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Peer Flipped Learning in Financial Mathematics: Enhancing Academic Achievement and Mathematical Thinking in College of Science Students

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Abstract

This research employed a quasi-experimental design to investigate the impact of Peer Flipped Learning (PFL) on academic achievement (AA) and mathematical thinking (MT) among mathematics department students at Imam Abdulrahman Bin Faisal University's College of Science, Saudi Arabia. Two assessment tools were employed: an AA test for the financial mathematics course and an MT test. A sample of 70 students was randomly divided into an experimental group (n=35) exposed to PFL and a control group (n=35) taught through traditional methods in a financial mathematics course. Results from post-assessment AA and MT tests revealed statistically significant differences, favoring the experimental group. These findings highlight the efficacy of PFL as a modern teaching strategy for enhancing university-level mathematics education and underscore the importance of training educators in its implementation to foster students' academic achievement and critical thinking skills.

Keywords: *peer flipped learning, academic achievement, mathematical thinking, financial mathematics, university students.*

Introduction

In our rapidly evolving world, marked by globalization and a technological revolution of unprecedented scale, the ability to navigate and harness information has become a critical skill for individuals of all ages. To prepare students for the challenges posed by this information-rich landscape, educational institutions worldwide are confronted with the urgent need to develop innovative teaching and learning strategies. These strategies aim not only to hone students' thinking skills but also to equip them with effective methods for information acquisition and processing, while nurturing a diverse set of skills to thrive in various contexts.

The development of critical thinking has emerged as a paramount educational concern in contemporary systems. The educational process is no longer merely about imparting knowledge and facts; it now extends to shaping students into discerning thinkers (Housart et al., 2005). Mathematics curricula and educational materials have emerged as fertile ground for fostering robust thinking methods and modes. They play a pivotal role in molding students' personalities and cultivating their capacity for mathematical insights and profound understanding. Furthermore, the enhancement of mathematical thinking (MT) is a central objective of modern mathematics curricula, as emphasized by the

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National Council of Teachers of Mathematics Standards (NCTM) (NCTM, 2000). The NCTM standards underscore the importance of fostering MT, critical thinking, deductive reasoning, creative thinking, mathematical proof, and utilizing mathematics as a tool for thought and communication, with the goal of cultivating learners who are thinkers rather than mere receivers.

MT development is envisioned as a goal spanning all age groups, with classrooms assuming a pivotal role in nurturing these skills. Hamada (2009) has outlined conditions conducive to promoting MT in classroom environments, including encouraging dialogues and discussions, aligning content with learner needs, facilitating meaningful activities, and fostering collaborative group work with individual learner progress tracking.

Abd Al-Kareem (2014) delineates the multifaceted nature of MT, encompassing skills such as induction, deduction, symbolic expression, guessing, modeling, generalization, and mathematical proof. Despite the importance of MT, numerous studies, including those by Abo Al-Haija (2004), Al-Zoghbi (2011), Al-Fadli (2016), and Al-Koroun (2019), have pointed to deficiencies in students' MT levels.

Concurrently, academic achievement (AA) serves as a critical metric for evaluating the quantitative and qualitative outcomes of the educational process. It is instrumental in effecting behavioral, perceptual, and social changes in students while bolstering their self-confidence and awareness of personal growth (Ismaili, 2011). Weak AA in mathematics education represents a significant challenge for educational and scientific institutions, as demonstrated by various studies, including Talfha (2006), Girgis (2012), Al-Ghamdi (2013), Swalha (2013), Abo Al-Ros (2016), and Al-Belwi (2020). Consequently, the research in mathematics teaching and learning has endeavored to enhance achievement by extending its focus from teachers and students to curricula, teaching methods, and strategies (Jahlan, 2009).

Within the realm of higher education, addressing AA among university-level students has assumed great importance. Several factors, including students' psychological and social conditions, subject matter complexity, and lecture timing, contribute to low AA levels. Consequently, novel teaching strategies and contemporary teaching technologies have been deployed to enhance AA among students (Al-Etibi, 2019).

One such teaching strategy that has gained prominence and demonstrated effectiveness is the flipped learning (FL) strategy. FL leverages modern technological resources to transform teaching methods, motivate students, and facilitate communication. It involves providing recorded lessons accessible to students outside the classroom, followed by in-class practical application. This reversal of classroom tasks and homework assignments, along with role exchange, characterizes the FL approach (Zahed, 2016).

The adoption of FL strategies in the teaching process has yielded numerous benefits, including the development of higher-order thinking skills (Lee & Lai, 2017), increased student participation and interaction (Fisher et al., 2021), enhanced cooperation, meaningful discussions, and opportunities for participation (Per & Arne, 2018). Additionally, FL has been linked to reduced mathematics anxiety, heightened mathematical achievement, improved attitudes toward mathematics learning, enhanced self-learning, bolstered MT, and increased self-efficacy in mathematics (Pierce, 2013; Dove & Dove, 2014; Hwang & Lai, 2017; Sun et al., 2017; 2018; AL-Hamdani & Al Breiki, 2018).

Previous studies have highlighted the effectiveness of the FL approach. For instance, Ruiz de Miras et al. (2022) reported that PFL contributes to self-learning, motivation, and AA enhancement among students. Agustina & Naphiah (2021) found that PFL allows for more classroom activities, encourages student participation, and fosters critical thinking skills, addressing scientific illiteracy. Khpre et al. (2021) observed PFL's positive impact

on AA and motivation. Finally, Al-Masri (2022) identified the positive effects of PFL on scientific inquiry and motivation among intermediate stage students.

In light of the critical importance of MT development and AA improvement in mathematics education, this research investigates the potential of the flipped learning strategy as a viable approach to achieving these educational objectives.

Problem of Research

The research problem addressed in this study revolves around two significant issues in mathematics education among undergraduate students. Firstly, there is a pervasive problem of poor academic achievement (AA) in mathematics, indicating that students are not attaining the desired level of proficiency in the subject. Secondly, there is a distinct lack of interest and focus on developing students' mathematical thinking (MT) skills, which are essential for their future success in mathematics and related fields.

Previous studies have emphasized the need to adopt contemporary methods and strategies that leverage modern technological resources to address these challenges effectively. Given this backdrop, the research problem at hand pertains to the examination of the impact of employing the flipped learning (FL) strategy in teaching the financial mathematics course. This research aims to investigate how FL can potentially contribute to enhancing both AA and MT among female students in the mathematics department at the College of Science, Imam Abdulrahman Bin Faisal University.

Questions of Research

The research sought to answer the following main question:

What is the effect of using the PFL in teaching the financial mathematics curriculum on developing the AA and the MT among the female students of mathematics department, college of science, Imam Abdulrahman Bin Faisal University?

To answer the main question, the following secondary questions were identified:

1- What is the effect of using the PFL in teaching the financial mathematics curriculum on developing the AA among the female students of mathematics department, college of science, Imam Abdulrahman Bin Faisal University?

2- What is the effect of using the PFL in teaching the financial mathematics curriculum on developing the MT among the female students of mathematics department, college of science, Imam Abdulrahman Bin Faisal University?

Objectives of Research

The research sought to identify the effect of using the PFL in teaching the financial mathematics curriculum on developing the AA and the MT among the female students of mathematics department, college of science, Imam Abdulrahman Bin Faisal University.

Significance of Research

Significance of research unfolds in the following:

1- The current research counts on one of the modern directions in the blended learning.

2- Usage of the PFL as this method is one of the modern educational directions in the teaching field.

3- It may benefit the teaching staff and the educationalists regarding utilizing the technological means in teaching.

4- It may be of much more benefit for the decision makers and those in charge in universities regarding planning.

5- It may help in increasing the effectiveness of female students in the university stage regarding mathematics teaching and learning as well as developing the positive directions amongst the students towards benefitting from the technological applications in learning process.

Delimitations of Research:

The research was confined to the following Delimitations:

1- The objective delimitations: they were restricted to the financial mathematics curriculum, which is one of the curricula in mathematics department, college of science, Imam Abdulrahman Bin Faisal University.

2- The spatial delimitations: they were restricted to Kingdom of Saudi Arabia, College of Science, Imam Abdulrahman Bin Faisal University, mathematics department.

3- The time delimitations: the research was applied in the second semester of the academic year 2021/2022.

4- The human delimitations: they were restricted to a sample of the female students in mathematics department of College of Science, Imam Abdulrahman Bin Faisal University in Saudi Arabia.

Hypotheses of Research

1- There is a statistically significant difference between mean scores of the experimental group students and the control group students in the post application of achievement test of the financial mathematics curriculum in favor of the experimental group students which was attributed to the teaching method (the PFL strategy – the regular teaching method).

2- There is a statistically significant difference between mean scores of the experimental group students and the control group students in the post application of the MT skills test in favor of the experimental group students which was attributed to the teaching method (the PFL strategy – the regular teaching method).

Terms of Research

Flipped learning strategy: Barseghian (2011) defines it as a form of blended education that includes any use of technology to benefit from classroom education, so that a teacher can spend more time interacting with students rather than giving lectures. This can be commonly carried out using the videos prepared by the teachers and watched by students outside the classroom time inside the classrooms.

Peer flipped learning: Mazur (2013) defines it as a pattern that involves providing students with video lectures via the web to be studied at home. At the time of class, teachers supply them with a series of questions about key concepts and basic skills associated with the annotated lesson so that each learner works to answer them or perform them individually, then the teachers revise all responses to identify learners who provided true responses and direct them towards helping their classmates to answer questions and achieve the target skills.

It is procedurally defined as a pattern of the blended learning, through which teacher of the financial mathematics starts to prepare the lessons through video clips and the other technological multimedia and make them available on Blackboard platform or WhatsApp application to be reviewed by the students before the lecture time in their houses at the appropriate time. During the lecture, the teacher provides the students with a set of activities which the students handle them separately. After that, the teacher specifies students with true answers and directs them towards helping their classmates to achieve their activities and the target skills.

Academic achievement: Talafha (2006) defined it as a group of facts, concepts, principles, laws, theories and acquired principles by the learners as a result of studying a topic or a specific unit.

It was procedurally defined as amount of knowledge and information gained by female students of Mathematics department as a result of their study for the set financial mathematics curriculum. This is measured by marks taken by female students in the AA test developed by the researchers for such purpose.

Mathematical thinking: Al-Khatib (2006) defined as a pattern of thinking patterns that learners engage in when they are exposed to a mathematical situation, which is represented in one of the following aspects: conclusion, induction, proof, logical thinking, modeling, conjecture, cause and effect, criticism, and prediction.

It was procedurally defined as the mental process practiced by the student to mentally solve mathematical problems based on the following skills: induction, conclusion, generalization, expression by symbols, mathematical proof, modeling. It is measured by the degree obtained by the student in the MT skills test prepared by the researchers. for such purpose.

Theoretical Framework and Literature Review

First Axis: The Flipped Learning

The FL is regarded as one of the blended learning approaches, owing to the fact that a large part of learning process and mastering the instructional content fall on the students' shoulders. In this approach, the role of the teacher is transformed from the one who has the main role in the classroom, and the primary and only source of information, to the guide, mentor and facilitator for learning of the students. Add to that, the general shape of the classroom environment also changes, i.e., from teacher-based teaching to student-based learning, where the student occupied the greatest interest from working within the classroom environment (Abaeian & Samedi, 2016).

Karlsson & Janson (2016) add that the basic idea of FL is to overturn the traditional order of teaching, i.e., the teaching lectures precede the teaching activities of the students. The students prepare their lessons as one of the home assignments before coming to the classroom, as well as the time allocated for teaching is spent performing active learning tasks under the direction and guidance of the teacher, instead of having the home assignment as one of the tasks that are required to be carried out.

The FL strategy derives its theoretical basis from some theories, such as the cognitive constructivist theory of Piaget, which focused on active and effective learning, emphasized that the meaningful learning occurs when the learner is active and in interaction with educational materials and practices science processes and this what is achieved in the FL. Moreover, the strategy relies on the social constructivist theory of Vygotsky which stressed the role of other members such as learners, peers and parents on the consideration that they are partners in helping learners gain their own experience, and novel knowledge. Add to that, the FL relies on the communication theory which is regarded a theory convenient for the digital age; own to the fact that this theory enables the learners to be in contact with teacher via the digital communication means, such as social networks and various Internet sites, which help the learner to exchange knowledge easily and quickly (Ng, 2015).

Steps of flipped learning strategy

Al-Kehili (2015) indicated that implementation of this strategy is according to the following steps:

1- Determination: at this step, the skill or the lesson to be implemented via the FL strategy is determined.

2- Analysis: this step implies analyzing the content with the aim of identifying values, knowledge, skills and the very important concepts that must be known.

3- Designing: this step refers to designing the interactive educational video, in a way that includes the scientific materials with audio and video components for a time period not exceeding ten minutes.

4- Direction: At this step, the students are to be directed towards watching videos uploaded on the educational platform whenever during the time period preceding the class period.

5- Application: this step is carried out during the class where the students apply the concepts they learn from the video during the learning activity.

6- Evaluation: At this step, students are to be evaluated during the class via suitable assessment tools.

Pillars of flipped learning

To apply the FL strategy effectively and efficiently, there is a necessity to provide four main pillars for FL (Metwali, 2015).

1- Flexibility Environment: The application of FL requires constant rearrangements of the learning environment in proportion to the educational situation and the students' levels and needs. This may include the formation of a special part for self-study, group study, doing research, application, or any type of rearrangement, and all of this can be in one learning environment. Therefore, there must be sufficient flexibility in the learning environment and amongst those in charge thereof to accommodate such dynamics and facilitate the task for the teacher to do so.

2- Learning Culture: There may be a change in learning process cycle through transferring from teacher centered learning philosophy as being the only source of knowledge to student centered learning philosophy. Consequently, the student turns from being the educational process product to the learning process center; owing to the fact that he constantly formulates the knowledge effectively and positively. During this process, the teacher intervenes and help students to get promoted from one knowledge level to another.

3- Intentional Content: This is to determine what content will be provided through direct teaching and what may be provided to students in other ways. This depends on the decisions taken by the teacher according to the nature of the subject and the students themselves.

4- Professional Educator: In contrary to what some people expect, the need for professional educator becomes an urgency in the FL. It is worth mentioning that this type of learning doesn’t aims to or even leads to doing without the teacher, but it increases the need for the teachers who are able to handle this type of teaching strategies. While using this type of teaching strategies, the teachers have miscellaneous decisions to take. Consequently, those decisions should be as much true as possible, such as moving between the direct instruction and the indirect instruction through technology.

Figure 1 shows the four main pillars of this strategy.

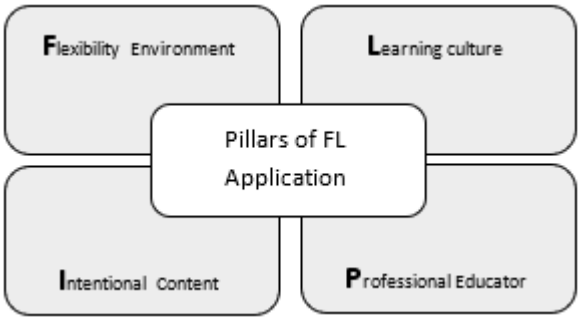

Figure 1. Pillars of Flipped Learning Application

Characteristics of flipped learning strategy

The FL strategy, regardless of other teaching strategies, is characterized with miscellaneous characteristics that regard, on the whole, the students' needs and their potentials; with the aim of reaching better learning based on what the technology provides in terms of optimal learning opportunities. Characteristics of FL can be briefed in the following ones:

1- Keeping pace with the requirements and data of the digital age: this is because the digital age has produced a new generation different from previous generations, and possessed miscellaneous technical tools that were not available beforehand. Those individuals born during the last decade of the twentieth century grew up in an environment replete with digital and non-digital technological devices and tools (Al-Sherman, 2013).

2- Flexibility: The way by which the educational content is presented in the FL environment, i.e., through educational videos uploaded on the Internet, gives students the opportunity to benefit from this feature, and this, in reality, suits the quality of students in the current era (Al-Mashni, 2015; Hamdallah, 2015).

3- Effectiveness: Rearranging the elements of the educational process and its time makes the interaction more enriching, and more useful. As is the case within blended teaching in general, the aim of flipped teaching is to take advantage of (electronic) learning capabilities and the direct traditional learning capabilities; as well as alleviating negatives of each teaching style if handled separately, Therefore, the learning process becomes more effective and able to achieve its long-term objectives and short term objectives more optimally and sustainably (Bishop & Verleger, 2013).

4- Academically retarded students: FL takes into account the individual differences among the students and leaves more room for the teacher during the class to take care of the students and guide them. Instead of being preoccupied with explaining the learning material, he involves all students in the lesson and discusses them, and helps them when needed (Al-Zein, 2015).

5- Increasing the interaction between the teacher and the student: owing to the fact that it is a type of blended learning, and combines the regular learning with the electronic learning, the FL increases the interaction between the teacher and the student. Besides, the increase of interaction between the teacher and the students broadens and includes the retarded students in a way that helps the teacher to perform a more substantive role (Alvarez, 2012).

6- Helping the teacher with the classroom management: the teachers face miscellaneous challenges inside the classroom, most notably; some students' unwanted behavior affects other students' learning, as well as their own lack of attention, and these students have long negatively affected the classroom environment in the traditional learning style. As a result of the fact that the teacher doesn't have the sufficient time and effort in the traditional learning to examine the reasons behind such actions by the

students, the problem is dealt with superficially especially the teacher is bound by a schedule which they should adhere by (Farag, 2010).

7- The FL strikes the balance between the direct and indirect teaching methods and styles. This, in turn, gives the student much more confidence in his learning. It is worth mentioning that acquiring knowledge and grasping the ideas through watching video outside the classroom supply the students with all the required knowledge and information to be applied during the class as well as discussing thereof with their classmates and the teacher. Moreover, the direct class time is necessary for the students; to be sure that they mastered the information to ascertain of what they learn through watching videos. There is not any ambiguity nor obscurity during conducting the FL strategy, and amongst the much more helpful matters that make FL an exciting experience are the discussions and dialogues among students themselves, or questions posed to the teacher, as well as direct applications that demonstrate a proper understanding of principles and ideas, are helpful (Al-Mashni, 2015).

Obstacles of flipped learning strategy application

There are a host of cases that should be taken into consideration; in order to be a stumbling block before applying the FL strategy and getting much of the experience, including the following ones (Al-Sharman, 2015; Al-Mashni, 2015; Valstad, 2011):

1- Appropriate technology level for adopting the FL style: They may be regarded amongst the key issues in the success or failure of this learning style. However, this case has nothing to do with this learning style, but it can be enlisted in the educational technology generally. This conclusion is corroborated by the previous studies asserting that availability of technology with the appropriate method and level is among the main facts determining success or failure of adopting a specific educational technology. Moreover, availability and speed of internet everywhere is regarded one of the fundamental matters that should be taken into consideration.

2- Availability belief system contradicts the FL philosophy: there are a lot of teachers who find it very difficult to give up a large part of their ego, This is when they move from the role of teaching students, which they find appropriate for them, to the role of mentoring and guiding them, so that the teacher is not the only knowledge source for the student, but he becomes one of the many sources that can be referred to for more information. There is a need for the teacher's skills to deal with programs; so, he can produce materials and videos for FL.

3- The student must accept his new responsibilities in learning, and abandon his dependence on the teacher, as is the case with the traditional learning style: in the traditional learning style, the students come to the classroom and have a perception in their minds of how the class will be, which will depend on what the teacher fundamentally says. However, in the FL style, the student shoulders a large and new responsibility, and without having such responsibilities, this learning style will lack in a basic pillar. Consequently, students must accept such learning style; for ensuring they adhere to their role. Moreover, the teacher should be enthusiastic about this new learning style, be prepared to answer students' questions, justify moving to this style instead of the traditional one, and clarify what benefit is to be achieved from this learning style in addition to and all related questions.

Types of flipped learning

The active learning strategies have contributed to formulating multiple types for the FL; in a way that each type reflects one of these strategies which are in use for processing, designing and presenting the curricular and extracurricular activities in a specific way. Amongst these types are those presented by Khalaf (2016), Abu Bakr (2020) as follows:

1- The Traditional FL: it is regarded the first and most widely used form of FL by teachers and starts with learners watching lessons recorded as educational videos from their homes, and then they go to the classroom to participate in the implementation of educational activities and assignments determined by the teacher as part of an integrated work project. This may entail working individually or collectively on the activities, then they are to be assessed at the end of the class period. The learners may participate in assessing themselves when the teacher provides a tool that describes the standards of targeted learning in a gradual manner, and the learners in the light of what they produced can determine their grade on the scale.

2- Flipped Mastery: it is similar to a traditional learning style but depends on the application of activities individually so that each learner's scores are calculated based on how successful they are in achieving the required goals according to the established criteria. Therefore, it focuses on achieving the degree of mastery by evaluating each learner: if he or she achieves the desired degree (the degree of mastery), he moves to the next lesson, and if he does not achieve it, he/ she re-studies the lesson again; so as to master his/ her experiences and skills.

3- Peer flipped learning. It is carried out by providing lectures in the form of recorded videos to be watched by the learners before heading to the class; and the educational videos are to explain in detail all the targeted concepts and educational experiences. When coming to the classroom, the learners find a series of questions that revolve around the main concepts in the explained lesson or the involved basic skills; so that the learner should answer thereof individually. Moreover, the teacher compares the learner's answers with the correct answer to determine if they are true or false, and then directs learners who have answered the correct answers to help their peers who have a misunderstanding or have not mastered the targeted skills. Consequently, every learner with a proper understanding and mastery of the targeted skills works to help their peers to reach the target level. Each learner participates with each other under the full supervision and control of the teacher to intervene if necessary to achieve the desired goals of the lesson not only at the level of remembrance or understanding but at the level of creativity.

4- Inquiry flipped learning: it is meant to provide recorded lectures including the basic information around the target topic. In the classroom, the educational assignments to be performed are provided to learners with research and inquiry tools that they can use to detect important details and required information to implement them. So, the learners can share information and the experiences they reach and work on explaining thereof to other people, with the aim of reaching to the prerequisite steps and performances as well as the most optimal pathways for achieving the target educational product. Consequently, the teacher’s role in such learning style is confined to presenting the research and enquiry tools for the learners as well as providing them with the final form of the targeted educational product according to the target criteria, so that the students can make inquiries and discover the required specifications which they should seek to follow in their products or use in assessing their products.

Steps of peer flipped learning strategy

In the current research the PFL strategy is carried out according to the following steps:

First: Stage of planning, designing the educational activities and sending them to the female students through the technological means:

During this stage, the teacher starts to analyze the curriculum to specify the most important values, knowledge, skills and concepts. After that he moves to record a video clip for the lesson or present a YouTube clip, detailing the targeted educational concepts and experiences.

Second: Learning using the technology outside the classroom:

During this stage, the students individually watch videos sent by the teacher at home before attending the lecture and solve the practices set by the teacher after watching the video clip. Each student writes down her inquiries and notes to show them to the teacher at the beginning of the next lecture.

Third: Learning inside the classroom

Inside the classroom, the teacher and the students start to execute the instructional activities with their different forms based on the students' activity and make them the focus of the educational process. This is according to the following procedures:

1. The teacher checks the students' notes and inquiries about the video that the students watched, answers them, and supplies them with his feedback.

2. Providing some activities and exercises on the lesson, and the teacher interacts with his students, and the students with each other to solve them.

3. Presenting a series of questions that revolve around the main concepts in the explained lesson or the basic skills, so that each student answers them individually through short test 1.

4. The teacher compares the learner's answers with the correct answer to determine if they are true or false, and then directs learners who have answered the correct answers (teacher's partner) to help their peers who have a misunderstanding or have not mastered the targeted skills (learner's partner).

5. The teacher asks each student (teacher's partner) to help her classmate (learner's partner), Thus, each learner who has the correct understanding and mastery of the targeted skills tries to help her classmate to reach the target level. It is worth mentioning that the participation takes place between the students with each other under the full supervision, guidance and control of the teacher to intervene If necessary to achieve the desired objectives of the lesson.

6. After teaching the peers, the teacher applies a short test 2; in order to ensure that the learner's peer understands the concepts of the lesson.

7. The teacher, based on the results of the second short test, identifies the misconceptions of the learner's peer, which were not answered in the second short test; so as to handle them in the next lecture.

Second Axis: Mathematical Thinking

Mathematical Thinking (MT) is deemed among the general terms that includes many meanings and directions, and this can be attributed to the fact that every researcher in the educational field has dealt with the term from his personal perspective, Despite the fact that there is contradictory points of view, there is an agreement amongst researchers and mathematicians that MT is meant to be a process that includes one or more of the following: logic, abstraction, conclusion, representation, analysis, synthesis, generalization, and proof. (Karadag, 2009).

Skills of the mathematical thinking

Mathematical thinking (MT) encompasses a diverse set of skills, as defined by Jaafar (2013) and Abd Al-Kareem (2014). These skills include deduction, which involves extracting specific details from general principles; induction, the process of deriving overarching rules from multiple individual cases; generalization, the ability to create general expressions from specific examples; and intuition, conscious guesswork without analytical rigor. Symbolism plays a crucial role, employing mathematical symbols for effective communication and translation. Modeling represents real-world scenarios mathematically, while relationship perception uncovers novel connections from existing

information. Visual spatial perception bridges the gap between visual observations and mathematical outcomes, enhancing problem-solving. Finally, mathematical proof comprises verbal or symbolic justifications, tracing each statement's logical progression and relying on established evidence, such as postulates and theories, to validate mathematical results. These multifaceted MT skills collectively empower individuals to approach mathematical challenges with creativity, precision, and rigor.

Significance of mathematical thinking

MT has an important position. This importance lies in being one of the main goals on which the learning process is based; owing to the fact that the ability to think mathematical and employ mathematics in solving problems is regarded as one of the salient goals of education. Consequently, the MT is a substantive factor in developing and facilitating aspects of economic, technical, and scientific life. The countries start to progressively realize that the economic progress of some society is derived from power of the MT (the mathematical culture) of such countries. Add to that, importance of the MT also lies in being a means of arming the students with the ability to use and employ the mathematical concepts (Stacey, 2006).

MT is an important tool in the school education process and an essential tool in teaching and learning mathematics. In addition, MT is an effective strategy through which various mathematical problems can be solved (Stacey, 2006).

The significance of Mathematical Thinking (MT) is underscored by its role in fostering essential educational outcomes, as highlighted by Katagiri (2004). Firstly, MT serves as a potent motivational force, guiding students towards the acquisition of specific mathematical skills and knowledge. It empowers students to discern which skills and knowledge are most suitable for solving various mathematical problems and encourages active engagement with diverse problem-solving situations. Additionally, MT plays a pivotal role in cultivating independent thinking and the ability to learn autonomously. By acting as this driving force, it elevates students to a level of independent thought and self-directed learning, enhancing their understanding of how to learn effectively and achieve self-sufficiency within the educational environment. In essence, MT empowers learners with the tools and motivation necessary to excel in mathematics and become autonomous, lifelong learners.

The significance of Mathematical Thinking (MT) becomes evident when considering the challenges identified by Ibrahim (2005) in developing this crucial skill in students. MT serves as a powerful antidote to mathematical anxiety, which often stems from a fear of mathematics, by providing students with the tools to approach mathematical problems with confidence and rationality. It offers an alternative to the indoctrination method in teaching mathematics, fostering a more flexible and exploratory mindset. In overcrowded classrooms, MT can empower students to think critically and independently, transcending the limitations of class size. Moreover, it encourages thoughtful reflection rather than rushed problem-solving, promoting a deeper understanding of mathematical concepts. Finally, the development of MT is essential for utilizing modern technological means effectively, even in schools with limited resources, as it equips students with problem-solving skills that can be applied across various contexts. In essence, MT serves as a valuable asset in overcoming obstacles to mathematical learning and empowers students to navigate the challenges of mathematics with confidence and resilience.

Methodology

The current research depended on the experimental approach with quasi experimental design for measuring the effect of the independent variable (PFL strategy) on the dependent variables (AA – MT skills) among the female students in mathematics department. Furthermore, the research depended on the equivalent groups through

choosing two groups as follows: an experimental group who were taught using the PFL strategy and the control group who were taught using the regular teaching method with the pre and the post application of the measurement tools to both groups.

Population:

The population for this research consists of female students enrolled in the mathematics department at the College of Science, Imam Abdulrahman Bin Faisal University, located in the Kingdom of Saudi Arabia, who are studying the financial mathematics curriculum during the second semester of the academic year 2021/2022.

Sample:

The sample for this research is comprised of 70 female students who were enrolled in the mathematics department at the College of Science, Imam Abdulrahman Bin Faisal University, during the second semester of the academic year 2021/2022. These students have been selected to participate in the research to assess the impact of using the flipped learning (FL) strategy in teaching the financial mathematics course on their academic achievement (AA) and mathematical thinking (MT) skills.

Procedures of Research:

The research procedures involved several key steps. Initially, the researchers focused on preparing teaching materials for the study. This phase began with a comprehensive analysis of the financial mathematics course to identify essential topics, concepts, and skills. Based on this analysis, the researchers developed educational lessons for the course, which were presented in the form of recorded videos and ready-made YouTube clips. To ensure the quality and integrity of the content, these materials were subjected to a rigorous review process. A group of experts in the field of curricula and instruction evaluated the scientific material and activities for scientific accuracy and linguistic appropriateness. Additionally, the materials were reviewed by a panel of educational technology experts to assess their technical validity. Following these reviews, the researchers made necessary modifications, including the addition of activities and exercises. Finally, the lessons were refined and prepared in their final form. Subsequently, the research process moved on to the preparation of instruments for the study.

Instruments:

Academic achievement test

The achievement test aims to measure the AA level among female students of the mathematics department of in the College of Science in the financial mathematics course content. The achievement levels to be measured were determined (remembering - understanding - application), and the initial form of the test was constructed based on multiple-choice questions in light of the table of specifications as a result of the course content analysis.

In order to ensure the validity of the test, it was presented in its initial form to a group of jurors specialized in mathematics education, with the aim of expressing what they deem appropriate, and presenting their suggestions of modifications. Then, the researchers made the necessary modifications in light of the opinions of the jurors. The test was administered to a pilot sample of (30) female students from the mathematics department in the College of Science at the end of the second semester for the academic year 2020/2021.

To verify the ability of the test items to discriminate, the correlation coefficient was calculated between the scores of each cognitive level and the total score of the achievement test obtained by the researchers from the pilot study of the test. The SPSS program was used to find the correlation coefficients, as shown in Table 1.

Table 1: The correlation matrix of the achievement test

Cognitive level correlation coefficient	Correlation Coefficient
Remembering	**0.81
Understanding	**0.85
Application	**0.83

Significant at 0.01

Table 1 shows that the correlation coefficients between the three cognitive levels and the total test score ranged from 0.81 to 0.85, all of which are statistically significant at the 0.01 level. These results indicate the test items' good ability to discriminate.

The reliability coefficient of the test was calculated using the Alpha Cronbach (α), resulting in a test reliability coefficient of 0.82. The total test time was determined to be 90 minutes, which includes 5 minutes for explaining the test instructions to the students. Additionally, difficulty coefficients were calculated for the test items and ranged from 0.25 to 0.70. Discrimination coefficients for the test items were computed based on the ease and difficulty coefficients and ranged from 0.21 to 0.25. Consequently, the final form of the achievement test comprises 15 multiple-choice questions and 3 open-ended questions, with each question containing 3 sub-questions that require students to provide solutions and steps.

Mathematical thinking test

The test aimed to measure the level of MT skills of female students of the mathematics department at the College of Science. To prepare the test, the researchers constructed a list of MT skills that can be developed among female students of the mathematics department, by reviewing some literature and previous studies and analyzing the financial mathematics course. The research identified the following list of MT skills: (deduction, induction, generalization, guessing, expression by symbols, and modeling).

The initial form of the test included multiple-choice questions with four choices designed to assess MT (Mathematical Thinking) skills. To validate the test, it was initially presented to a panel of experts specializing in mathematics education. The researchers then made necessary modifications based on the feedback and opinions of the experts. To assess the discriminative ability of the test items, it was administered to a pilot sample, and the correlation coefficient was calculated between each MT skill's level and the total score of the MT test, as illustrated in Table 2.

Table 2: The correlation matrix of the Mathematical Thinking test

Serial No.	Skills	Total Score
1	Induction	0.75**
2	Deduction	0.78**
3	Expression with symbols	0.85**
4	Generalization	0.81**
5	Guessing	0.74**
6	**Modeling**	0.83**

** Significant at 0.01

Table 2 reveals that the correlation coefficients between the sub-skills of MT and the total MT score ranged from 0.74 to 0.85, all of which are statistically significant at the 0.01 level. These results indicate the test items' strong ability to discriminate.

The reliability coefficient of the test was calculated using the Alpha Cronbach (α), resulting in an overall test reliability coefficient of 0.80. The total test duration was determined to be 80 minutes, which includes an additional 5 minutes allocated for explaining the test instructions to the students. Consequently, the final version of the MT test comprises 27 multiple-choice questions.

Experimental Design and Procedural Steps of the Research:

After preparing the research instruments and confirming their validity and reliability, the research experiment was executed and put into practice. Figure 2 illustrates the experimental design of the research along with its procedural steps.

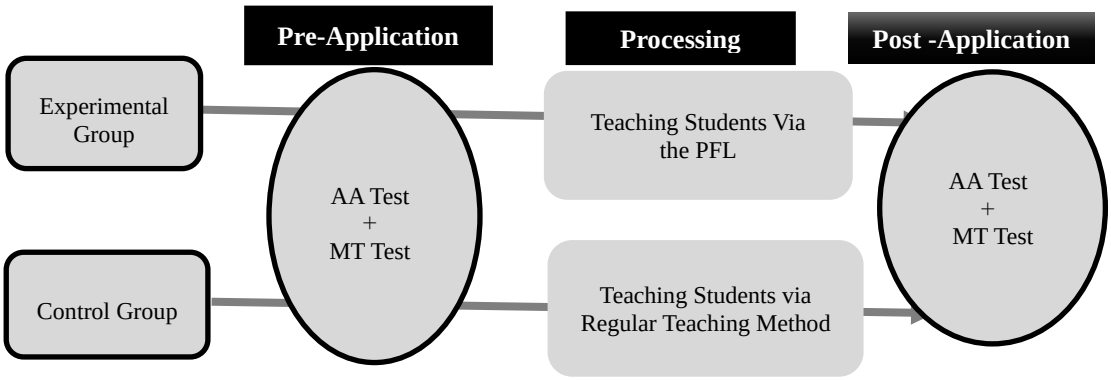

Figure 2. The experimental Design of the Research and Procedural Steps

Results and Discussion

Results of the Achievement Test

To address the first question, "What is the effect of using the FL strategy in teaching the financial mathematics course on the development of AA among students of the College of Science at Imam Abdul Rahman Bin Faisal University?" the first hypothesis, which asserts "there is a statistically significant difference between the mean scores of the experimental group students and the control group students in the post-application of the achievement test in favor of the experimental group students," was tested and verified (refer to Table 3).

Table 3: Results of the post application of the achievement test

Achievement Levels	Group	N	M	SD	DF	t	η^2	Size Effect
Remembering	Experimental	35	8.15	2.21	68	0.24	0.001	No
	Control	35	8.03	2.32				
Understanding	Experimental	35	7.43	1.98	68	4.96**	0.266	Large
	Control	35	5.41	1.77				
Application	Experimental	35	9.78	2.62	68	8.02**	0.4866	Large
	Control	35	6.07	1.40				
Total Test degree	Experimental	35	25.40	5.52	68	5.03**	0.2716	Large
	Control	35	19.60	5.66				

** Means Significant at 0.01

Table 3 indicates a statistically significant difference at the 0.01 significance level between the mean scores of the experimental group students and the control group in the post-application of the AA test, favoring the experimental group students in both the overall test and specifically in the levels of understanding and application. This confirms the validity of the first hypothesis. Furthermore, Table 3 reveals that the impact of employing the PFL strategy on the development of AA is substantial.

In general, the previous results indicate the significant impact of the PFL strategy on the development of mathematics achievement. This outcome can be attributed to several factors:

1. Comfortable Learning Environment: In the flipped classroom setting, students feel at ease and confident asking questions, which helps them overcome obstacles in understanding the material.

2. Flexibility in Learning: Female students can learn at their own pace, in a way that suits them, and in a location of their choice. They can also review missed information and benefit from contributions made by their classmates.

These findings align with the work of Al-Zein (2015), who reported that learners in flipped classrooms outperformed their peers using traditional teaching methods. Bergman & Sams (2015) also noted that flipped classrooms facilitate a deeper understanding of mathematical concepts and encourage students to put in the effort needed to grasp complex topics. Additionally, this research's results are consistent with other studies, including those conducted by Feledichuk & Wong (2015), Abu Bakr (2020), Abdul Salam (2020), Khapre et al. (2021), and Ruiz de Miras et al. (2022).

The effectiveness of the PFL strategy can be understood in light of the following aspects:

1. Active Learning Outside the Classroom: Students engage in activities such as watching videos, reading texts, and reviewing materials independently, focusing on remembering, understanding, and application.

2. Facilitated Classroom Learning: Instructors guide students in understanding and applying concepts during classroom sessions. Students have ample time to practice activities and solve exercises, fostering deep and meaningful learning, as per Bloom's cognitive goals classification (Bergman & Sams, 2015; Al-Sherman, 2015).

3. Constructivist Learning: The PFL strategy promotes a constructivist approach, facilitating social knowledge-building and understanding, which enhances academic achievement.

Moreover, the results can be explained by Bergman & Sams (2015), who noted that the flipped classroom encourages deeper learning. In traditional classes, more time is typically devoted to remembering and understanding, with limited time for higher-order thinking like application, analysis, evaluation, and synthesis. In contrast, the PFL strategy emphasizes lower-order thinking outside the classroom using technology while focusing on higher-order thinking inside the classroom. This results in more profound learning and a greater impact on achievement.

Researchers attribute these outcomes to the efficient use of lecture time, the integration of PFL with peer teaching, the formation of a clear mental image of course content, and the enhanced impact of technology in reinforcing concepts and improving AA (Academic Achievement).

Results of the Mathematical Thinking Test

To address the second question, "What is the effect of using the FL strategy in teaching the financial mathematics course on the development of MT (Mathematical Thinking) among students of the College of Science at Imam Abdul Rahman Bin Faisal University?" the second hypothesis, which posits "there is a statistically significant difference between the mean scores of the experimental group students and the control group students in the post-application of the MT skills test, favoring the experimental group students," was tested and confirmed (refer to Table 4).

Table 4: Results of the post application of the mathematical thinking skills test

MT Skills	Group	N	M	SD	DF	t	η^2	Size Effect
induction	Experimental	35	3.44	0.84	68	**8.95	0.541	Large
	Control	35	2.24	0.89				
Deduction	Experimental	35	3.58	1.03	68	**6.57	0.388	Large
	Control	35	2.28	1.12				
Expression with symbols	Experimental	35	3.89	0.70	68	**7.50	0.453	Large
	Control	35	2.50	1.32				
Generalization	Experimental	35	3.80	1.06	68	**7.78	0.471	Large
	Control	35	2.50	0.85				
Guessing	Experimental	35	3.77	0.95	68	**5.12	0.278	Large
	Control	35	2.68	1.24				
Modeling	Experimental	35	3.25	0.83	68	**4.86	0.258	Large
	Control	35	2.23	1.22				
Total Test degree	Experimental	35	21.80	3.07	68	**13.12	0.717	Large
	Control	35	14.50	3.89				

** Means Significant at 0.01

It is evident from Table 4 that a statistically significant difference exists at the 0.01 significance level between the mean scores of the experimental group students and the control group in the post-application of the MT (Mathematical Thinking) test, favoring the experimental group students across the entire test and all its skills. This confirms the validity of the second hypothesis. Additionally, Table 4 highlights that the impact of using the PFL strategy in the development of MT and its associated skills—induction, deduction, symbolic expression, guessing, generalization, and modeling—is substantial.

In general, the preceding results underscore the significant impact of the PFL strategy on the development of MT skills among female students in the Mathematics Department of the College of Science. Researchers attribute this effect to several factors:

1. Allocating Sufficient Time for Classroom Activities: Allowing ample time for classroom activities and applications motivates students to think flexibly in various directions.

2. Exchange of Ideas with Peers: Collaborative interactions with peers enhance students' critical thinking abilities. Additionally, one of the learning objectives in the flipped classroom is to encourage students to actively solve problems and draw upon previous experiences to tackle current challenges.

These findings align with the results of previous studies, such as those conducted by Marlowe (2012), Talley & Scherly (2013), Al-Maadi (2014), Khawarishida (2017), Ahmed (2018), Zou & Xie (2019), and Al-Shehri (2021).

The aforementioned results can be interpreted in the following ways:

1. Student as Researcher: In the flipped classroom, students take on a researcher's role, effectively using technology outside the classroom. This enhances their self-learning skills, independence in learning, and experience-building, all of which contribute to the development of MT skills.

2. Changing Perceptions of Learning: The PFL strategy alters students' perceptions of the learning process, increasing motivation, comfort in asking questions, and opportunities for interaction with teachers. Communication means through the internet and beyond the classroom enable students to share their work, obtain comments, and receive feedback from peers and teachers, enhancing MT skills (Alharbi & Alshumaimeri, 2016).

Furthermore, these results can be explained in light of what Ibrahim (2005) pointed out as obstacles to MT, including mathematical anxiety, traditional lecture-based teaching methods, hasty problem-solving, lack of reflection, and the absence of modern technological tools for MT development. All of these challenges can be addressed through peer learning strategies.

Researchers attribute these outcomes to the focus of the PFL strategy on providing feedback, reinforcing correct thinking processes, and correcting misconceptions. Peer teaching improves the peer's mastery of the subject matter, as they must anticipate and answer questions from their peers during the explanation process. Moreover, the use of peer teaching encourages students to test, experiment, and research independently and in collaboration with their peers. This deeper engagement leads to better understanding of educational materials, the ability to model concepts and generalizations, comprehension of relationships, formulation of logical conclusions, generation of hypotheses and justifications, and the development of non-traditional problem-solving skills. All these elements contribute to the enhancement of students' MT skills.

Conclusion:

This research investigated the impact of Peer Flipped Learning (PFL) on the academic achievement (AA) and mathematical thinking (MT) of female students in the mathematics department at the College of Science, Imam Abdulrahman Bin Faisal University. The results, as evidenced by post-assessment AA and MT tests, demonstrated significant improvements in favor of the experimental group, emphasizing the effectiveness of PFL as a contemporary teaching strategy for enhancing university-level mathematics education. It also highlights the vital need for educator training in implementing this approach to nurture students' academic achievement and critical thinking skills. Based on these findings, the study recommends the adoption of updated teaching strategies in mathematics education, emphasizing the move away from traditional methods, and encourages the organization of training courses and workshops for faculty members to effectively utilize PFL. Additionally, integrating technological tools and modern programs into mathematics courses is suggested. Future research should further explore the effects of PFL on various aspects of mathematics education, including creative thinking, critical thinking, visual thinking, and numerical sense among students. Furthermore, investigating the potential synergies between PFL and other teaching strategies, such as cooperative learning, differentiated education, and programmed education, could provide valuable insights into enhancing MT development among learners.

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