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Stress Fracture of Non-Reinforced Concrete: Analytical Solutions and Experimental Development

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Abstract

To analyze the behavior of non-reinforced concrete subjected to stresses, the analytical solutions proposed by Griffith and Irwin regarding the mechanics of linear elastic fracture for brittle materials were evaluated. These analyses involve criteria such as plastic deformation, cohesive force, fracture effort, elastic potential energy, and critical energy release rate. These analytical solutions were compared with experimental results carried out during the investigation and other authors' results. In this way, it was possible to establish trends in terms of mechanical behaviors. It was shown that the Griffith analytical solution, based on which was formulated by the Irwin solution, generates a greater correlation with the actual behavior of concrete specimens at the specified design resistance. Finally, Griffith's analytical solution was adjusted based on the flexural stress tests. The mix to manufacture the cylinders and test beams was designed according to the procedures of the American Concrete Institute, ACI Committee 211 and the Road Note Laboratory (RNL).

Keywords: *fracture; brittle materials; concrete; flexural stress tests.*

1. Introduction

Deformations of structures are evaluated from the perspective of solid mechanics. This type of analysis can be carried out by establishing that the work done by external forces acting on a structural element is finally transformed into deformation energy; this is the same internal work that develops as the structure deforms. If its elastic limit is not exceeded, the elastic deformation energy will return the structure to its initial geometric shape if the loads that cause the deformation are removed. The displacements and slopes developed in the structure can lead to fractures in the material, causing the collapse of the structure, that is, failure due to total separation. Thus, by evaluating the elastic behavior, it is essential to enter into the Linear Elastic Fracture Mechanics (LEFM), which is the science that studies the mechanisms and processes by which cracks are propagated into solids. It also evaluates the general and particular distribution of stresses and deformations that occur in a cracked material or discontinuities, subject to a certain external voltage condition. Moreover, it allows determining the fracture resistance of materials and provides a methodology to study crack growth processes.

El estudio de la Mecánica de Fractura Elástica Lineal (MEFL) ha avanzado en las últimas décadas, con el desarrollo de nuevas técnicas y metodologías que mejoran la comprensión del comportamiento de los materiales ante cargas extremas. Recientes investigaciones en

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el campo de la MEFL han arrojado resultados de gran relevancia para la evaluación de la integridad estructural. En particular, se ha demostrado que la aplicación de modelos numéricos basados en la teoría de la MEFL permite predecir con precisión el comportamiento de estructuras en presencia de fisuras, lo cual es muy útil para prevenir fallos catastróficos en la ingeniería civil. Además, las investigaciones sobre la influencia de factores como la velocidad de propagación de las grietas en la fractura de materiales han mejorado la comprensión de los procesos de crecimiento de grietas y la resistencia a la fractura de los materiales, lo que ha permitido mejorar la comprensión de los procesos de crecimiento de grietas y la resistencia a la fractura de los materiales. Ahora bien, es esencial evaluar las deformaciones de las estructuras desde la perspectiva de la mecánica de sólidos para garantizar su estabilidad y seguridad. En este tipo de análisis, se considera que el trabajo realizado por las fuerzas externas que actúan sobre un elemento estructural se transforma en energía de deformación, la cual es devuelta a la estructura si no se supera su límite elástico. Sin embargo, los desplazamientos y pendientes desarrollados en la estructura pueden provocar fracturas en el material y eventualmente, el colapso de la estructura. En este trabajo, se evaluará el comportamiento elástico de una estructura utilizando la Mecánica de Fractura Elástica Lineal (MEFL) y se discutirán los resultados de investigaciones recientes en este campo, así como su relevancia para la evaluación de la integridad estructural

2. Theoretical bases

The fracture can be defined as the separation or fragmentation of a solid under the action of stresses. This results in two new surfaces undergoing three stages: initiation or crack nucleation, propagation of those cracks and final separation . Depending on the amount of plastic deformation preceding the fracture, it is classified as ductile if there is a noticeable plastic deformation and brittle if there is little or no plastic deformation. From the point of view of fracture propagation, there are two forms of the crack path through the microstructure of polycrystalline materials: intergranular trajectory and transgranular trajectory (Figure 1).

La fractura es el proceso mediante el cual un sólido se separa o fragmenta bajo la acción de cargas externas. Este proceso consta de tres etapas: la nucleación o iniciación de la grieta, la propagación de esta y la separación final en dos superficies distintas [1]. La clasificación de la fractura se basa en la cantidad de deformación plástica que precede al fallo. Si se produce una deformación plástica significativa, se habla de una fractura dúctil; en cambio, si no hay deformación plástica o es mínima, se trata de una fractura frágil. Desde el punto de vista de la propagación de la fractura, existen dos trayectorias de la grieta a través de la microestructura de los materiales policristalinos: la trayectoria intergranular y la trayectoria transgranular (Figura 1).

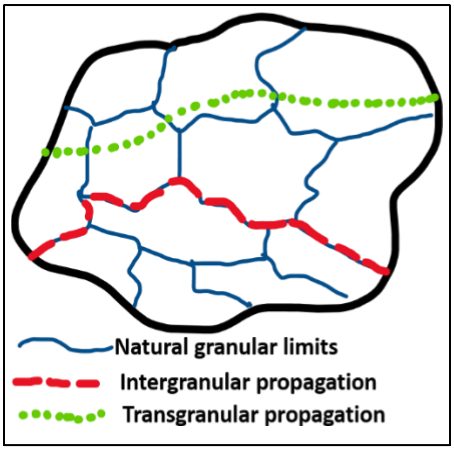

Figure 1 Atomic model of cleavage fracture

In the particular case of transgranular propagation, the crack can be shown in two ways: crystallina, where the crack path is along a well-defined crystal direction and not crystallina, where the crack path follows a path independent of any crystallographic direction. Brittle materials have faults where the dominant mechanism of their fracture is cleavage, which is the direct separation of a plane by the breakage of the atomic links under stress, as seen in Figure 2.

En la propagación transgranular de una fisura, se pueden distinguir dos tipos de trayectorias de fisuración: cristalina, en la que la fisura sigue una dirección cristalográfica bien definida, y no cristalina, en la que la fisura sigue una trayectoria independiente de cualquier dirección cristalográfica. Los materiales frágiles tienen fallas donde el mecanismo dominante de fractura es la clivaje, que es la separación directa de un plano por la rotura de los enlaces atómicos bajo tensión, tal como se muestra en la Figura 2. Además, la propagación transgranular de las fisuras también puede ocurrir a través de un proceso de estricción, donde el material se estrecha en la región de la punta de la fisura debido al aumento de la tensión en esa zona. Este proceso puede generar un aumento significativo en la concentración de tensiones, lo que a su vez acelera la propagación de la fisura y, en última instancia, conduce a la fractura del material. Es importante destacar que la elección del modo de propagación de la fisura (intergranular o transgranular) está influenciada por varios factores, incluyendo la orientación cristalina, la geometría de la fisura, la temperatura y el tipo de carga aplicada.

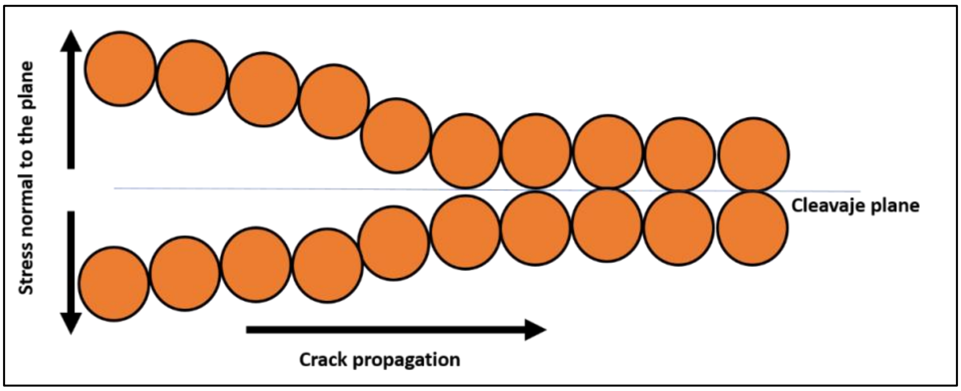

Figure 2 Routes of propagation of the crack in polycrystalline materials

In terms of the Linear Elastic Fracture Mechanics (LEFM), it is essential to ask yourself and resolve the following questions: What is the value of the tensile stress that causes the fracture? What is the crack size that produces fracture when a given load is applied? And finally, how long does that crack take to grow? In this order of ideas, there are two ways to answer these questions: Experiences in the laboratory with related methods and materials [2,3] or mathematically modeling of this type of fracture mechanics, from which behavior can be predicted. For various years, several authors have presented to the scientific community different modellings of the mechanics of linear elastic fracture, which have allowed to evaluate their scopes and predict the behavior of some materials. For this case, the projections are compared from Griffith and Irwin's modellings and the Dugdale-Barenblatt solution, which were presented to the world various years ago; it is also mentioned, but they have been the subject of studies by several authors in the field of materials. For a better understanding, a description of the basic principles of fracture mechanics is presented below, starting with the estimation of the theoretical force, going through the introduction of the stress intensity factor and the fracture hardness and ending with the application in calculating fracture force.

En términos de Mecánica de Fractura Elástica Lineal (LEFM, por sus siglas en inglés), es fundamental plantear y resolver las siguientes preguntas: ¿Cuál es el valor del esfuerzo de tracción que provoca la fractura? ¿Cuál es el tamaño de la grieta que produce la fractura cuando se aplica una carga determinada? Y finalmente, ¿cuánto tiempo tarda en crecer esa grieta? En este orden de ideas, existen dos formas de responder a estas preguntas: mediante experimentos en laboratorio con métodos y materiales relacionados [2,3] o mediante la modelización matemática de este tipo de mecánica de fractura, a partir de la cual se pueden predecir los comportamientos. Durante varios años, autores han presentado a la comunidad científica diferentes modelos de la mecánica de fractura elástica lineal, los cuales han permitido evaluar sus alcances y predecir el comportamiento de algunos materiales. Para este caso, se comparan las proyecciones de los modelos de Griffith e Irwin y la solución de Dugdale-Barenblatt, los cuales fueron presentados al mundo hace varios años, y se menciona que han sido objeto de estudio por varios autores en el campo de los materiales. Para una mejor comprensión, a continuación, se presenta una descripción de los principios básicos de la mecánica de fractura, comenzando con la estimación de la fuerza teórica, pasando por la introducción del factor de intensidad de esfuerzos y la dureza de fractura y terminando con su aplicación en el cálculo de la fuerza de fractura.

2.1. Cohesive force

Depending on the cleavage mechanism, fracture stress must be sufficient to separate the atomic planes; based on this, the fracture force has the magnitude of the cohesive force. The binding force of the atom (σ) varies depending on the separation of the atom (x), as shown in Figure 3.

Dependiendo del mecanismo de fractura por clivaje, la tensión de fractura debe ser suficiente para separar los planos atómicos; a partir de esto, la fuerza de fractura tiene la magnitud de la fuerza cohesiva. La fuerza de enlace de los átomos (σ) varía dependiendo de la separación de los átomos (x), como se muestra en la Figura 3.

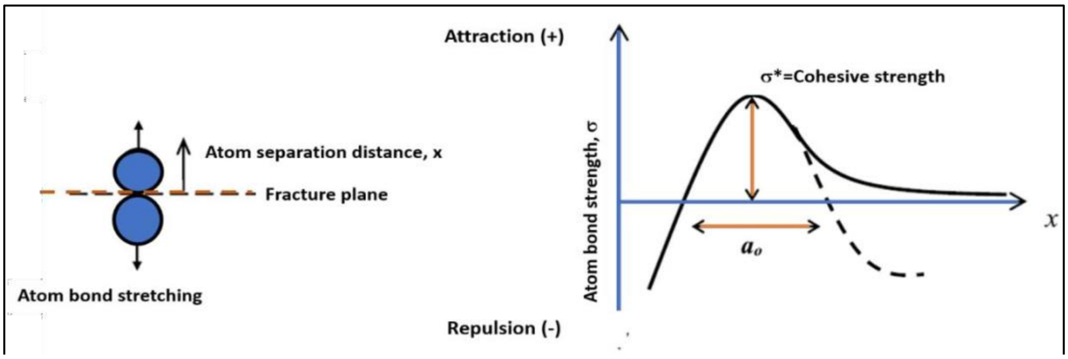

Figure 3 Cohesive force model. Source: Adapted from [1].

The theoretical cohesive force σ^* is the maximum of the curve σ versus x, reached in $x = a_0/2$; As the separation of the atom increases further, the binding force of the atom decreases, so the fracture process is irreversible. The variation of the binding force of the atom as a function of its separation is given by the equation (1):

$$\sigma = \sigma^* \sin\left(\frac{2\pi x}{a_0}\right) \tag{1}$$

If the term $(2\pi x)/(a_0)$ is close to zero, the approximation in equation (2) is valid.

$$\sigma = \sigma^* \left(\frac{2\pi x}{a_0}\right) \tag{2}$$

Assuming that the separation of the planes produces an elastic deformation, given by Hooke's law $\sigma = E\epsilon$, where E is the module of elasticity and ϵ is the strain, equal to x/a_0, it is reasonable to present equation (3).

$$\sigma^* \left(\frac{2\pi x}{a_0}\right) = E \frac{x}{a_0} \quad (3)$$

By clearing cohesive force, the equation (4) is obtained.

$$\sigma^* = \frac{E}{2\pi} \quad (4)$$

By replacing the typical E values, which can become high values, the evidence shows that the cohesive force is much greater than the experimental stresses to produce cleavage, which are in the range of hundreds of MPa, being able to reach up to about 50 times greater. However, since the material's tendency to divide along crystallographic planes is a real phenomenon, the logical conclusion is that there must be a condition that reduces fracture effort or concentrates that effort high enough to achieve cohesive resistance in solid materials. These reasonings occurred at the beginning of the twentieth century, but it was Inglis [4], in 1913, who mathematically demonstrated that the fracture was related to the presence of defects in the material.

A partir del reemplazo de los típicos valores E, que pueden alcanzar valores elevados, se ha evidenciado que la fuerza cohesiva es mucho mayor que las tensiones experimentales necesarias para producir la clivación (fenómeno que se refiere a la tendencia de algunos materiales cristalinos a fracturarse siguiendo planos de debilidad), como los planos de enlace atómico más débiles en su estructura cristalina, las cuales se encuentran en el rango de cientos de MPa, pudiendo llegar hasta unas 50 veces mayores. Sin embargo, dado que la tendencia del material a dividirse a lo largo de planos cristalográficos es un fenómeno real, la conclusión lógica es que debe existir una condición que reduzca el esfuerzo de fractura o que concentre ese esfuerzo lo suficiente para alcanzar la resistencia cohesiva en materiales sólidos. Estos razonamientos surgieron a principios del siglo XX, pero fue Inglis [4], en 1913, quien demostró matemáticamente que la fractura estaba relacionada con la presencia de defectos en el material. En este contexto, en el campo de la mecánica de la fractura, se ha investigado exhaustivamente la influencia de los defectos en los materiales en la reducción de los esfuerzos en la fractura. Se ha comprobado que la presencia de defectos, ya sean naturales o artificiales, es un factor determinante en la propagación de las fisuras en los materiales sólidos. Los defectos naturales pueden incluir porosidades e inclusiones, mientras que los defectos artificiales son aquellos que son creados durante el procesamiento y la fabricación del material.

2.2. Griffith's analytical solution to evaluate linear elastic fracture mechanics in brittle materials

In 1920, years after Inglis' approach, Alan Arnold Griffith [5], an English engineer, made a significant contribution by predicting the fracture stress, claiming that this fracture results from an energy transfer process.

En 1920, años después del planteamiento de Inglis, Alan Arnold Griffith [5], un ingeniero inglés, realizó una importante contribución al predecir el estrés de fractura, sosteniendo que esta se produce por un proceso de transferencia de energía.

Griffith demonstrated that a crack will propagate when the elastic energy stored in the body is released at a rate that is equal to the creation of surface energy as the crack grows (Figure 4).

Griffith demostró que una fisura se propagará cuando la energía elástica almacenada en el cuerpo se libere a una tasa igual a la creación de energía superficial a medida que la fisura crece (Figura 4). Este fenómeno es conocido como la teoría de la energía de fractura y proporciona una herramienta útil para predecir la resistencia a la fractura de materiales frágiles.

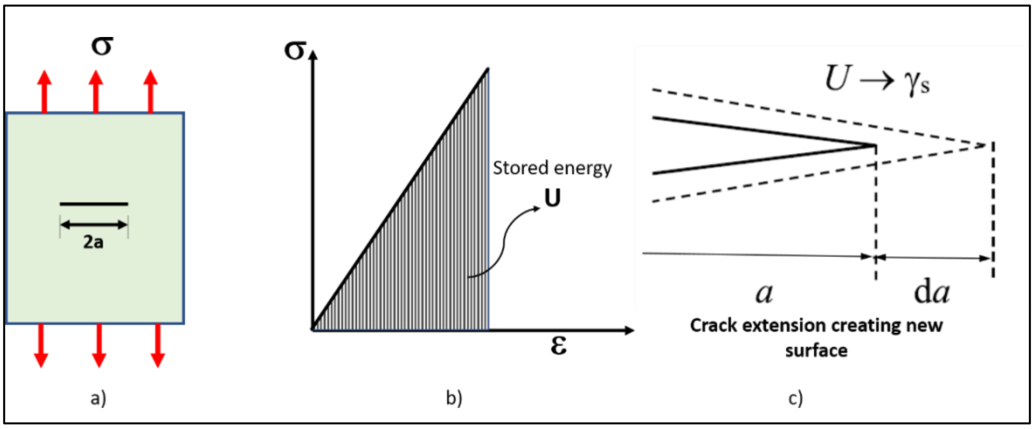

Figure 4 Griffith's brittle fracture model. a) Thin plate with a central crack. b) Stress-deformation register. c) Converting stored energy U into surface energy γ_s . Source: Adapted from [1].

Griffith's reasoning was based on the following: In a flat, homogeneous and isotropic plate, subjected to an elastic deformation, with a central crack as shown in Figure 4, the deformation energy can be modeled as presented in equation (5).

El razonamiento de Griffith se basó en lo siguiente: En una placa plana, homogénea e isotrópica, sometida a una deformación elástica, con una fisura central como se muestra en la Figura 4, la energía de deformación puede ser modelada como se presenta en la ecuación (5).

$$U = \frac{\pi\sigma^2 a^2}{E} \tag{5}$$

Where: σ is the tension applied on the plate, a is the size of the crack and E is the module of elasticity (Young's modulus). In a completely brittle fracture (without plastic deformation), the work supplied by the loads is stored as potential elastic energy and consumed to create two new fracture surfaces. If the stored energy is U and the surface energy is γ_s, the energy balance is:

$$\Delta U = U + 4a\gamma_s \tag{6}$$

Here, Griffith took the surface area as 4a which corresponds precisely to the cracking surface area considering a unit thickness of length 2a and multiplied by two because that is the number of faces that the crack has. When the crack increases its length, it enters an unstable equilibrium state under the applied stress, before which the mechanical system transfers energy from the non-fractured area to the fractured area, converting elastic energy into surface energy. To calculate the fracture energy or critical energy release rate, it is only necessary to derive the energy with respect to the length of the crack. This is a property of the material, and it is considered constant while in the elastic range.

En este caso, Griffith utilizó el área de superficie como 4a, lo que corresponde precisamente al área de la superficie de fractura considerando una longitud de 2a y multiplicado por dos, ya que es el número de caras que tiene la grieta. Cuando la grieta aumenta de longitud, entra en un estado de equilibrio inestable bajo el esfuerzo aplicado, antes de que el sistema mecánico transfiera energía desde el área no fracturada al área fracturada, convirtiendo la energía elástica en energía superficial. Para calcular la energía de fractura o la tasa crítica de liberación de energía, solo es necesario derivar la energía con respecto a la longitud de la grieta. Esto es una propiedad del material y se considera constante mientras está en el rango elástico.

$$\frac{dU}{da}=\frac{dW}{da} \tag{7}$$

By deriving each term, the following is obtained:

$$\frac{dU}{da}=\frac{2\pi\sigma^2 a}{E} \tag{8}$$

$$\frac{dW}{da}=4\gamma_s \tag{9}$$

Replacing (8) and (9) in (7), clearing the tension stress σ, which in this case corresponds to the stress for which the prolongation of the crack is initiated, and it is known as Griffith's fracture stress σ_c (10):

$$\sigma_c=\sqrt{\frac{2\gamma_s E}{\pi a}} \tag{10}$$

The most important term is the Critical deformation energy release speed, represented in the equation as $2\gamma_s$, known as the Griffith´s Module.

2.3. Irwin's analytical solution

Irwin reiterated Griffith's postulate by proposing that the rate of energy release in a cracked body should be equal to the demand for fracture work to cause a crack to spread spontaneously. Irwin named this condition as instability, so when the energy release rate is insufficient, the crack does not propagate, so it is said to be stable. As under Griffith's criterion, the energy available in a cracked body under the action of a charge comes from the work done by the loads, which is stored in the body as elastic energy. In honor of Griffith, Irwin used the letter G as a symbol of the energy release rate, expressed as follows:

Irwin amplió la teoría de Griffith proponiendo que la tasa de liberación de energía en un cuerpo con fisura debe igualar la cantidad de energía necesaria para que la fisura se propague de forma espontánea. Esta condición, que Irwin denominó inestabilidad, implica que si la tasa de liberación de energía no es suficiente, la fisura no se propagará y, por lo tanto, se considerará estable. Al igual que en el criterio de Griffith, la energía disponible en un cuerpo con fisura bajo la acción de una carga proviene del trabajo realizado por la carga, el cual se almacena en el cuerpo como energía elástica. En honor a Griffith, Irwin utilizó el símbolo G para representar la tasa de liberación de energía, que se expresa de la siguiente manera:

$$G=\frac{d(F-U)}{da} \tag{11}$$

where U is the stored energy and F is the work supplied by loads. If W is the work needed to grow the crack and R is the amount of work needed to cause the extension of the crack, the fracture resistance can be written as $R=dW/da$. Based on this analysis, Irwin postulated the Fracture Energy Criterion: If $G>R$, the crack spreads. Although the Irwin approach allows experimental determination of fracture strength, its practical application remains complicated, as the value of G depends on the geometry, crack size and loading conditions.

2.4. Model Dugdale-Barenblatt

Dugdale [6] suggested that plastic deformation occurs only in a narrow area ahead of the crack end for ductile materials. This model can also be interpreted as a particular case of the Barenblatt model for brittle materials, which is why the Dugdale-Barenblatt model emerges as a contribution to classical linear elastic theory. For example, Yi et al. [7] have addressed the problem of a Dugdale crack between different materials, numerically studying the size of the plastic zone and the displacement of the crack end of the interface under uniform loads but representing an elastoplastic behavior. Researchers like Ferdjani [8] have made comparisons to establish that the critical stress is not a constant of the material. But on a practical level, and thanks to what has been established by Willis and Rice [9], the equivalence of the Dugdale-Barenblatt approach has been demonstrated with Griffith's results, which can be written as follows:

Según el autor Dugdale [6], la deformación plástica ocurre únicamente en una zona estrecha en la punta de la fisura en materiales dúctiles. Este modelo se considera una variante del modelo de Barenblatt para materiales frágiles, lo que da lugar al modelo Dugdale-Barenblatt como una contribución a la teoría elástica lineal clásica. Un estudio numérico realizado por Yi et al. [7] abordó el problema de la fisura Dugdale en diferentes materiales, determinando el tamaño de la zona plástica y el desplazamiento de la punta de la fisura en la interfaz bajo cargas uniformes, pero teniendo en cuenta un comportamiento elastoplástico. El investigador Ferdjani [8] ha realizado comparaciones para establecer que el esfuerzo crítico no es constante en todos los materiales. Sin embargo, en términos prácticos, se ha demostrado que el enfoque Dugdale-Barenblatt es equivalente a los resultados obtenidos por Griffith, tal como se ha indicado por Willis y Rice [9]. Este resultado se puede expresar matemáticamente de la siguiente manera:

$$G = -\frac{\partial U}{\partial a} = \int_{0}^{\partial t} \sigma(\delta) d\delta \tag{12}$$

3. Methodology for experimentation

Normally concrete structure designs do not involve the low resistance to tensile forces of this material. However, some load systems can produce stress on elements that usually work at compression (columns or walls) when there are important geometric asymmetries in the structural system or during earthquakes [10]. Generally, it is complex to develop an appropriate process for measuring concrete stress resistance through direct testing, as there are difficulties for optimal sample assembly and uncertainties regarding stress concentrations developed in the vicinity of couplings. The NTC 722 [11] in Colombia proposes an indirect method by loading a standard cylinder of 0.15 m in diameter by 0.30 m in height to compression along two diametrically opposed axial axes (splitting test). Indirect stress resistance was thus calculated:

$$RT = \frac{2P}{\pi L D} \tag{13}$$

Where RT is Tensile or indirect voltage resistance of a cylinder [kg/cm2; 1 MPa 10.2 kg/cm2], P is the maximum applied load [kg], L is the cylinder length [cm] and D is the cylinder diameter [cm]. The design of mixtures for experimental samples was based on the American Concrete Institute procedures developed by the ACI 211 Committee [12] and the Road Note Laboratory (RNL). The design parameters were: 36 MPa, 34.5 MPa, 30 MPa and 21 MPa with water/cementing material ratios (w/c) of 0.36, 0.40, 0.45 and 0.48, respectively and without air included. Six cylinders with dimensions of 0.150 × 0.300 m were manufactured for each of the established ratios. In addition, three 0.150 × 0.150 × 0.300 m beams were manufactured. All specimens were cured by immersion and tested at 28 days of age. The axial compression [13] and compression stress [14] tests were used, cylindrical specimens were used, while the three beams for each w/c ratio were used for bending stress testing [15]. Compression and bending stress tests (Table 1) were calculated using equations (14) y (15),

$$\sigma = \frac{P}{A} \tag{14}$$

$$\sigma_f = \frac{Pl}{bd^2} \tag{15}$$

where P is the maximum fracture load, A is the cross-sectional area of the specimen, l is the length between supports, b and d are the average width and height of the specimen.

Table 1 Results of axial compression (A. Comp), splitting test (D. Comp) and bending stress test

Tests	Water/cementing material ratio 0.36				0.4				0.45				0.58			
				AVG				AVG				AVG				AVG
A. Comp. (MPa)	35.0	36.0	36.0	35.7	33.0	35.0	35.0	34.3	29.7	30.0	31.4	30.4	23.0	20.6	21.0	21.5
D. Comp. (MPa)	3.70	3.60	3.70	3.67	3.40	3.30	3.40	3.37	2.71	2.64	2.83	2.73	2.20	2.23	2.00	2.14
Bending (MPa)	4.80	4.90	4.60	4.77	4.30	4.70	4.70	4.57	4.18	4.21	4.09	4.16	3.80	3.91	3.80	3.84

4. Comparison with the analytical solution

Luis A. de Béjar [16] made an estimate of Griffith's module using generalized simple stress tests. His article reported a value of G-135.04 N/m, equivalent to 0.13504 N/mm. These values are consistent with those reported by Hillerborg [17] for relatively low w/c ratios. For this reason, the value of 0.13504 N / mm will be assumed for calculating the fracture stress according to the Griffith and Irwin modeling, and a value of a = 50 according to the Béjar report. The value of a varies depending on the resistance, but it will be taken the same for all mixtures to evaluate trends. Based on the aforementioned, the estimates of fracture efforts according to analytical solutions (equation 10) are presented in Table 2. In contrast, experimental results are presented, taking as the value of E the established in C.8.5.1 of the Colombian regulation NSR-10 [18], in agreement with ACI 318 [19].

Table 2 Fracture stress from analytical solution vs. experimental tests

A. Comp. [MPa]	G [N/mm]	E [MPa]	a [mm]	Analytical. [MPa]	Bending [MPa]	D. Comp. [MPa]
35.7	0.13504	28 100	50	6.95	4.77	3.67
34.3	0.13504	27 500	50	6.88	4.57	3.37
30.4	0.13504	25 900	50	6.67	4.16	2.73
21.5	0.13504	21 800	50	6.12	3.84	2.14

The fracture stress values calculated by indirect compression and bending are close to the value of 10% and 13%, respectively, of the maximum experimental compressive stress. However, they are considerably removed from the value calculated by the analytical solution. As seen in Figure 5, the fracture stress values calculated using the splitting test are almost half of the projections raised by Griffith and Irwin. This means that, in practice, the bending test allows for a better assessment of the tensile behavior of a concrete element and that the greater the axial compressive strength, the greater the similarity between the analytical solution and the experimental value. According to the tendency found, a mathematical correction is proposed to Griffith's fracture stress from a value calculated through experimentation. For this case, Figure 5 shows that for design compression values less than 30 MPa, the behavior in all three cases is linear, while for values greater than 30 MPa, it tends to form an exponential curve.

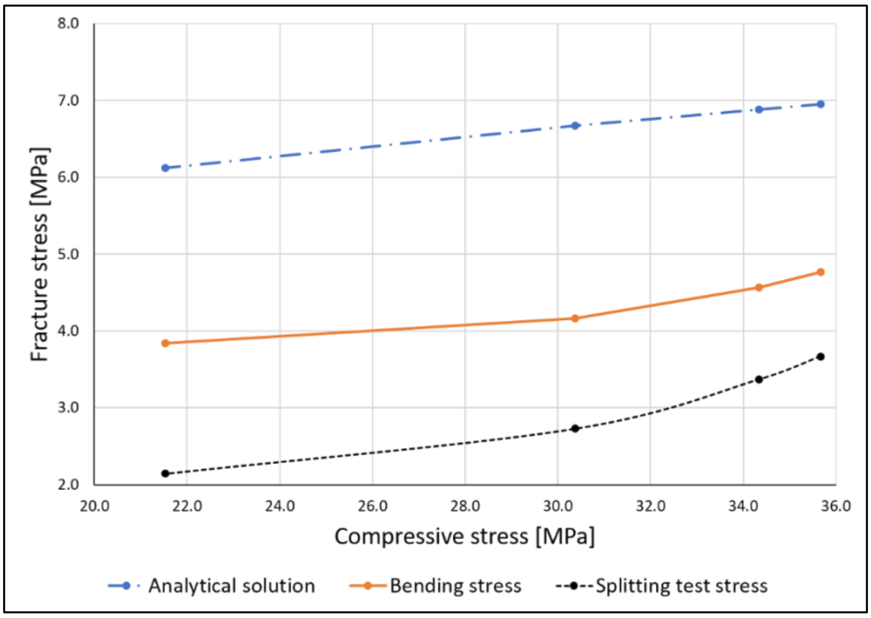

Figure 5 Graphical comparison of fracture stress

Based on this analysis, we propose two types of settings: a linear factor for f'c values less than 30 MPa and another exponential factor for higher values, as follows:

If f'c <30 MPa, then, $\sigma_{exp} = 0.625\ \left[\sqrt{\frac{2E\gamma_s}{\pi a}}\right]$ (16)

If f'c > 30 MPa, then, $\sigma_{exp} = 0.0079\ \left[\sqrt{\frac{2E\gamma_s}{\pi a}}\right]^{3.3036}$ (17)

5. Conclusions

According to the quasi-brittle nature of the concrete, its resistance to tensile forces is not used in many structural designs. Still, to the extent that great modifications have been made to its mixture, basically with the decrease of fine aggregates and the addition of micro and nanofibers, this stretching capacity has been gaining great importance. Strength evaluation is an important process for understanding the behavior of this material. Tests that allow their evaluation under stress force conditions can become complex, and they are often not achieved under similar conditions to what an element experiences once it becomes an integral part of a real structure. From this perspective, analytical solutions are a valuable tool to achieve values as close as possible to reality. Irwin's analytical solution, reiterating Griffith's postulate, considers the energy transfer process and material defects. Precisely, these defects produce some uncertainty when testing a material, so it has been necessary to propose a mathematical adjustment. For the case of this research, the UHPC study was taken as a reference, where its stretching capacity is high compared to traditional concretes. According to all this, it is clear in Figure 5 that the trend is that the higher the resistance to the compression of concrete, the greater the tenacity, so the higher the energy transfer, the greater the Griffith Module. With this trend, the analytical solution and the experimentation curves begin to converge. More research on this behavior should be conducted. Finally, it is necessary to clarify that, at more tensile force, the crack size is also greater, a fact that was not considered in the present investigation since it was worked on from what was reported by other authors.

References

[1] González-Velázquez JL. Mechanical behavior and fracture of engineering materials. Structural Integrity. 2019.

[2] Cantillo V, Guzmán A. Fluid-Pressured Test to Measure Tensile Strength of Concrete. Journal of Materials in Civil Engineering 2014;26:776–80. https://doi.org/10.1061/(ASCE)MT.1943-5533.0000849.

[3] Resan SF, Chassib SM, Zemam SK, Madhi MJ. New approach of concrete tensile strength test. Case Studies in Construction Materials 2020;12:e00347. https://doi.org/10.1016/j.cscm.2020.e00347.

[4] Inglis CE. Stresses in a plate due to the presence of cracks and sharp corners. Trans Inst Naval Archit 55, 219-241 1913.

[5] Griffith AA. The phenomena of rupture and flow in solids, Phil. I Trans. R. Soc. Series A 1921:22L-I63-198 (1920-L92L).

[6] Dugdale DS. Yielding of steel sheets containing slits. Journal of the Mechanics and Physics of Solids 1960;8:100–4.

[7] Yi DK, Xiao ZM, Tan SK. On the plastic zone size and the crack tip opening displacement of an interface crack between two dissimilar materials. International Journal of Fracture 2012;176:97–104. https://doi.org/10.1007/s10704-012-9713-3.

[8] Ferdjani H, Marigo J-J. Dugdale model applied to a mixed mode loading of a semi infinite cracked structure. 22ème Congrès Français de Mécanique, AFM, Association Française de Mécanique, 2015.

[9] Willis JR. A comparison of the fracture criteria of Griffith and Barenblatt. Journal of the Mechanics and Physics of Solids 1967;15:151–62.

[10] Moreno EI, Varela-Rivera J, Solis-Carcaño RG, Gómez López MA. Resistencia a tensión del concreto elaborado con agregado calizo de alta absorción [Tensile strength of concrete with high absorption limestone aggregate]. Concreto y cemento. Investigación y Desarrollo 2016;8:35–45.

[11] Instituto Colombiano de Normas Tecnicas y Certificacion (ICONTEC). NTC 722 - Método de ensayo para determinar la resistencia a la tensión indirecta de especímenes cilíndricos de concreto [Test method for determining the indirect tensile strength of cylindrical concrete specimens] 2000.

[12] Dixon DE, Prestrera JR, Burg GR, Chairman SA, Abdun-Nur EA, Barton, S. G., ... & Lee SH. Standard Practice for Selecting Proportions for Normal , Heavyweight , and Mass Concrete (ACI 211 . 1-91) 1991.

[13] ASTM. ASTM C39, Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens 2014.

[14] ASTM. ASTM C496-96, Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens 2017.

[15] ASTM. ASTM-C78, Standard Test Method for Flexural Strength of Concrete 2021.

[16] de Béjar LA. Virtual estimation of the Griffith ' s modulus and cohesive strength of ultra-high performance concrete. Engineering Fracture Mechanics 2019;216:106488.

[17] Hillerborg A. Results of three comparative test series for determining the fracture energy Gf of concrete. Materials and Structures 1985;18:407–13.

[18] Ministerio colombiano de ambiente vivienda y desarrollo territorial. NSR-10, Reglamento Colombiano de Construcción Sismo Resistente [Colombian Construction Regulations for Earthquake Resistance] 2010.

[19] ACI Committee 318. ACI 318-19: Building code requirements for structural concrete and commentary. 2019.