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Sumudu Homotopy Analysis Method to solved Generalized Hirota-Satsuma Coupled Kdv System

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Abstract

In this work, we use the homotopy analysis method (HAM) and combine it with the sumudu transform method (STM). This connection between the two methods is called the Sumudu homotopy analysis method (SHAM), and we use the method to solve generalized Hirota-Satsuma coupled kdv systems. And we compare the approximate solutions of this method with the Sumudu transform method. Comparison tables and graphics showed that SHAM is much closer than STM to the exact solution.

Keywords: *hirota-satsuma coupled kdv systems, homotopy analysis method, sumudu transform method.*

1. Introduction

Some engineering problems require the use of mathematical models directly in order to be understood and solved. To comprehend and analyze these mathematical models, it is typically necessary to use elements of statistics, linear algebra, or differential and integral calculus [1].

Let's start with the metamorphosis of Sumudu: Watugala discovered the Sumudu transformation in 1993, which was regarded as innovative for this century and solved control engineering problems [2]. For this transition, Weerakoon provided a challenging inverse formula [3]. Asiru employed the Sumudu transform to resolve systems of discrete dynamic equations and integral equations in his studies [4], [5]. Mohamed Z. Mohamed, Amjad E. Hamza, and Abdelilah Kamal H. Sedeeg, outlined a novel technique known as the conformable double Sumudu composition method for solving one-dimensional regular and singular conformable functional Burger's equations [6].

In 1992, Liao proposed the homotopy analysis method as a broad analytic approach for nonlinear problems using the fundamental notions of homotopy in topology (HAM) [7]. Numerous nonlinear problems have been successfully solved using this approach in the sciences and engineering.

The ongoing study in this area served as motivation and inspiration for the sumudu homotopy analysis technique (SHAM), which we provide in this paper as an approximation method for solving the nonlinear equations. It is crucial to remember that the suggested method is an elegant combination of the Sumudu transform method and the homotopy analysis method. In many different domains, the homotopy analysis method (HAM) is frequently employed to resolve strong non-linear issues. The outcomes of the HAM are independent of both tiny and big physical parameters, unlike the perturbation approach. The fact that the homotopy analysis technique (HAM) gives users a choice in

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the expression format for higher-order approximation series solutions is another benefit. The homotopy analysis method (HAM) allows for the simple control of a series solution's convergence [8].

In 2020, the author A.K. Alomari used the sumudu transforms and homotopy analysis method (SHAM) for solving a system of fractional partial differential equations [9]. In (2017), the authors, Rishi Kumar Pandey and Hradyesh Kumar Mishra, solved time fractional third-order dispersive type PDE equations using a combination of sumudu transforms and the homotopy analysis method. [10]. In 2022, the authors R. K. Bairwaa and Karan Singh solved the time-fractional Schrödinger equations under initial conditions by using a newly developed analytical method known as the Sumudu transform iterative method [11]. Also in 2022, the authors Shrooq M. Azzo and Saad A. Manaa solved the generalized hirota-satsuma coupled kdv systems by used sumudu decomposition method [12]. And in 2023, the authors Rafał Brociek, Agata Wajda, Marek Błasik and Damian Słota, used of the homotopy analysis method (HAM) to solve the fractional heat conduction equation [13].

In this work, we used the generalized Hirota–Satsuma coupled Korteweg–de Vries (KdV) equation [14] : as

$$U_\tau = \frac{1}{2}U_{\chi\chi\chi} - 3UU_\chi + 3(VW)_\chi,$$

$$V_\tau = -V_{\chi\chi\chi} + 3UV_\chi,$$

$$W_\tau = -W_{\chi\chi\chi} + 3UW_\chi. \qquad (1)$$

The exact solitary solution of equation (1) as in [14]–[16] is:

$$U(\chi,\tau) = \frac{1}{3}(\gamma - 2a^2) + 2a^2 \tanh^2\big(a(\chi + \gamma\tau)\big),$$

$$V(\chi,\tau) = \frac{-4a^2c_0(\gamma+a^2)}{3c_1^2} + \frac{4a^2(\gamma+a^2)}{3c_1}\tanh\big(a(\chi+\gamma\tau)\big),$$

$$W(\chi,\tau) = c_0 + c_1\tanh\big(a(\chi+\gamma\tau)\big). \qquad (2)$$

and initial conditions are:

$$U(\chi,0) = \frac{1}{3}(\gamma - 2a^2) + 2a^2\tanh^2(a\chi),$$

$$V(\chi,0) = \frac{-4a^2c_0(\gamma+a^2)}{3c_1^2} + \frac{4a^2(\gamma+a^2)}{3c_1}\tanh(a\chi)\ ,$$

$$W(\chi,0) = c_0 + c_1\tanh(a\chi). \qquad (3)$$

where $a, c_0, c_1 \neq 0\ and\ \gamma$ arbitrary constants.

2. Basic idea of sumudu homotopy analysis method (SHAM)

To illustrate the basic idea of this method, we consider an equation $N[\omega(\chi)] = g(\chi)$, where N represents a general nonlinear ordinary or partial differential operator including both linear and nonlinear terms. The linear terms are decomposed into L+R, where L is the highest order linear operator and R is the remaining linear operator. Thus, the equation can be written as [17]

$$L\omega + R\omega + N\omega = g(\chi). \qquad (4)$$

where $N\omega$, indicates the nonlinear terms.

by applying the sumudu transform on both sides of equation (4), we get

$$S[L\omega] + S[R\omega] + S[N\omega] = S[g(\chi)]. \qquad (5)$$

using the differentiation property of the sumudu transform, we have

$$\frac{S[\omega]}{s^n} - \sum_{k=0}^{n-1} \frac{\omega^{(k)}(0)}{s^{(n-k)}} + S[R\omega] + S[N\omega] = S[g(\chi)]. \quad (6)$$

on simplifying

$$S[\omega] - s^n \sum_{k=0}^{n-1} \frac{\omega^{(k)}(0)}{s^{(n-k)}} + s^n\big[S[R\omega] + S[N\omega] - S[g(\chi)]\big] = 0. \quad (7)$$

we define the nonlinear operator

$$N[\varphi(\chi,\tau;q)] = S[\varphi(\chi,\tau;q)] - s^n \sum_{k=0}^{n-1} \frac{\varphi^{(k)}(\chi,\tau;q)(0)}{s^{(n-k)}} + s^n\big[S[R\varphi(\chi,\tau;q)] + S[N\varphi(\chi,\tau;q)] - S[g(\chi)]\big]. \quad (8)$$

where $q \in [0,1]$ and $\varphi(\chi,\tau;q)$ is a real function of χ, τ and q. we construct a homotopy as follows

$$(1-q)S[\varphi(\chi,\tau;q) - \omega_0(\chi,\tau)] = \hbar q H(\chi,\tau) N[\omega(\chi,\tau)]. \quad (9)$$

where S denotes the sumudu transform, $q \in [0,1]$ is the embedding parameter, $H(\chi,\tau)$ denotes a nonzero auxiliary function, $\hbar \neq 0$ is an auxiliary parameter, $\omega_0(\chi,\tau)$ is an initial guess of $\omega(\chi,\tau)$ and $\varphi(\chi,\tau;q)$ is a unknown function. Obviously, when the embedding parameter $q = 0$ and $q = 1$, it holds

$$\varphi(\chi,\tau;0) = \omega_0(\chi,\tau), \ \varphi(\chi,\tau;1) = \omega(\chi,\tau). \quad (10)$$

respectively. Thus as q increases from 0 to 1, the solution $\varphi(\chi,\tau;q)$ varies from the initial guess $\omega_0(\chi,\tau)$ to the solution $\omega(\chi,\tau)$. Expanding $\varphi(\chi,\tau;q)$in Taylor series with respect to q, we have

$$\varphi(\chi,\tau;q) = \omega_0(\chi,\tau) + \sum_{m=1}^{\infty} \omega_m(\chi,\tau) q^m. \quad (11)$$

where

$$\omega_m(\chi,\tau) = \frac{1}{m!} \frac{\partial^m \varphi(\chi,\tau;q)}{\partial q^m}\bigg|_{q=0}. \quad (12)$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, and the auxiliary function are properly chosen, the series (11) converges at $q = 1$, then we have

$$\varphi(\chi,\tau;q) = \omega_0(\chi,\tau) + \sum_{m=1}^{\infty} \omega_m(\chi,\tau). \quad (13)$$

which must be one of the solutions of the original nonlinear equations. According to the equation (13), the governing equation can be deduced from the zero-order deformation (9). define the vectors

$$\vec{\omega}_0 = \{\omega_0(\chi,\tau), \omega_1(\chi,\tau), \dots, \omega_m(\chi,\tau)\}. \quad (14)$$

differentiating the zeroth-order deformation equation (9) m-times with respect to q and then dividing them by $m!$ And finally setting $q = 0$, we get the following mth-order deformation equation:

$$S[\omega_m(\chi,\tau) - X_m \omega_{m-1}(\chi,\tau)] = \hbar H(\chi,\tau) \mathcal{R}_m(\vec{\omega}_{m-1}). \quad (15)$$

applying the inverse sumudu transform, we have

$$\omega_m(\chi,\tau) = X_m \omega_{m-1}(\chi,\tau) + \hbar S^{-1}[H(\chi,\tau) \mathcal{R}_m(\vec{\omega}_{m-1})]. \quad (16)$$

where

$$\mathcal{R}_m(\vec{\omega}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\varphi(\chi,\tau;q)]}{\partial q^{m-1}}\bigg|_{q=0}. \quad (17)$$

and

$$X_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1 \end{cases} . \qquad (18)$$

3. Applying SHAM to the equation

By applying the aforesaid method subject to equation (1), we have

$$S[U] - U_0 - s\left[S\left[\frac{1}{2}U_{\chi\chi\chi} - 3UU_\chi + 3(VW)_\chi\right]\right] = 0,$$

$$S[V] - V_0 - s\left[S\left[-V_{\chi\chi\chi} + 3UV_\chi\right]\right] = 0,$$

$$S[W] - W_0 - s\left[S\left[-W_{\chi\chi\chi} + 3UW_\chi\right]\right] = 0. \qquad (19)$$

the nonlinear operator is

$$N_1[\varphi_1(\chi,\tau;q)] = S[\varphi_1(\chi,\tau;q)] - \varphi_1(\chi,\tau;0) - s\left[S\left[\frac{1}{2}\big(\varphi_1(\chi,\tau;q)\big)_{\chi\chi\chi} - 3\varphi_1(\chi,\tau;q)\big(\varphi_1(\chi,\tau;q)\big)_\chi + 3\big(\varphi_2(\chi,\tau;q)\varphi_3(\chi,\tau;q)\big)_\chi\right]\right],$$

$$N_2[\varphi_2(\chi,\tau;q)] = S[\varphi_2(\chi,\tau;q)] - \varphi_2(\chi,\tau;0) - s\left[S\left[-\big(\varphi_2(\chi,\tau;q)\big)_{\chi\chi\chi} + 3\varphi_1(\chi,\tau;q)\big(\varphi_2(\chi,\tau;q)\big)_\chi\right]\right],$$

$$N_3[\varphi_3(\chi,\tau;q)] = S[\varphi_3(\chi,\tau;q)] - \varphi_3(\chi,\tau;0) - s\left[S\left[-\big(\varphi_3(\chi,\tau;q)\big)_{\chi\chi\chi} + 3\varphi_1(\chi,\tau;q)\big(\varphi_3(\chi,\tau;q)\big)_\chi\right]\right]. \qquad (20)$$

and thus

$$\mathcal{R}_{1,m}\big(\vec{U}_{m-1}\big) = S[U_{m-1}] - (1 - X_m)U_0 - s\left[S\left[\frac{1}{2}(U_{m-1})_{\chi\chi\chi} - 3\big(\textstyle\sum_{n=0}^{m-1} U_n (U_{m-1-n})_\chi\big) + 3\big(\textstyle\sum_{n=0}^{m-1} V_n W_{m-1-n}\big)_\chi\right]\right],$$

$$\mathcal{R}_{2,m}\big(\vec{V}_{m-1}\big) = S[V_{m-1}] - (1 - X_m)V_0 - s\left[S\left[-(V_{m-1})_{\chi\chi\chi} + 3\big(\textstyle\sum_{n=0}^{m-1} U_n (V_{m-1-n})_\chi\big)\right]\right],$$

$$\mathcal{R}_{3,m}\big(\vec{W}_{m-1}\big) = S[W_{m-1}] - (1 - X_m)W_0 - s\left[S\left[-(W_{m-1})_{\chi\chi\chi} + 3\big(\textstyle\sum_{n=0}^{m-1} U_n (W_{m-1-n})_\chi\big)\right]\right]. \qquad (21)$$

the mth-order deformation equation are given by

$$S[U_m(\chi,\tau) - X_m U_{m-1}(\chi,\tau)] = \hbar\mathcal{R}_{1,m}\big(\vec{U}_{m-1}\big),$$

$$S[V_m(\chi,\tau) - X_m V_{m-1}(\chi,\tau)] = \hbar\mathcal{R}_{2,m}\big(\vec{V}_{m-1}\big),$$

$$S[W_m(\chi,\tau) - X_m W_{m-1}(\chi,\tau)] = \hbar\mathcal{R}_{3,m}\big(\vec{W}_{m-1}\big). \qquad (22)$$

applying the inverse sumudu transform, we have

$U_m(\chi,\tau) = X_m U_{m-1}(\chi,\tau) + \hbar S^{-1}[\mathcal{R}_{1,m}(\vec{U}_{m-1})],$

$V_m(\chi,\tau) = X_m V_{m-1}(\chi,\tau) + \hbar S^{-1}[\mathcal{R}_{2,m}(\vec{V}_{m-1})],$

$W_m(\chi,\tau) = X_m W_{m-1}(\chi,\tau) + \hbar S^{-1}[\mathcal{R}_{3,m}(\vec{W}_{m-1})].$ (23)

now, solving the system (23) to find $U_1(\chi,\tau), V_1(\chi,\tau), W_1(\chi,\tau), U_2(\chi,\tau), V_2(\chi,\tau)$ and $W_2(\chi,\tau)$ respectively, $m = 1, 2, 3, ...$, we get

$\mathcal{R}_{1,1}(\vec{U}_0) = S[U_0] - (1-0)U_0 - s\left[S\left[\frac{1}{2}(U_0)_{\chi\chi\chi} - 3\big(U_0(U_0)_\chi\big) + 3(V_0W_0)_\chi\right]\right],$

$\mathcal{R}_{2,1}(\vec{V}_0) = S[V_0] - (1-0)V_0 - s\left[S\left[-(V_0)_{\chi\chi\chi} + 3\big(U_0(V_0)_\chi\big)\right]\right],$

$\mathcal{R}_{3,1}(\vec{W}_0) = S[W_0] - (1-0)W_0 - s\left[S\left[-(W_0)_{\chi\chi\chi} + 3\big(U_0(W_0)_\chi\big)\right]\right].$ (24)

thus

$U_1(\chi,\tau) = \hbar S^{-1}\left[S[U_0] - U_0 - s\left[S\left[\frac{1}{2}(U_0)_{\chi\chi\chi} - 3\big(U_0(U_0)_\chi\big) + 3(V_0W_0)_\chi\right]\right]\right],$

$V_1(\chi,\tau) = \hbar S^{-1}\left[S[V_0] - V_0 - s\left[S\left[-(V_0)_{\chi\chi\chi} + 3\big(U_0(V_0)_\chi\big)\right]\right]\right],$

$W_1(\chi,\tau) = \hbar S^{-1}\left[S[W_0] - W_0 - s\left[S\left[-(W_0)_{\chi\chi\chi} + 3\big(U_0(W_0)_\chi\big)\right]\right]\right].$ (25)

$\mathcal{R}_{1,2}(\vec{U}_1) = S[U_1] - (1-1)U_0 - s\Big[S\Big[\frac{1}{2}(U_1)_{\chi\chi\chi} - 3\big(U_0(U_1)_\chi + U_1(U_0)_\chi\big) +$
$3(V_0W_1 + V_1W_0)_\chi\Big]\Big],$

$\mathcal{R}_{2,2}(\vec{V}_1) = S[V_1] - (1-1)V_0 - s\left[S\left[-(V_1)_{\chi\chi\chi} + 3\big(U_0(V_1)_\chi + U_1(V_0)_\chi\big)\right]\right],$

$\mathcal{R}_{3,2}(\vec{W}_1) = S[W_1] - (1-1)W_0 - s\left[S\left[-(W_1)_{\chi\chi\chi} + 3\big(U_0(W_1)_\chi + U_1(W_0)_\chi\big)\right]\right].$

thus

$U_2(\chi,\tau) = U_1(\chi,\tau) + \hbar S^{-1}\Big[S[U_1] - s\Big[S\Big[\frac{1}{2}(U_1)_{\chi\chi\chi} - 3\big(U_0(U_1)_\chi + U_1(U_0)_\chi\big) +$
$3(V_0W_1 + V_1W_0)_\chi\Big]\Big]\Big],$

$V_2(\chi,\tau) = V_1(\chi,\tau) + \hbar S^{-1}\left[S[V_1] - s\left[S\left[-(V_1)_{\chi\chi\chi} + 3\big(U_0(V_1)_\chi + U_1(V_0)_\chi\big)\right]\right]\right],$

$W_2(\chi,\tau) = W_1(\chi,\tau) + \hbar S^{-1}\left[S[W_1] - s\left[S\left[-(W_1)_{\chi\chi\chi} + 3\big(U_0(W_1)_\chi + U_1(W_0)_\chi\big)\right]\right]\right].$
(26)

Now if $\hbar = -1$ then the solution in equations (25) and (26) give same rustle as the SDM and SHPM

4. Application

All numerical results were obtained using *Mathematica* software, utilizing all of the above approaches. This is due to its ease of use and ability to manipulate data.

The approximate solution by SHAM of order-three to system (1) with an initial condition in equation (3) and an exact solution in equation (2) can be seen below:

$$U(\chi,\tau) = \frac{1}{3}(\gamma - 2a^2) + \tau^2(-4\gamma^2\hbar^2 a^4 \text{sech}^4(a\chi) + 2\gamma^2\hbar^2 a^4 \cosh(2a x)\text{sech}^4(a\chi)) + 2a^2\tanh^2(a x) + \tau(-8\gamma\hbar a^3\text{sech}^2(a\chi)\tanh(a x) - 4\gamma\hbar^2 a^3 \text{sech}^2(a\chi)\tanh(a x))$$

$$V(\chi,\tau) = -\frac{4c_0 a^2(\gamma + a^2)}{3{c_1}^2} + \tau\left(-\frac{8\gamma\hbar a^3(\gamma + a^2)\text{sech}^2(a\chi)}{3c_1} - \frac{4\gamma\hbar^2 a^3(\gamma + a^2)\text{sech}^2(a\chi)}{3c_1}\right) + \frac{4a^2(\gamma + a^2)\tanh(a x)}{3c_1} - \frac{4\gamma^2\hbar^2 a^4(\gamma + a^2)\tau^2\text{sech}^2(a\chi)\tanh(a x)}{3c_1}$$

$$W(\chi,\tau) = c_0 + \tau(-2\gamma c_1\hbar a\text{sech}^2(a\chi) - \gamma c_1\hbar^2 a\text{sech}^2(a\chi)) + c_1\tanh(a x) - \gamma^2 c_1\hbar^2 a^2\tau^2\text{sech}^2(a\chi)\tanh(a x)$$

Tables 1-3 show the difference in absolute error between the exact and approximate solutions by SHAM and STM for $U(\chi,\tau), V(\chi,\tau)$, and $W(\chi,\tau)$ respectively, (when we plot the $\hbar$ -curves we can find that the valid region for $\hbar$ = -0.99 for $U(\chi,\tau)$, $\hbar$ = -0.909 for $V(\chi,\tau)$ *and* $W(\chi,\tau)$). From these tables, there is a clear change in the results that shows the accuracy of the solution by SHAM, when $\chi = 1, a = 0.1, c_0 = 1.5, c_1 = 0.1$. $\gamma = 1.5$, and $\tau \in [\,0,1\,]$.

TABLE 1: absolute error of STM and SHAM with Exact for $U(\chi,\tau)$

(χ,τ)	$\lvert STM - Exact\rvert$	$\lvert \text{SHAM} - Exact\rvert$
(1,0)	0	0
(1,0.1)	4.4325×10^{-6}	7.38259×10^{-8}
(1,0.2)	1.77249×10^{-5}	2.06141×10^{-7}
(1,0.3)	3.98586×10^{-5}	2.70208×10^{-7}
(1,0.4)	7.08013×10^{-5}	1.25332×10^{-7}
(1,0.5)	1.10507×10^{-4}	3.82625×10^{-7}
(1,0.6)	1.58917×10^{-4}	1.42068×10^{-6}
(1,0.7)	2.15961×10^{-4}	3.16812×10^{-6}
(1,0.8)	2.81555×10^{-4}	5.81588×10^{-6}
(1,0.9)	3.55606×10^{-4}	9.56585×10^{-6}
(1,1)	4.3801×10^{-4}	1.46302×10^{-5}
Least Square Error	4.88622×10^{-8}	3.51711×10^{-11}

TABLE 2: absolute error of STM and SHAM with Exact for $V(\chi,\tau)$

(χ,τ)	$\lvert STM - Exact\rvert$	$\lvert \text{SHAM} - Exact\rvert$
(1,0)	0	0
(1,0.1)	4.45179×10^{-6}	2.37668×10^{-5}
(1,0.2)	1.86676×10^{-5}	4.46848×10^{-5}
(1,0.3)	4.39241×10^{-5}	6.14754×10^{-5}
(1,0.4)	8.14779×10^{-5}	7.28797×10^{-5}
(1,0.5)	1.32563×10^{-4}	7.76613×10^{-5}
(1,0.6)	1.9839×10^{-4}	7.46082×10^{-5}
(1,0.7)	2.80143×10^{-4}	6.25344×10^{-5}
(1,0.8)	3.78977×10^{-4}	4.02817×10^{-5}
(1,0.9)	4.96018×10^{-4}	6.72184×10^{-6}
(1,1)	6.32363×10^{-4}	3.92423×10^{-5}
Least Square	9.33888×10^{-8}	3.03682×10^{-9}

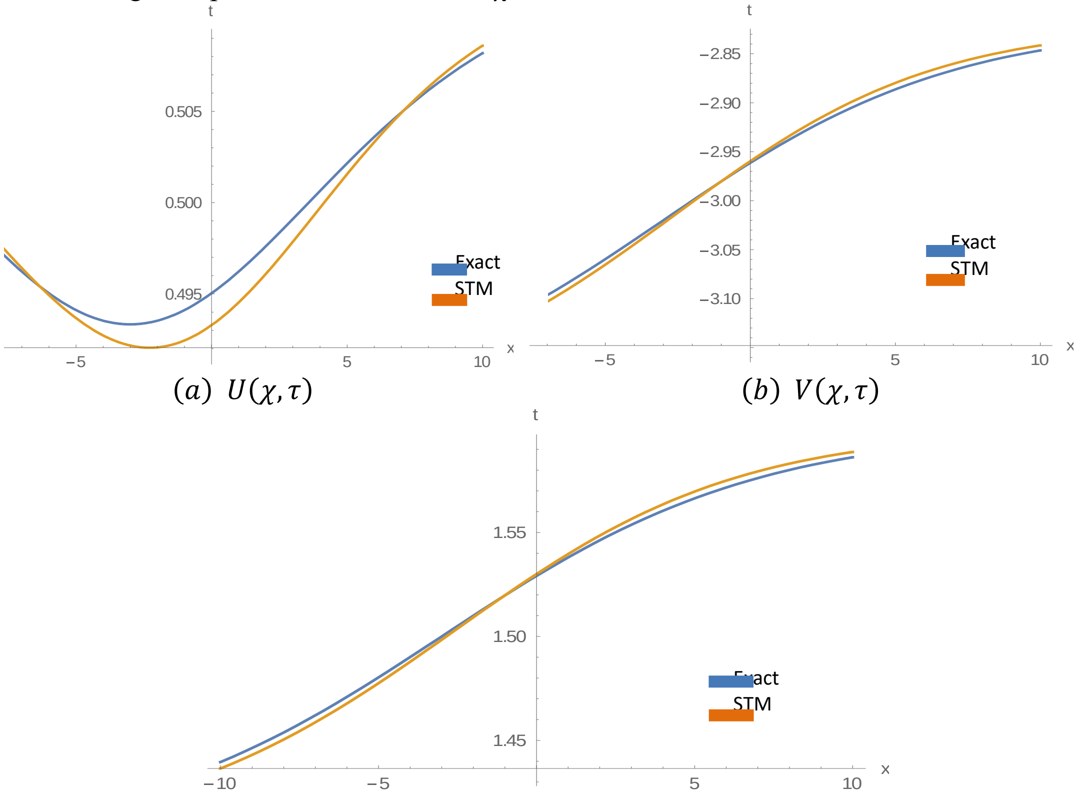
Error		

TABLE 3: absolute error of STM and SHAM with Exact for $W(\chi,\tau)$

(χ,τ)	$\lvert STM - Exact \rvert$	$\lvert \mathrm{SHAM} - Exact \rvert$
(1,0)	0	0
(1,0.1)	2.21115×10^{-6}	1.18047×10^{-5}
(1,0.2)	9.27198×10^{-6}	2.21945×10^{-5}
(1,0.3)	2.18166×10^{-5}	3.05341×10^{-5}
(1,0.4)	4.04692×10^{-5}	3.61985×10^{-5}
(1,0.5)	6.58427×10^{-5}	3.85735×10^{-5}
(1,0.6)	9.85383×10^{-5}	3.70571×10^{-5}
(1,0.7)	1.39144×10^{-4}	3.10601×10^{-5}
(1,0.8)	1.88234×10^{-4}	2.00075×10^{-5}
(1,0.9)	2.46367×10^{-4}	3.33866×10^{-6}
(1,1)	3.14087×10^{-4}	1.94912×10^{-5}
Least Squar Error	2.3039×10^{-8}	7.49183×10^{-10}

The curves in Figure 1 and Figure 2 show that how the SHAM curves are close to the solitary solution curve when $\chi \in [-10, 10]$ and $\tau \in [0,10]$, at $\tau = 2$.

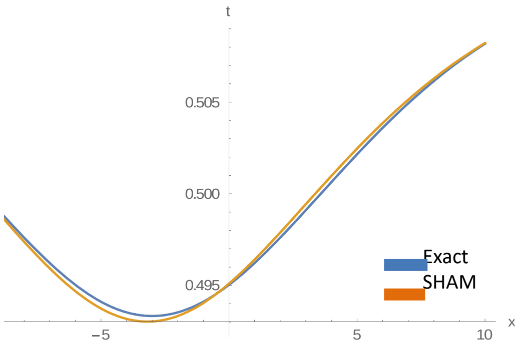
Where Figure 3 plot the $\hbar$ -curves when $\chi=0.2, \tau=0.01, -5 \leq \hbar \leq 0.5$.



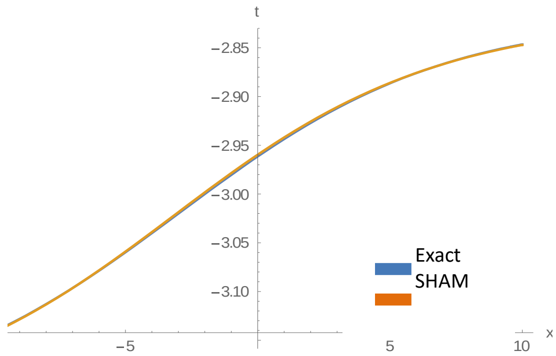
$(a)\ U(\chi,\tau)$ $(b)\ V(\chi,\tau)$

$(c)\ W(\chi,\tau)$

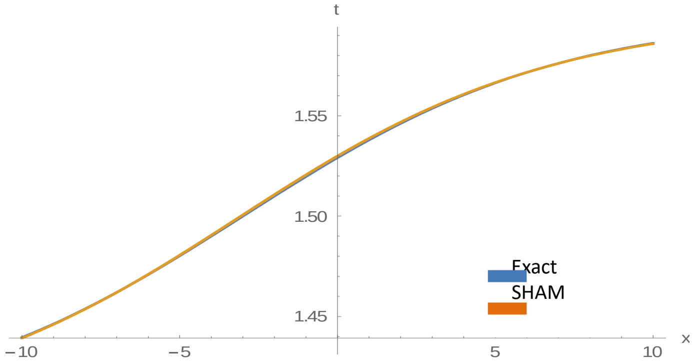
Figure 1 Curves of STM with exact for $\chi \epsilon [-10,10], \tau \epsilon [0,10]$



$(a)\ U(\chi,\tau)$

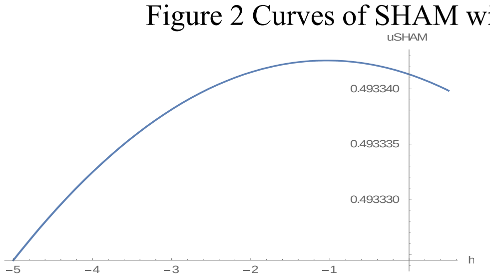


$(b)\ V(\chi,\tau)$

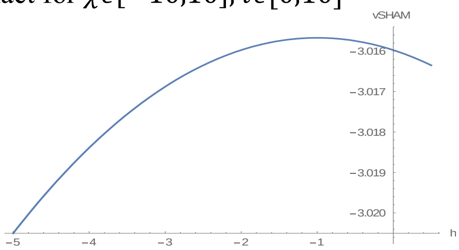


$(c)\ W(\chi,\tau)$

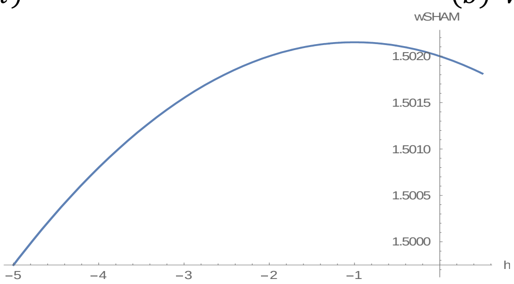
Figure 2 Curves of SHAM with exact for $\chi\epsilon[-10,10], \tau\epsilon[0,10]$



$(a)\ U(\chi,\tau)$



$(b)\ V(\chi,\tau)$



$(c)\ W(\chi,\tau)$

Figure 3 the $\hbar$ -curves when $(\chi,\tau) = (0.2,0.01), -5 \leq \hbar \leq 0.5$

5. Conclusion

The sumudu homotopy analysis method is used in this study to solve generalized Hirota-Satsuma coupled KdV systems under particular beginning conditions. The findings demonstrate that the sumudu homotopy analysis method is a more potent and effective strategy for locating an approximative solution for generalized Hirota-Satsuma coupled KdV equations than the sumudu transform method. When nth approximation numerical results are compared to the known precise solution, the results demonstrate good approximation to the true solution of the equations with only three iterations. Obviously, for $\hbar$=-1 theobtained solution are as the same SHPM and SADM, while if we plot the curve $\hbar$ we find the best value of $\hbar$ are $\hbar$ = -0.99 for U(χ,τ), $\hbar$ = -0.909 for V(χ,τ), and W(χ,τ). Also, the tables and figure show the differences between the methods .

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