strategy to relate mathematical objects with real-world situations students encounter. As Barraza (2018) notes, documentary studies belong to the bibliographic research category, with the principal characteristic being using secondary data as an information source to direct the research process in two ways. First, relating the available information to the object of study and associating it to establish a panoramic view of the phenomenon or situation under investigation.

Categorías de Estudio

Based on the research methods delineated previously, specifically the primacy of the qualitative approach and the partial review of the literature associated with the object of study, four categories have been proposed to elucidate the underlying causes or factors impeding the integration and proper implementation of modeling in the mathematics classroom. Table 1 enumerates these proposed categories.

Table 1 Operationalization of study categories

Category	Subcategory	Dimension	Indicators
	Social factors	Students' socioeconomic background characteristics	- Strengths and weaknesses of the context - Collaboration of parents in the formation process of the students.
		Pedagogy (teaching methods)	Integrated teaching methodology to guide mathematics learning.

Factors limiting the integration and adequate use of modeling in the reinforcement of mathematical thinking	Pedagogical Factors	Didactics	Examination of the predominant resources utilized by educators and students during mathematics instruction.
	Cognitive factors	Cognitive aspects that affect mathematical learning	Difficulties limiting student learning in the area of mathematics
	Psychological factors	Students' likes, interests, emotions, and motivations in disciplinary learning.	Activities most liked or disliked by students in the development of mathematics classes.

Note. The above table delineates the study categories through which the factors or causes limiting the integration and appropriate implementation of modeling in mathematics pedagogy.

The proposed categories facilitate the analysis of studies comprising the research tradition to determine the potential factors or causes impeding the integration of modeling in the mathematics classroom and the resultant repercussions. Examining the literature through these categories provided a systematic framework to elucidate the constraints around modeling integration and their downstream effects.

Research Techniques and Instruments

A systematic literature review was conducted to synthesize the current state of knowledge within the disciplinary field. Data were extracted from relevant studies identified through a comprehensive search of databases and other sources. Identified studies were screened for relevance based on pre-defined eligibility criteria. Data from included studies were extracted using a standardized form, capturing key study characteristics, methods, and findings. Extracted data were managed in Microsoft Excel to facilitate synthesis and analysis, as shown in Figure 1.

Figure 1 Documentary Corpus Model

Campo	Peso	Investigaciones consultadas													
		Ord.	Número Documento	Título de la investigación	País	Año	Tipo de material	Objetivo de la Investigación	Pregunta de la Investigación	Tipo de Investigación	Resultados de la investigación	ME SIRVE PARA	Conceptos tratados en la investigación Concepto	Aportes a mi investigación	Referencia dirección sitio web
		1													
		2													
		3													
		4													
		5													

Note: The figure shows the documentary corpus model used to retrieve information derived from the review process.

Developing a documentary corpus was integrated into the research process to collect and organize information for subsequent interpretation systematically and to gain an in-depth understanding of the object of study. In particular, the compiled documentary corpus facilitated expanding the understanding of the objectives, methods, findings, conclusions, and contributions of the studies included in the analyzed research overview. Developing the corpus enabled a comprehensive synthesis of the existing literature to extract critical insights into the factors constraining the integration of modeling in mathematics classrooms. The corpus-based approach enabled a robust synthesis of the existing literature to extract critical insights into the factors constraining modeling integration in mathematics classrooms.

Phases in the research process

The research process in this study was carried out in three phases directly related to the objectives formulated.

- Phase One - Document Retrieval

The initial phase of this study comprised a systematic review of the literature on mathematical modeling in the classroom context. Searches were conducted to identify relevant studies from peer-reviewed journals, dissertations/theses, books, and other scholarly sources. Details of the search methodology are provided in the results section.

- Phase Two - Analysis and Reflection

This research phase comprised a qualitative synthesis and critical appraisal of the studies identified through the systematic review. The aim was to analyze and interpret the collective evidence on using mathematical modeling to strengthen thinking skills in the classroom context. Extracted data were analyzed using an inductive, thematic approach to identify key benefits and challenges related to modeling pedagogies.

- Phase Three - Improvement

The third phase of the research process aims to formulate guidelines and strategies to integrate and correctly use modeling in mathematics learning.

Results

This section presents the results obtained from the research process, which focuses on analyzing the study categories to identify the factors that impede the integration of modeling into mathematics teaching contexts. The findings presented in this section are the product of fulfilling the research objectives and executing each of the phases previously delineated in the methodological design.

Results Document Retrieval Phase

Specific methods were employed to design a work plan to recover relevant documents for the research to fulfill the first objective of this study, which focused on reviewing the research tradition of integrating and utilizing modeling in the mathematics classroom. To this end, two stages of searching for and selecting bibliographic references were conducted. In the first stage, an exhaustive search was undertaken for documents concerning modeling in mathematics education, utilizing various tools, including databases, university repositories, and unpublished digital sources. Once the bibliographic material was assembled in the second stage, we classified it concerning the study categories delineated in Table 1.

In order to retrieve the relevant references consulted, specific inclusion and exclusion criteria were established for selecting bibliographic material pertinent to the present study.

- Academic productions that link the study of modeling in mathematics learning.
- Articles, theses, books, and book chapters, among others, analyze situations or factors that affect modeling learning.
- Bibliographic material related to the methods and didactics used by teachers who guide mathematics teaching.
- Peer-reviewed academic publications published in peer-reviewed journals with recognized indexing.

In addition, for the bibliographic sources exclusion, the following criteria were taken into account:

- Eliminate documentary sources in which topics related to the object of study are not linked
- Not taking into account articles in which, although allusion is made to mathematical modeling, possible factors that influence its use by educators who guide the teaching of mathematics are not analyzed.
- It omitted documentary material that does not offer the expected scientific rigor in the results.

The bibliographic material related to the study was organized through a documentary corpus, in which about 60 research studies related to the object of study were registered.

Results Analysis and Reflection Phase

After configuring the documentary corpus of research references, we analyzed the information, framing it in each established study category. The following results were derived from this analysis:

- Social Factors Category

A review of the extant research literature reveals that the students' social context impacts their learning process. As Guerra and Grino (2013) underscored, mathematics learning is shaped by various factors, including students' socioeconomic status, social milieu, and the family dynamics they navigate.

Regarding the low academic achievement of students in the aforementioned disciplinary field, researchers such as Reardon (2012) and Duarte and Bos (2012) assert that students from lower socioeconomic backgrounds exhibit poorer school performance. Moreover, they highlight that the existence of socioeconomic inequalities favors minority groups since families with higher incomes invest in their children's education. At the same time, those with fewer resources can barely afford to cover basic needs, let alone purchase school supplies. Additionally, the authors note cases in which parents have low levels of education, with some having merely completed primary school, which hinders student learning since they cannot rely on adequate support at home for academic activities.

Based on the scholarly literature, authors including Chaparro et al. (2016), Abuya et al. (2018), and Rodríguez and Guzmán (2019) argue that parents' knowledge and skill levels are a determining factor in their children's academic performance, especially in reading comprehension and mathematics. Students tend to perform better academically when their parents have higher education than peers whose parents did not complete primary schooling. Therefore, parents' academic background is crucial to their children's educational success.

The Organization for Economic Cooperation and Development (OECD, 2016) has pointed out that the low academic performance of Colombian students is related to socioeconomic problems. Therefore, establishing public policies that reduce the gap between strata is fundamental to improving school performance and promoting social cohesion.

From a sociological perspective, socioeconomic factors directly impact learning processes and academic performance. As evidenced by several studies, students from lower strata face difficulties derived from the scarcity of educational resources and limited access to knowledge. In contrast, those from affluent backgrounds have facilities for learning, such as the Internet, tutoring, and educated parents. Consequently, teachers need to consider the social context of learners in order to reduce gaps and provide a comprehensive and inclusive education.

- Pedagogical Factors Category

Mathematical modeling is considered essential for developing students' mathematical thinking. However, its effective integration is commonly hindered by teachers' application of conventional teaching methods and the limited availability of suitable resources and materials in the educational environment. This unfavorable reality can lead to poor student performance in modeling and understanding mathematical concepts.

According to research by González et al. (2014), conventional teaching methods can restrict students' ability to apply modeling to real-world situations and develop their mathematical thinking skills. In line with these findings, Maaß and Artigue (2017) argue that the traditional approach to teaching mathematics focuses on problem-solving but does not sufficiently emphasize a deep understanding of mathematical concepts and objects or the application of modeling to everyday situations.

Various experts have investigated how educators apply methodology and didactics in the classroom for teaching the modeling cycle. Per these investigations, the predominance of traditional approaches can limit student participation, impeding the proper acquisition of skills necessary for the modeling process. In particular, Blum and Borromeo (2009), Doerr et al. (2012), Ernest (2014), and Kaiser and Sriraman (2015) have shown that conventional teaching methods based on information transmission and memorization do not encourage the development of students' mathematical skills. Regarding modeling specifically, they have highlighted that the traditional approach does not promote students' ability to model real-world situations and apply mathematics to meaningful contexts. Moreover, the traditional methodology focused on problem-solving and algorithm application does not encourage creativity and mathematical thinking. Using conventional didactic resources like textbooks and blackboards can also make a deep understanding of disciplinary concepts and constructing accurate mathematical models difficult.

It is important to note that the unsuccessful incorporation of modeling in the educational context is not solely due to inadequate methodology and didactics used by some teachers. According to Mejía (2019), Lara and Peralta (2019), Gómez and Sánchez (2019), Espinoza and Gallegos (2019), and Bautista and Forero (2021), teaching modeling is a complex task requiring specialized knowledge and careful planning. Some teachers may feel intimidated and unsure of how to effectively approach modeling in their pedagogical practice due to a lack of competence. This situation can negatively impact students' understanding and ability to apply modeling to real-world situations. Suppose students do not have an adequate knowledge of modeling. In that case, they cannot develop the necessary skills to apply mathematics in solving real-world problems. Consequently, they may struggle to establish connections between mathematical concepts and situations requiring their use.

- Cognitive Factors Category

The review of scientific literature has shown that the cognitive disabilities present in students vary in severity and scope, which can negatively impact their ability to process complex mathematical information. Research conducted by subject matter experts such as Campbell et al. (2017), Hsieh et al. (2018), and Krawec et al. (2020) reveals that some students with these challenges may have limitations in understanding and applying abstract mathematical concepts, making it difficult for them to participate in mathematical modeling activities. This can prevent students from connecting mathematical concepts and real-world situations requiring modeling. As a result, learners may struggle to relate mathematical concepts to situations in their environment and improperly use modeling.

Additionally, strengthening mathematical modeling and promoting mathematical thinking poses challenges in teaching and learning these skills. Considering each student's particularities, these challenges are intrinsically tied to cognitive factors, which can restrict progress and understanding of mathematical modeling processes if not addressed. Through an exhaustive review of academic literature, four specific cognitive factors have been identified as obstacles to acquiring fundamental mathematical modeling knowledge, negatively impacting the development of mathematical thinking.

The cognitive factors are 1) working memory capacity, 2) inhibitory control, 3) cognitive flexibility, and 4) abstract reasoning. Students with limitations in these areas may find it difficult to retain information, switch between different models, connect models to abstract concepts and comprehend theoretical modeling frameworks. Educators must account for these cognitive constraints and provide targeted scaffolding to facilitate modeling competency development. A personalized, student-centered approach can mitigate the adverse effects of individual cognitive challenges on mathematical thinking and modeling skill acquisition.

- Limited prior knowledge

According to researchers such as Vanbecelaere et al. (2020), Wei et al. (2020), Carrillo (2023), and Pérez et al. (2023), one of the critical cognitive factors influencing the learning process of modeling and development of mathematical thinking is the lack of established prior knowledge. Making connections between concepts and applying them in concrete situations requires a solid foundation of mathematical knowledge. Suppose students have not acquired a deep understanding of the fundamental concepts. In that case, they will find understanding and applying mathematical modeling challenging.

- Logical reasoning skills

Mathematical modeling involves the ability to abstract real-world situations and represent them using mathematical symbols and equations. However, according to studies by Flegas and Charalampos (2013), Mirzaxolmatovna (2021), Purwitaningrum and Prahmana (2021), and Bronkhorst et al. (2021), some students may find it challenging to make the transition from the concrete to the abstract, which hinders their understanding of mathematical modeling. Furthermore, generalizing and applying mathematical concepts across different contexts requires an advanced level of cognitive abstraction that not all students fully develop.

- Abstraction difficulties

Another cognitive factor that constrains the learning process of mathematical modeling is the insufficient development of abstraction ability, which involves the skill to represent mathematical concepts and relationships through symbols and equations. It is essential to highlight that mathematical modeling requires symbolic language to express mathematical relationships and construct models representing real-world situations. However, in studies conducted by Davydov (2020), Ramos et al. (2020), and Aguilar et al. (2021), the researchers have pointed out that some students struggle to understand and manipulate such symbols in the school setting, which hinders their ability to carry out the mathematical modeling cycle adequately. This limitation in symbolic abstraction can harm their comprehension of equations and restrict their ability to solve mathematical modeling problems accurately.

- Problem-solving limitations

The ability to mathematically model involves tackling complex problems, identifying relevant variables, formulating assumptions, and validating obtained results. These capabilities are closely tied to mathematical thinking. However, despite the aforementioned, research conducted by Siagan et al. (2019), Fuentes et al. (2019), and Santos (2020) reveals that some students face challenges in problem-solving due to limitations in their ability to plan strategies, organize information, and evaluate achieved outcomes. These cognitive constraints can impede the modeling process, negatively impacting the development of advanced competencies within mathematical thinking.

In summary, the cognitive factors previously outlined, including limitations in problem-solving, difficulties in abstraction, logical reasoning skills, and limited prior knowledge, pose considerable challenges in acquiring mathematical modeling skills. These obstacles can adversely affect students' aptitude to comprehend, apply, and efficiently develop this discipline. Consequently, it is essential to address and overcome these cognitive factors by implementing suitable pedagogical strategies that promote more profound, more flexible mathematical thinking, thereby facilitating substantial improvements in learning and competency in this area. Targeted scaffolding, explicit instruction, and opportunities for repeated practice may help students develop the higher-order cognitive skills required for expert mathematical modeling and problem-solving. Additional research is needed to design and assess instructional interventions to support students' cognitive growth in this domain.

- Psychological Factors Category

In the context of the modeling learning process and the promotion of mathematical thinking, it is crucial to consider not only the cognitive aspects but also the psychological difficulties that can have a negative impact on student learning. After thoroughly analyzing existing research, it has been identified that the psychological factors that most significantly influence the acquisition of disciplinary modeling knowledge are related to a lack of enthusiasm, interest, and satisfaction with the learning process. These attitudes represent a significant obstacle to developing the mathematical skills to properly carry out the modeling cycle (Mutlu, 2019; Heyder, et al., 2020; Szücs & Mammarella, 2020).

According to research by Sierra (2019), El Adl and Alkharusi et al. (2020), and García et al. (2021), it has been corroborated that students can experience levels of affliction and apprehension associated with their performance and achievements in mathematics. The demand for favorable results and comparison with classmates can induce a feeling of tension that negatively affects their ability to carry out mathematical modeling effectively. These emotional difficulties can lead to cognitive obstacles and difficulties in maintaining concentration and developing clear thinking, which limits students' ability to face and solve mathematical problems that require modeling processes.

The way students perceive difficulty and their belief that they are incapable of understanding or mastering mathematical concepts constitute an additional psychological factor that can influence the process of learning mathematical modeling. According to the perspective presented by Alhassora (2017), Mahecha, et al. (2021), Mu and Guo (2022), and Yaftian and Barghamadi (2022), those students who rate themselves as "not very competent" in mathematics may experience a decrease in their sense of self-efficacy and a lack of confidence in their abilities. These limiting beliefs can obstruct their active participation in mathematical modeling and hinder their progress and development in this discipline.

In summary, based on a thorough review and analysis of the scientific literature, it can be concluded that psychological challenges such as lack of attraction and enthusiasm, performance-related anxiety, and perceptions of difficulty and limited self-efficacy constitute significant barriers in the process of acquiring skills in mathematical modeling and developing mathematical thinking. It is relevant to apply pedagogical strategies that promote student motivation, confidence, and emotional well-being to create an environment conducive to learning and encourage a favorable attitude towards mathematics.

Results Improvement Phase

Based on the findings that emerged from the analysis of the research categories, it has been possible to determine that the most prominent obstacles to the integration of modeling in the representation and development of mathematical thinking are found in the social, pedagogical, cognitive, and psychological domains. Consequently, proposing some guidelines to overcome the identified problems is opportune.

Following the above, this study proposes measures to mitigate the obstacles identified in each study category examined. The objective is to effectively address the detected difficulties and achieve significant progress in the field of study.

- Social dimension

The collective construction of knowledge in the mathematics classroom through modeling requires an approach that values and promotes the social dimension as a crucial factor. Establishing a conducive environment where students feel motivated and supported to participate in modeling activities is imperative. This process implies encouraging collaboration among peers, dialogue, and interaction, fostering a climate of trust and mutual respect. In this way, the development of social skills and the joint construction of mathematical knowledge are facilitated.

Among the essential guidelines, it is convenient to highlight the promotion of active and collaborative student participation in integrating modeling in the mathematics classroom. Conducive environments are necessary where students can interact jointly, share ideas, propose hypotheses, and solve problems as a team. Encouraging collaboration fosters an enriching exchange of knowledge and experiences, which contributes to the modeling process and boosts the development of social skills, such as effective communication, negotiation, and teamwork.

It is also fundamental to consider organizing projects or challenges involving the entire educational community because by connecting mathematical modeling with authentic everyday life situations, students can appreciate the relevance and applicability of mathematical concepts in authentic contexts. This link allows them to go beyond the school environment's boundaries and use modeling to address real-world problems and challenges. In this way, meaningful learning strengthens the connection between mathematics and students' daily lives.

Consequently, the guidelines aimed at overcoming the difficulties identified in the social sphere must be aware that making progress requires time, perseverance, and an unwavering commitment from educators and the educational community. However, the long-term benefits, both in terms of learning mathematics and developing social skills, fully justify the effort made. It is necessary to understand that positive results in the social aspect will not be achieved overnight. However, their achievement constitutes a valuable investment for the integral growth of students.

- Pedagogical Dimension

In the pedagogical dimension, it is essential to carry out concrete measures to improve the integration of modeling in the educational environment of mathematics. To achieve this goal, one of the primary guidelines lies in providing training and education to educators about the effective use of strategies and didactic approaches that effectively incorporate modeling. In this sense, it is essential to provide teachers with updated and relevant pedagogical tools and resources that allow them to design learning experiences in which students can actively participate in activities related to mathematical modeling.

In another vein, it is relevant to focus on creating and using educational resources that facilitate understanding mathematical foundations through modeling. These resources can range from interactive simulations and educational games to digital applications that promote experimentation and problem-solving linked to real situations. Using this resource allows students to explore and apply mathematical concepts in a practical and meaningful way, strengthening their understanding and interest in learning modeling.

Another of the guidelines to consider is promoting creativity and critical thinking in students through the modeling approach. Teachers must encourage an educational environment that stimulates the generation of concepts, the formulation of assumptions, and cooperative work. Students must be encouraged to ask questions, investigate, and seek solutions to mathematical problems using modeling as a central resource. This way, advanced cognitive skills are stimulated, and thoughtful and analytical thinking is encouraged in the mathematics teaching-learning process.

Consequently, in the pedagogical field, it is essential to provide training to educators in the use of approaches and didactic techniques that integrate modeling, in addition to developing and using educational resources that facilitate the understanding of mathematical concepts through this methodology. At the same time, promoting originality, analytical thinking, and collaboration within the classroom is crucial. These measures will help transform the educational process of mathematics, offering students an enriching and meaningful experience.

- Cognitive Dimension

In the cognitive dimension, it is essential to implement improvements that boost the development of mathematical thinking through modeling. To this end, one essential guideline is establishing meaningful connections between mathematical concepts and their applicability in real situations. This strategy involves providing students with examples of problematic situations that require modeling for resolution. By facing these challenges, students can apply their mathematical knowledge in a tangible and contextualized way, reinforcing their understanding of concepts and their aptitude to transfer them to different contexts.

During the mathematical modeling process, it is essential to encourage critical and reflective thinking in students, especially those with dysfunctional conditions such as autism (ADHD), dyscalculia, or cognitive disability. In this context, it is necessary to adapt both the teaching resources and the methodologies used to stimulate their ability to question, justify, and evaluate the solutions they generate through modeling.

In summary, the guidelines to improve the cognitive dimension must promote the development of solid mathematical competencies through modeling. This implies establishing meaningful connections between mathematical concepts and their practical application while stimulating critical and reflective thinking in students. These guidelines will strengthen the understanding of mathematics and develop fundamental cognitive skills that will benefit the students' school education process and other aspects of their lives.

- Psicological Dimension

The crucial role of the psychological dimension in successfully integrating modeling in the mathematics classroom cannot be underestimated. Providing students with solid emotional and motivational support is indispensable to enrich this dimension, establishing a safe and respectful environment that actively promotes exploration and learning by applying the mathematical modeling cycle.

In the academic sphere, it is fundamental to provide emotional support to students when addressing the resolution of mathematical challenges, with it being essential to recognize their emotional states and provide them with the necessary support to manage possible levels of anxiety or frustration. Aware of this premise, employing techniques that foster empathy, positive encouragement, and constant positive reinforcement is indispensable. It is important to remind students that the mathematics learning process is gradual and that errors constitute valuable opportunities to grow and improve in this discipline.

On the other hand, it is pertinent to establish a conducive environment where students can generate trust and feel comfortable sharing their ideas, asking questions, and actively participating in mathematical modeling. This purpose is achieved by promoting open communication and cultivating an environment of mutual respect, where diversity of thought is valued, and individual contributions are recognized as significant elements.

Likewise, in the psychological dimension, offering constructive feedback and appropriate recognition of student achievements is relevant. It is essential to highlight individual efforts and advances, emphasizing demonstrated strengths and providing precise guidance for continuous improvement. The feedback should be transparent, impartial, and framed within a growth mindset approach, allowing students to identify areas for improvement and set achievable goals in their learning process.

From this perspective, implementing these improvements in the psychological dimension of mathematics teaching and learning engenders a conducive environment that favors the development of mathematical thinking and the acquisition of modeling skills. By providing students with motivation, confidence, and security, they actively participate, explore new ideas, and face mathematical challenges positively. As a result, their self-esteem, sense of self-efficacy, and enjoyment in the mathematics learning process enhance students' comprehensive development.

Discussion

Through a review and analysis of the relevant scientific literature on the topic in question, it was possible to determine that the elements with the most significant impact on the effective acquisition and application of mathematical modeling are mainly related to social, pedagogical, cognitive, and psychological aspects.

It is relevant to highlight, according to the statements of researchers, that in the social sphere, not all students possess the necessary resources to achieve the objectives related to modeling. According to Reardon (2012) and Duarte and Bos (2012), in households with low economic resources, there is a tendency for students to exhibit low academic performance in mathematics, while in privileged households, where parents mostly hold university degrees, there is a tendency to replicate the process of academic training with their children. In addition, by having sufficient economic resources, students can access various learning resources, such as the Internet, books, and private lessons.

Based on the above, it is relevant that schools, through the implementation of programs and projects, focus on closing the existing socioeconomic gap in the educational environment. It is advisable to establish forceful actions that support students from rural areas and low socioeconomic backgrounds to facilitate the acquisition of school resources that encourage their learning. Likewise, educational institutions could train parents who have not completed their primary education since some lack this education level.

Concerning the pedagogical dimension, the analysis of studies revealed that the teaching methods and approaches used by educators in their teaching practice have a negative impact on the appropriation of modeling as a tool to represent mathematical thinking. In terms of teaching methodology, authors such as Kaiser and Sriraman (2015) and Maaß and Artigue (2017) argue that a traditional approach prevails that focuses mainly on problem-solving and the mechanical mastery of operations, formulas, and algorithms, which limits the analytical and reflective capacity of students. Specifically, traditional teaching methods in mathematical modeling do not encourage a complete understanding of the mathematical concepts and objects that allow adequate modeling application in authentic contexts. In addition, the didactic materials educators use for this purpose are not always sufficient to meet the individual needs of each student.

Facing the pedagogical challenges that arise requires educational institutions to adopt a reflective and committed approach to address them effectively. To achieve this, it is essential to promote continuous and enriching teacher training that provides educators with new tools, strategies, and pedagogical perspectives related to mathematical modeling. In the didactic field, it is urgent to develop appropriate teaching materials adapted to students' needs and learning styles. These materials should promote a deep and contextualized understanding of the mathematical concepts and processes involved in modeling. Additionally, they should be accessible, interactive, and designed to arouse students' curiosity and interest to encourage their active participation in the learning process.

Likewise, educational institutions must recognize and appreciate the diversity of their students, putting into practice inclusive and personalized strategies that adjust to the particularities of each student during the modeling process. Promoting a reflective and innovative educational culture in schools is crucial to effectively transforming pedagogy and didactics. Promoting spaces for collective reflection and traditional educational practices are critically questioned to seek new pedagogical alternatives. Furthermore, experimentation, project-based learning, STEAM education, gamification, and integrating digital technologies as enriching resources for mathematical modeling should be encouraged. Only through these measures can quality education be fostered, in which students can develop solid mathematical skills and apply modeling in real situations, preparing them to face the challenges of the 21st century.

In the context of mathematics classes, some students face obstacles when approaching modeling problems due to limitations in their ability to establish planning strategies, organize relevant information, and evaluate results. These cognitive difficulties harm the development of advanced skills in mathematical thinking. Therefore, teachers and schools must implement effective pedagogical strategies, provide individualized support, promote critical thinking and problem-solving, use educational resources and technology, and encourage collaboration among students. These actions will contribute to improving students' cognitive deficiencies in modeling and the development of mathematical thinking.

Moreover, it is essential to recognize that students with significant differences in their cognitive structure converge in school classrooms, as some are more agile than others. There are also students with functional diversity, such as those with autism, ADHD, and cognitive disabilities, among many other disorders that limit their intellectual development. Teachers and schools must, therefore, implement inclusive and personalized strategies that adjust to the particularities of each student during the modeling process.

Finally, studies show that in the teaching and learning process, some educators do not consider the psychological component relevant, as they focus mainly on developing curricular topics and ignore students' interests, tastes, emotions, and expectations. Promoting a reflective and innovative educational culture in schools is necessary to address this.

According to Szücs and Mammarella (2020), the lack of attention paid by educators to the psychological dimension of students in the mathematics classroom has led to a low level of intrinsic motivation for learning mathematics. As a result, students lack enthusiasm and curiosity to explore and understand concepts related to modeling, which are fundamental to representing real-world mathematical objects. This lack of interest can generate a negative attitude towards appropriating disciplinary knowledge, creating an emotional barrier that hinders students' active participation and willingness to learn.

Schools and educators must address students' lack of motivation and interest in mathematical modeling. First, educators must recognize the importance of the psychological dimension in the mathematics learning process and implement pedagogical strategies that foster students' intrinsic motivation. This involves valuing individual achievement and fostering curiosity and exploratory skills. In addition, educators can employ didactic approaches that integrate mathematical modeling in relevant and meaningful contexts so that students understand the usefulness and applicability of mathematical concepts in situations that are relevant to them.

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It is also essential to provide opportunities for students to actively participate in the modeling process, making decisions and solving problems autonomously. Schools can also organize extracurricular activities, such as mathematical modeling competitions, cross-curricular projects, visits to places related to the practical application of mathematics, and motivational talks by specialized professionals. Ultimately, a more significant appropriation of disciplinary knowledge will be promoted by creating an environment in which students feel valued, motivated, and emotionally engaged in learning mathematics.

Generally speaking, when it comes to learning modeling as a strategy for developing mathematical thinking, schools and critical actors in the educational process must consider various aspects. This includes the social reality in which students live, the didactic methods and approaches used in the classroom to foster modeling, and the possible cognitive and psychological challenges that students may face. The educational process and the academic formation of students should not be viewed in isolation but approached holistically, taking into account the multiple factors that influence the acquisition of disciplinary knowledge.

Conclusions

The conclusions referred to result from achieving the objectives set out in the study. Based on this, it can be concluded that the fundamental factors that hinder the integration of modeling in the educational context of mathematics, which in turn harm the development of mathematical thinking, are social, pedagogical, cognitive, and psychological. Therefore, it is concluded that:

Globally, various studies show a socioeconomic gap in almost all countries. This situation affects the school scenario since not all students have the same economic opportunities to access education, especially the means and resources that facilitate mathematics learning, mainly modeling. From this perspective, it is concluded that it is essential that educational institutions adopt a critical stance toward this situation and develop strategies that contribute to reducing socioeconomic inequalities among students. In this sense, it is advisable to promote campaigns focused on seeking economic resources that benefit students with less economic capacity to guarantee their permanence in the educational system.

In addition, it is essential to provide primary education to parents who have not been able to complete their school education so that they can support their children in performing tasks and duties related to mathematics.

Given the pedagogical elements that restrict knowledge acquisition in modeling, the research highlights the traditional approaches to teaching mathematics as the most critical and the didactics used by the teachers in charge of guiding this area. Regarding the first aspect, the prevalence of mechanical, mathematical teaching is evident, in which the simple memorization of formulas and algorithms is prioritized without encouraging students' analytical and reflective development. Regarding didactics, numerous investigations agree that mathematics teachers tend to privilege using conventional resources, such as the board, the textbook, and, sometimes, photocopies. Although these resources can be adequate for learning, they fail to awaken students' interest and motivation toward understanding and appropriation of the knowledge inherent to mathematical modeling.

From this perspective, it is concluded that it is inevitable for educators to implement new teaching approaches in the school environment, where the protagonism of students and their active participation in the educational process are prioritized. In addition, teachers in charge of teaching mathematics must become guides and orientators for students instead of mere information transmitters. Likewise, a resignification of the didactics used in the transmission of mathematical knowledge is required so that the different learning styles of students are respected. Providing a pleasant educational environment that promotes a taste for learning is essential.

In this sense, it would be beneficial to integrate information and communication technologies (ICT) to create school environments that foster the appropriation of knowledge related to the modeling process and, consequently, promote the development of mathematical thinking.

Not only social and pedagogical factors influence the acquisition of knowledge related to the modeling cycle. It is important to note that most of the studies reviewed in this analysis point to the presence of cognitive factors related to a low level of development of mathematical thinking and insufficient appropriation of the disciplinary knowledge of modeling. In school classrooms worldwide, students do not present the same characteristics, as some face limitations of a cognitive nature that hinder the understanding of modeling processes. It is possible to find students with disorders such as autism, attention deficit hyperactivity disorder (ADHD), cognitive disability, and other conditions that affect the development of mathematical thinking.

Given this scenario, teachers must be trained to face the challenge of teaching mathematics since they will always encounter students who present particularities. Therefore, they must act effectively in adapting the educational environment to serve all types of students, respecting their learning styles and possible cognitive dysfunctionalities that they may manifest.

Finally, another factor influences the development of mathematical thinking and the understanding of modeling processes, and it is related to the assumptions of the students' learning styles. Unfortunately, some have developed aversion, rejection, and little interest in learning mathematics due to previous experiences. Therefore, educators must design strategies to engage students' interest so that they perceive mathematics learning and modeling processes as an enjoyable experience that allows them to understand situations in their environment.

References

Abuya, B., Mumah, J., Austrian, K., Mutisya., M.and Kabiru, C. (2018). Mothers' Education and Girls' Achievement in Kibera: The Link With Self-Efficacy. SAGE Open, 3-13(43).653-665. https://journals.sagepub.com/doi/full/10.1177/2158244018765608

Aebli, H. (2002). Doce formas básicas de enseñar. Una didáctica basada en la psicología. Ed. Narcea.

Aguilar Salinas, W. E., Fuentes Lara, M. D., Justo López, A. C., y Martínez Molina, A. D. (2021). Propuesta para el tratamiento de problemas de tasas de variación relacionadas mediante el uso de GeoGebra: Un estudio de casos. Formación universitaria, 14(5), 95-106. https://www.scielo.cl/scielo.php?pid=S0718-50062021000500095&script=sci_arttext

Alhassora, N. S. A., Abu, M. S., and Abdullah, A. H. (2017). Inculcating higher-order thinking skills in mathematics: Why is it so hard. Man in India, 97(13), 51-62. http://eprints.utm.my/id/eprint/76878/

Álvarez, E y Álvarez de Zayas, C. (2004) Elementos Epistemológicos de la Matemática y su enseñanza. Kipus.

Ander Egg, E. (1995). Técnicas de Investigación Social. 24. Edición. Editor. Lumen, Buenos Aires.

Barraza, C. (2018). Manual para la Presentación de Referencias Bibliográficas de Documentos Impresos y Electrónicos. [Archivo PDF]. http://www.utemvirtual.cl/manual_referencias.pdf

Bautista, A., y Forero, A. (2021). La modelación matemática: un camino hacia la contextualización y significatividad del aprendizaje. Revista Tecnológica-Educativa Digital, 8(16), 49-58. https://www.edutec.es/revista/index.php/edutec-e

Blomhøj, M. (2004). Mathematical modelling - A theory for practice. International Perspectives on Learning and Teaching Mathematics. National Center for Mathematics Education.

Blum, W., and Borromeo Ferri, R. (2009). Mathematical Modelling: Can It Be Taught And Learnt?.Journal of Mathematical Modelling and Application , 1 (1), 45-58.

https://www.researchgate.net/publication/279478754_Mathematical_Modelling_Can_It_Be_Taught_And_Learnt

Borromeo Ferri, R. (2003). Mathematical thinking styles- An empirical study. 08.11.2019 tarihinde. https://www.researchgate.net/profile/rita_borromeo_ferri/publication/252220599_mathematical_thinking_styles_-_an_empirical_study/links/55017e1f0cf24cee39f79363.pdf

Bronkhorst, H., Roorda, G., Suhre, C. and Goedhart, M. (2021). Student Development in Logical Reasoning: Results of an Intervention Guiding Students Through Different Modes of Visual and Formal Representation. Can. J. Sci. Math. Techn. Educ. 21, 378–399. https://doi-org.biblio.uptc.edu.co/10.1007/s42330-021-00148-4

Campbell, J. R., Sharpe, L., and Fellows, M. (2017). Math and Science for Students with Disabilities: The Promise, the Challenge, and the Practice. Routledge.

Capone, R. (2022). Interdisciplinarity in Mathematics Education: From Semiotic to Educational Processes. Eurasia Journal of Mathematics, Science and Technology Education, 18((2), 1-27. https://www.ejmste.com/article/interdisciplinarity-in-mathematics-education-from-semiotic-to-educational-processes-11508

Carrillo González, D. A. (2023). El juego como estrategia didáctica para potencializar las habilidades lógico matemáticas. [Tesis de Licenciatura. Universidad Nacional Abierta y a Distancia. Colombia]. https://repository.unad.edu.co/handle/10596/55723

Chaparro, A., González, C., y Caso, J. (2016). Familia y rendimiento académico: configuración de perfiles estudiantiles en secundaria. Revista Electrónica de Investigación Educativa, 18(1), 53-68. https://www.scielo.org.mx/pdf/redie/v18n1/v18n1a4.pdf

Córdoba, F. (2011). La modelación en matemática educativa: Una práctica para el trabajo de aula en ingeniería. [Tesis de maestría. Instituto Politécnico Nacional, México D.F.]. https://www.repositoriodigital.ipn.mx/bitstream/123456789/11708/1/cordoba_2011.pdf

Davydov, V. V. (2020). The psychological characteristics of the formation of elementary mathematical operations in children. In Addition and subtraction (pp. 224-238). Routledge.

De Almeida Luna, A.V., Souza, E.G., de Souza Lima, L.B. (2015). Mathematical Texts in a Mathematical Modelling Learning Environment in Primary School. In: Stillman, G., Blum, W., Salett Biembengut, M. (eds) Mathematical Modelling in Education Research and Practice. International Perspectives on the Teaching and Learning of Mathematical Modelling. Springer, Cham. https://doi.org/10.1007/978-3-319-18272-8_45

Doerr, H. M., delMas, R., and Livengood, R. (2012). A modeling perspective on problem solving and problem posing. In G. A. Stillman, G. Kaiser, W. Blum, and J. P. Brown (Eds.), Teaching mathematical modeling: Connecting to research and practice (pp. 65-78). Springer.

Duarte, J. y Bos, M. S. (2012). Desigualdad en los Aprendizajes de los Alumnos Colombianos. [Archivo PDF]. https://mapeal.cippec.org/wp-content/uploads/2014/05/duarte-y-bos-desigualdad-en-los-azjes-de-alumnos-colombianos.pdf

El Adl, A. y Alkharusi, H. (2020). Relaciones entre las estrategias de aprendizaje autorregulado, la motivación para el aprendizaje y el rendimiento en matemáticas. Revista chipriota de ciencias de la educación, 15 (1), 104-111.

Ernest, P. (2014). Problems with research by teachers on their own practice: Some potential pitfalls and alternative pathways. Educational Action Research, 22(1), 45-60.

Espinoza, G., y Gallegos, M. (2019). Modelación matemática en el aula. Revista Científica de Eduación y Desarrollo Social, 10(2), 17-26.

Flegas, K., and Charalampos, L. (2013) Exploring Logical Reasoning and Mathematical Proof in Grade 6 Elementary School Students. Can J Sci Math Techn 13, 70–89. https://link.springer.com/article/10.1080/14926156.2013.758326

Fuentes López, C. O., Páez Gómez, P. A., y Prieto Monroy, D. E. (2019). Dificultades de la resolución de problemas matemáticos de estudiantes de grado 501 Colegio Floresta Sur, sede b, jornada tarde, Localidad de Kennedy.[Tesis de maestría. Universidad Cooperativa. Colombia]. https://repository.ucc.edu.co/server/api/core/bitstreams/55eb4339-e1e2-4625-83b8-f7c94759ef6b/content

García González, M. S., Ramírez Gómez, B., y Navarro Sandoval, C. (2021). Situaciones que originan emociones en estudiantes de matemáticas. Bolema: Boletim de Educação Matemática, 35, 39-62. https://www.scielo.br/j/bolema/a/NBr5FmkQnRMrdCQLRW8RGpz/

Gómez, L. A., y Sánchez, F. M. (2019). Estrategias didácticas para la enseñanza de la modelación matemática en educación media superior. Revista Científica de Educación y Desarrollo Social, 10(2), 69-81.

González Martín, A. S., Carreira, P., y Biza, I. (2014). Modelación matemática y desarrollo del pensamiento matemático en estudiantes universitarios. Enseñanza de las Ciencias, 32(3), 97-111.

González Rojas, D.Y. (2021). La modelación, un recurso pedagógico para el pensamiento numérico y el aprendizaje significativo. Revista Scientific, 6(19), 102-121. https://doi.org/10.29394/Scientific.issn.2542-2987.2021.6.19.5.102-121

Greefrath, G., and Vorhölter, K. (2016). Home Teaching and Learning Mathematical Modelling Chapter Teaching and Learning Mathematical Modelling: Approaches and Developments from German Speaking Countries. Springer International Publishing AG Switzerland. https://link.springer.com/chapter/10.1007/978-3-319-45004-9_1

Guerra, N., y Grino, J. (2013). Principales factores que inciden en el rendimiento de los estudiantes de la universidad Austral de Chile, puerto montt [Tesis de pregrado. Universidad Austral de Chile]. https://n9.cl/u0c59

Hammersley, M. y Atkinson, P. (1994). Etnografía. Paidós.

Heyder, A., Weidinger, A. F., Cimpian, A., and Steinmayr, R. (2020). Teachers' belief that math requires innate ability predicts lower intrinsic motivation among low-achieving students. Learning and Instruction, 65, 101220. https://www.semanticscholar.org/paper/Teachers%E2%80%99-belief-that-math-requires-innate-ability-Heyder-Weidinger/4e401304d0b95c499d0d818cb27c8023d05188a6

Hıdıroğlu, Ç., and Can, B. (2020). Effect of HTTM (History/ Theory/ Technology/ Modeling) Learning Environment on Preservice Science Teachers' Perceptions of Mathematical Thinking and Mathematical Modelling Skills / HTTM (History/ Theory/ Technology/ Modeling) Öğrenme Ortamının Fen Bilgisi Öğretmeni Adaylarının Matematiksel Düşünmelerine İlişkin Algılarına ve Matematiksel Modelleme Becerilerine Etkisi. 11(2), 239-272. https://dergipark.org.tr/tr/download/article-file/1089895

Hsieh, M. J., Yang, Y. C., Hsu, C. H., and Chiu, F. C. (2018). Learning and Motivation of Students with Intellectual Disabilities: Teaching Practices and Technology-Assisted Learning. Journal of Educational Computing Research, 56(8), 1309-1331.

Kaiser, G., and Sriraman, B. (2015). A global survey of international perspectives on modelling in mathematics education. In G. A. Stillman, W. Blum, and M. S. Biembengut (Eds.), Mathematical modelling in education research and practice: Cultural, social and cognitive influences (pp. 37-54). Springer.

King, R.D., Garrett, S. M. and Coghill, G. M. (2005). On the use of qualitative reasoning to simulate and identify metabolic pathways. Bioinformatics, 21,2017-2026. http://dx.doi.org/10.1093/bioinformatics/bti255

Krawec, J., Schenker, J., Shurrab, M., and Bousquet, J. (2020). Enhancing Problem Solving and Metacognition for Students With Intellectual Disability Through Mathematical Modeling. Journal of Learning Disabilities, 53(2), 139-149.

Lara, J. L., y Peralta, J. (2019). El ciclo de modelación como estrategia de enseñanza en el aprendizaje de la matemática. Revista Científica de Educación y Desarrollo Social, 10(2), 49-68.

León, A.P., Casas, J.C. y Restrepo, G. (2016). Desarrollo del pensamiento lógico basado en resolución de problemas en niños de 4 a 5 años, Panorama 10 (19), 98-107. https://journal.poligran.edu.co/index.php/panorama/article/view/831/672

Li, Y., and Schoenfeld, A.H. (2019). Problematizing teaching and learning mathematics as "given" in STEM education. International Journal of STEM Education. 6(44), 1-13. https://stemeducationjournal.springeropen.com/articles/10.1186/s40594-019-0197-9

Lingefjärd, T. y Holmquist M. (2005). To assess students' attitudes, skills and competencies in mathematical modeling. Teaching Mathematics and its Applications: An International Journal of the IMA, 24(2-3), 123–133. https://doi.org/10.1093/teamat/hri021

Maaß, K., and Artigue, M. (2017). Teaching and learning mathematics through modelling: Advances and perspectives. Cham: Springer.

Mahecha Escobar, J. C., Viancha-Rincón, E. L., Conejo-Carrasco, F., y Soler-Cifuentes, D. C. (2021). El juego como estrategia pedagógica para la autorregulación del aprendizaje en matemáticas. Revista Electrónica en Educación y Pedagogía, 5(9), 68-82.

Mejía, R. (2019). Enseñanza de la modelación matemática en el aula. Revista Internacional de nvestigación en Educación Matemática, 12(3), 79-98.

Méndez, M., Céspedes, N. y Flórez, I. (2020). Caracterización de ambientes y tareas de modelación matemática en la formación de docentes de matemáticas. Repositorio institucional Universidad de Santo Tomás, Colombia. https://repository.usta.edu.co/handle/11634/28141

Mirzaxolmatovna, X. Z. (2021). The role of logical issues in teaching mathematics to primary school pupils. ACADEMICIA: An International Multidisciplinary Research Journal, 11(5), 465-467.

Mu, D., and Guo, W. (2022). Impact of Students' Online Learning Burnout on Learning Performance–the Intermediary Role of Game Evaluation. International Journal of Emerging Technologies in Learning (iJET), 17(2), 239-253.

Mutlu, Y. (2019). Math Anxiety in Students with and without Math Learning Difficulties. International Electronic Journal of Elementary Education, 11(5), 471-475.

Newton, K.J., Leonard, J., Evans, B.R. and Eastburn, J.A. (2012), Preservice Elementary Teachers' Mathematics Content Knowledge and Teacher Efficacy. School Science and Mathematics, 112. 289-299. https://doi.org/10.1111/j.1949-8594.2012.00145.x

Olarte, J. (2020). Homogeneizar la práctica de la modelación: un reto del sistema educativo colombiano. Standardizing the Use of Mathematical Models: A Challenge Faced by the Colombian Educational System. Revista Educación, 44(1),124.https://www.redalyc.org/journal/440/44060092024/html/

Ortiz, M. y Camelo, F. (2020). Un panorama de la modelación matemática en los encuentros colombianos de matemática educativa entre 2012-2015. Góndola, Enseñanza y Aprendizaje de las Ciencias, 15(2), 251–267. https://doi.org/10.14483/234.

Pérez Benítez, A. A., García Rodríguez, M. L., y Flores Cervantes, C. (2023). Acercamiento a la comprensión de textos en la resolución de problemas en matemáticas considerando los procesos inductivos y deductivos: el caso de Bobby. RIDE Revista Iberoamericana para la Investigación y el Desarrollo Educativo, 13(26). 1-33. https://www.ride.org.mx/index.php/RIDE/article/view/1423

Pólya, G. (1963). Sobre el aprendizaje, la enseñanza y la enseñanza del aprendizaje. El Estadounidense Matemática Mensual, 70(6), 605-619. doi:10.2307/2311629

Purwitaningrum, R., and Prahmana, R. C.(2021). Developing instructional materials on mathematics logical thinking through the Indonesian realistic mathematics education approach. International Journal of Education and Learning, 3(1), 13-19.

Ramos Barrera, M. G., y Bello Rodríguez, S. P. (2020). Enseñanza de matemática en economía: El problema de la abstracción. Catálogo editorial, 84-103.

Reardon, S. (2012). The widening academic achievement gap between the rich and the poor: New evidence and possible explanations. R. Murnane y G. Duncan (Eds.), New York: Russell Sage Foundation Press, 50p.

Rendón Mesa, P., Duarte, P., y Villa Ochoa, J. (2015) La Modelación Matemática y su función articuladora entre saberes en la formación de un ingeniero. XIV CIAEM-IACME, Chiapas, México, Conferencia interamerica de educación Matemática.

Reverand, E. (2003). Una aproximación a la teoría de la actividad. En: Mora, D. (Ed.), Tópicos en educación matemática. Caracas: Ediciones de la Universidad Central de Venezuela.

Rodríguez, R. D., y Guzmán, R. R. (2019). Rendimiento académico de adolescentes declarados en situación de riesgo. Revista de Investigación Educativa, 37(1), 147-162. https://revistas.um.es/rie/article/view/303391

Ron, J. (2007) Una Estrategia Didáctica para el proceso de enseñanza-aprendizaje de la Resolución de Problemas en las clases de Matemática en la educación Secundaria Básica. Tesis Doctorado. Instituto Superior Pedagógico "Enrique José Varona", La Habana.

Santos Trigo, M. (2020). Problem-solving in mathematics education. Encyclopedia of mathematics education, 686-693.

Schoenfeld, A. (2016). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint). Journal of Education. 196. 1-38. https://journals.sagepub.com/doi/10.1177/002205741619600202

Siagan, M. V., Saragih, S., and Sinaga, B. (2019). Development of Learning Materials Oriented on Problem-Based Learning Model to Improve Students' Mathematical Problem Solving Ability and Metacognition Ability. International electronic journal of mathematics education, 14(2), 331-340.

Sierra Castillo, S. L. (2019). Las emociones al interior de la clase de matemáticas (Doctoral dissertation, Universidad Externado de Colombia).

Stacey, K., Burton, L., y & Mason, J. (2010). Thinking mathematically. Pearson Education Limited.

Szücs, D., & Mammarella, I. C. (2020). Ansiedad hacia las matemáticas.

Takahashi, A. (2007). Planning a lesson for students to develop mathematical thinking through problem solving. DePaul University. Vol 26, p.2.

Vanbecelaere, S., Van den Berghe, K., Cornillie, F., Sasanguie, D., Reynvoet, B., & Depaepe, F. (2020). The effects of two digital educational games on cognitive and non cognitive math and reading outcomes. Computers & Education, 143, 103680. https://doi.org/10.1016/j.compedu.2019.103680.

Vargas, W. (2020) Metodología de la enseñanza aprendizaje de la matemática orientada al desarrollo del pensamiento lógico matemático para estudiantes de Segundo de secundaria. Tesis Doctoral. Universidad Autónoma del Beni.

Villa-Ochoa, J. A. (2009). Modelación en educación matemática: Una mirada desde los lineamientos y estándares curriculares colombianos. Revista Virtual Universidad Católica del Norte, 27, 1-21.

Villa Ochoa, J., y Alencar, E. (2019). Un panorama de investigaciones sobre Modelación Matemática: Colombia y Brasil. Revista de Educação Matemática. 16 (21). 18-37. http://funes.uniandes.edu.co/12701/1/215-644-1-PB.pdf

Villa Ochoa, J., Sánchez Cardona, J., y Parra Zapata, M.M. (2022). Modelación matemática en la perspectiva de la educación matemática. En Rodríguez, M., Pochulu, M.D., Espinoza, F. (Eds.), Educación matemática: aportes a la formación docente desde distintos enfoques teóricos, (pp. 67-89). Argentina: Ediciones UNGS.

Wei, X., Cheng, I. L., Chen, N. S., Yang, X., Liu, Y., Dong, Y., ... & Kinshuk. (2020). Effect of the flipped classroom on the mathematics performance of middle school students. Educational Technology Research and Development, 68, 1461-1484.

Yaftian, N., and Barghamadi, S. (2022). The Effect of Teaching Using Multimedia on Mathematical Anxiety and Motivation. Journal of Research and Advances in Mathematics Education, 7(2), 55-63.

Zaldívar, J., Quiroz, S. y Medina, G. (2018). La modelación matemática en los procesos deformación inicial y continua de docentes. IE Revista de investigación educativa de la REDIECH, 8(15), 87-110. http://www.scielo.org.mx/scielo.php?script=sci_arttext&pid=S2448-85502017000200087&lng=es&tlng=es

Zimmerman, W. y Cunningham, S. (eds.) (1991). Visualization in Teaching and Learning Mathematics. Mathematical Assn of Amer (1 Enero 1991).