On the Investigation of Derivation Pairs in Relation to Semi-Rings

Shaheed Jameel al-Dulaimi[1], Mustafa I. Hameed[2], Israa A. Ibrahim[3], Hussaini Joshua[4]

Abstract

The purpose of this article is to give some applications of the variations subordination concept on subclasses. The key objectives of the preceding paper are to investigate the dependency principle and to attempt to add a further group over polyvalent works via an extra operator related to derivative goods with a greater order. Several investigations were carried out to look into variations algebra musical notation by using algebraic frameworks like a ring or semi-ring for investigating some of its characteristics. The findings from the analysis of were additionally extended in this article through including the notation of $(\alpha, \beta) - \hat{\delta}$ −derivation pair and studying some of its features. Furthermore, a couple of instances are provided to demonstrate that any $(\alpha, \beta) - \hat{\delta}$ −derivation combination is a Jordan $(\alpha, \beta) - \hat{\delta}$ −derivation combine but not vice versa.

Keywords: *Derivation, Derivation Pair, Semi-ring, Analytic Function, Ring Theory, Generalized Derivation.*

1. INTRODUCTION

The Jordan, for example, derivation combination notation was first introduced by [1] and [2] via the concept about involution. To be more specific, the first paper introduced the syntax of calculation pair as an expansion of the Jordan -derivation notion discussed in [3]. While the second had investigated the same musical notation on the complicated - algebra and demonstrated numerous results. The author introduced the notation of (ϑ_1,ϑ_1)-derivation pair on rings in [4], which was further extended from the results of [5]. He demonstrated that any (ϑ_1,ϑ_1)-derivation pair is a Jordan, for example, (ϑ_1,ϑ_1)-derivation combine but not the other way around. He also demonstrated a number of additional findings concerning this notation in terms of the prime sound and the 2-torsion permitted ring. Yass stipulated and studied the code of fervently derivation pairs on prepare and semi-prime rings in [6]. She demonstrated some fundamental properties of this notation using the notion of prime as well as semi-prime rings. She also looked into the connection among this notation as well as derivations. In a nutshell, the author demonstrated that under certain conditions, firmly derivation connects become derivations. Kadhim [7] established the notation for ζ-derivation pairs as well as Jordan ζ-derivation france on the ζ-ring. The authors addressed some of the characteristics of this note-taking. Thiruveni et al. [8] investigated the syntax of semi-ring derivation pairs. The

[1] Department of computer science, Al- Maarif University College, Ramadi, Iraq, s.jkharbeet@uoa.edu.iq
[2] University of Anbar, College of Education for Pure Sciences, Department of Mathematics, Ramadi-Iraq, mustafa8095@uoanbar.edu.iq
[3] Department of Science College of Open Education, Kirkuk Education Directorate, Kirkuk, Iraq, esraabdullah3@gmail.com
[4] Department of Mathematical Sciences, Faculty of Science, University of Maiduguri, Nigeria, jhussaini@unimaid.edu.ng

same author defined and investigated the characteristics of ζ-derivation connects upon semi-rings [9]. Motivated by these studies, we provided the notation of $(\alpha, \beta) - \hat{\delta}$ −derivation pair as well as studied a few of its properties in this paper, where α as well as β are two auto orphisms consisting $\hat{\delta}$.

Definition 1.1 [6]: The ring R is considered to have a regular when there is present a component $\mu \in$ R where $b\mu b = b$ for every component an $b \in$ R.

Example 1.1 [10]: According to normal multiplication and addition circumstances, the rings $\mathbb{Q}$, the collection of all numbers that are rational, as well as $\mathbb{R}$, the set for every number that is real, have been regular rings.

Definition 1.2 [13]: A somewhat ring constitutes a triple $(\hat{\delta}, +, \cdot)$ if $(\hat{\delta}, +)$ as well as $(\hat{\delta}, \cdot)$ have been semi groups as well as . is equitable over +.

Example 1.2 [6]:

(1) According to normal operations of addition and multiplication, $\mathbb{N}$, the set that includes all numbers that are natural, kinds a semi-ring.

(2) According to normal multiplication and addition conditions, $\mathbb{Z}$, the set that includes all integers, types a semi-ring.

(3) According to normal multiplication and addition, the numbers both $\mathbb{Q}$ and $\mathbb{R}$ have been semi rings.

Definition 1.3 [6]: If $\hat{\delta}$ is a semi-ring, then it's known as 2-torsin permitted if $2\mu = 0$ indicates $\mu = 0$, when $\mu \in \hat{\delta}$.

Definition 1.4 [11]: Assume $(\hat{\delta}, +, \cdot)$ is a semi-ring. A self-guided tour $Y: \hat{\delta} \to \hat{\delta}$ it is claimed to have been Jordan derivation if $Y(\mu^2) = Y(\mu)\mu + \mu Y(\mu)$ over any of them $\mu \in \hat{\delta}$.

Definition 1.5 [6]: If $\hat{\delta}$ is a semi-ring, then it's known as semi-prime if $\mu\hat{\delta}\mu = 0$ indicates $\mu = 0$, when $\mu \in \hat{\delta}$.

Definition 1.6 [11]: Assume $(\hat{\delta}, +, \cdot)$ is a semi-ring. A self-guided tour $Y: \hat{\delta} \to \hat{\delta}$ it is claimed to have been derivation if $Y(\mu\tau) = Y(\mu)\tau + \mu Y(\tau)$ over any of them $\mu, \tau \in \hat{\delta}$.

Definition 1.7 [12]: Assume $(\hat{\delta}, +, \cdot)$ is a semi-ring. The left-hand-semi-module represents an abelian monoid $(\phi, +, 0_\phi)$ in which it has a function $\hat{\delta} \times \phi \to \phi$ considering as $(\mu, \tau) \to \mu\tau$ that meets the requirements provided underneath.

1- $1 \times \eta = \eta, \forall \eta \in \phi$

2- $(\mu\mu')\eta = \mu(\mu'\eta), \forall \mu, \mu' \in \hat{\delta}$ as well as $\eta \in \phi$

3- $\mu(\eta + \eta') = \mu\eta + \mu\eta', \forall \mu \in \hat{\delta}$ as well as $\eta, \eta' \in \phi$

4- $(\mu + \mu')\eta = \mu\eta + \mu'\eta, \forall \mu, \mu' \in \hat{\delta}$ as well as $\eta \in \phi$

5- $\mu 0_\phi = 0_\phi = 0_{\mu\eta}, \forall \mu \in \hat{\delta}$ as well as $\eta \in \phi$. If $\chi(\phi) = \phi$, ϕ is a $\hat{\delta}$-module, as well as $\chi(\phi)$ is the set that's components all have additive inverses.

Definition 1.8 [11]: Assume $\hat{\delta}$ is a semi-ring as well as ϕ is a $\hat{\delta}$-module. Adding maps $Y, \Gamma : \hat{\delta} \to \phi$: M are referred to as $\hat{\delta}$-Derivation combination if $Y(\mu\tau) = Y(\mu)\tau + \mu\Gamma(\tau)$ over any of them $\mu, \tau \in \hat{\delta}$ and $\Gamma(\mu\tau) = \Gamma(\mu)\tau + \mu Y(\tau)$ for any $\mu, \tau \in \hat{\delta}$. They also claimed to be Jordan. $\hat{\delta}$-Derivation combination if $Y(\mu^2) = Y(\mu)\mu + \mu\Gamma(\mu)$ for every value as well as $\Gamma(\mu^2) = \Gamma(\mu)\mu + \mu Y(\mu)$ over any $\mu \in \hat{\delta}$.

2. THE MAIN FINDINGS

Definition 2.1: The company semi ring is considered to be Bourne regular when there is present $\mu, \tau \in \hat{\delta}$ for all $b \in \hat{\delta}$ that result in $b + b\mu b = b\tau b$.

Example 2.1:

1. Allow $\mathbb{R}_0^+$ equal $\{\mu \in \mathbb{R} / \mu \geq 0\}$ and $\mathbb{Q}_0^+$ equal $\{\mu \in \mathbb{Q} / \mu \geq 0\}$. These constitute the Bourne semi rings.

2. In the event $\hat{\delta} = \{0, 1\}$ as well as $0 + 0 = 0, 0 + 1 = 1 + 0 = 1 + 1 = 1, 0 \cdot$

$0 = 0 \cdot 1 = 1 \cdot 0 = 0$ and $1 \cdot 1 = 1$, subsequently $(\hat{\delta}, +, \cdot)$ constitutes a Bourne regular semi ring.

3. Allow $\hat{\delta}$ to have been a subset that is not empty of $\mathbb{R}$ where min $\hat{\delta}$ is present. As an all $\mu, \tau \in \hat{\delta}$, define $\mu \oplus \tau = max\{\mu, \tau\}$ as well as $\mu \odot \tau = min\{\mu, \tau\}$. Following that there's Bourne's regular semi-ring $(\hat{\delta}, \oplus, \odot)$.

Theorem 2.1: Each of the Von Neumann regular within a semi ring $(\hat{\delta}, +, \cdot)$ is a Bourne regular, as well as the opposite is true if $\hat{\delta}$ is a group under additional.

Proof: Allow $\hat{\delta}$ become a Von Neumann regular as well as $(\hat{\delta}, +, \cdot)$ become a semi ring. Following that there's an $\mu \in \hat{\delta}$ with the value

$$b = b\mu b, \quad (b \in \hat{\delta}). \tag{1}$$

Let $\tau \in \hat{\delta}$. Following that,

$$b = b\tau b$$

Including $b\tau b$ to both ends of (1) yields

$$b + b\tau b = b\mu b + b\tau b$$
$$b + b\tau b = b(\mu + \tau)b$$
$$b + b\tau b = b\lambda b, \quad \text{where } \lambda = \mu + \tau \in \hat{\delta}$$

As a result, for every an $b \in \hat{\delta}$, it has $\lambda, \tau \in \hat{\delta}$ that correspond to $b + b\tau b = b\lambda b$. Therefore $\hat{\delta}$ Bourne constitutes a regular. Assume $\hat{\delta}$ is a group according to addition as well as $\hat{\delta}$ is a Bourne regular.

As a result, for every an $b \in \hat{\delta}$, it has $\mu, \tau \in \hat{\delta}$ that correspond to $b + b\mu b = b\tau b$.

Because $b\mu b \in \hat{\delta}$ as well as $\hat{\delta}$ are a group according to in addition $-b\mu b \in \hat{\delta}$

Adding $-b\mu b$ on each side of (1) yields

$$b + b\mu b - b\mu b = b\tau b - b\mu b$$
$$b = b(\tau - \mu)b, \quad \big(\text{The additive opposite of } b\mu b \in \hat{\delta} \text{ is } -b\mu b.\big)$$
$$b = b\lambda b, \quad \text{such that } \lambda = \tau - \mu \in \hat{\delta}.$$

As a result $\widehat{\delta}$, Von Neumann has become a regular.

Definition 2.2: Allow $\hat{\delta}$ become a semi-ring and ϕ to become a $\hat{\delta}$-module. Let $\alpha, \beta : \hat{\delta} \to \hat{\delta}$ denote two automorphisms. If both additive mappings $\Upsilon, \Gamma : \hat{\delta} \to \phi$ satisfy the equations further down, they are deemed to be $(\alpha, \beta) - \hat{\delta}$ −derivation pairs

$$\Upsilon(\mu\tau) = \Upsilon(\mu)\alpha(\tau) + \beta(\mu)\Gamma(\tau), \quad (\mu, \tau \in \hat{\delta})$$
$$\Gamma(\mu\tau) = \Gamma(\mu)\alpha(\tau) + \beta(\mu)\Upsilon(\tau), \quad (\mu, \tau \in \hat{\delta}).$$

As Well As they stated it was $J - (\alpha, \beta) - \hat{\delta}$ −derivation pair, if there is

$$\Upsilon(\mu^2) = \Upsilon(\mu)\alpha(\mu) + \beta(\mu)\Gamma(\mu), \quad (\mu \in \hat{\delta})$$
$$\Gamma(\mu^2) = \Gamma(\mu)\alpha(\mu) + \beta(\mu)\Upsilon(\mu), \quad (\mu \in \hat{\delta}).$$

Example 2.2: Assume $\hat{\delta}$ is a non-commutative semi-ring and ϕ is a $\hat{\delta}$-module. In addition, let $\mu, \tau \in \hat{\delta}$ be a value that $\alpha(\mu)r = \beta(\mu)v = 0$ for any $\mu, r, v \in \hat{\delta}$. When we establish, we get $\Upsilon, \Gamma : \hat{\delta} \to \phi$ by $\Upsilon(\mu) = r\alpha(\mu)$ and $\Gamma(\mu) = v\alpha(\mu)$, in which α as well as β are auto orphisms for $\hat{\delta}$. Then, (Υ, Γ) is a $(\alpha, \beta) - \hat{\delta}$ −derivation pair. Allow $\mu, \tau \in \hat{\delta}$, then

$$\Upsilon(\mu\tau) = r\alpha(\mu\tau) = r\alpha(\mu)\alpha(\tau)$$
$$= \Upsilon(\mu)\alpha(\tau) + \beta(\mu)v\alpha(\tau) = \Upsilon(\mu)\alpha(\tau) + \beta(\mu)\Gamma(\tau).$$

And

$$\Gamma(\mu\tau) = v\alpha(\mu\tau) = v\alpha(\mu)\alpha(\tau)$$
$$= \Gamma(\mu)\alpha(\tau) + \beta(\mu)r\alpha(\tau) = \Gamma(\mu)\alpha(\tau) + \beta(\mu)\Upsilon(\tau).$$

Hence, (Υ, Γ) is a $(\alpha, \beta) - \hat{\delta}$ −derivation pair.

Theorem 2.2: Consider to $\hat{\delta}$ be a semi-ring, ϕ become a $\hat{\delta}$-module and α, β are two mappings for $\hat{\delta}$. Then, in the event (Υ, Γ) be a $(\alpha, \beta) - \hat{\delta}$ −derivation pair, then $(\Upsilon + \Gamma)$ is a $(\alpha, \beta) - \hat{\delta}$ −derivation.

Proof: Allow (Υ, Γ) be a $(\alpha, \beta) - \hat{\delta}$ −derivation pair. Then, according to Definition 2.2, there is

$$\Upsilon(\mu\tau) = \Upsilon(\mu)\alpha(\tau) + \beta(\mu)\Gamma(\tau), \quad (\mu, \tau \in \hat{\delta}) \tag{2}$$

$$\Gamma(\mu\tau) = \Gamma(\mu)\alpha(\tau) + \beta(\mu)\Upsilon(\tau), \quad (\mu, \tau \in \hat{\delta}) \tag{3}$$

When we combine (1) and (2), we obtain

$$(\Upsilon + \Gamma)(\mu\tau) = (\Upsilon + \Gamma)(\mu)\alpha(\tau) + \beta(\mu)(\Upsilon + \Gamma)(\tau).$$

Hence, $(\Upsilon + \Gamma)$ is a $(\alpha, \beta) - \hat{\delta}$ −derivation.

Theorem 2.3: Consider to $\hat{\delta}$ be a prime semi-ring, ϕ become a $\hat{\delta}$-module, where α and β are two mappings for $\hat{\delta}$. If $\hat{\delta}$ admitting $(\alpha, \beta) - \hat{\delta}$ −derivation pair so that $\Upsilon(\mu) = 0$ for an $\mu, r \in \hat{\delta}$, then $r = 0$ or $\Upsilon(\mu) = 0$.

Proof: Allow $\mu, r \in \hat{\delta}$ and $r \neq 0$, we get

$$r\Upsilon(\mu) = 0 , \quad (\mu, r \in \hat{\delta}) \tag{4}$$

Replacing μ by $\mu\tau$ in (4), we obtain

$$r\Upsilon(\mu\tau) = 0 , \quad (\mu, \tau, r \in \hat{\delta}) \tag{5}$$

Because $\hat{\delta}$ we admit $(\alpha, \beta) - \hat{\delta}$ −derivation pair, we are covered by Definition 2.2

$$r(\Upsilon(\mu)\alpha(\tau) + \beta(\mu)\Gamma(\tau)) = 0 , \quad (\mu, \tau, r \in \hat{\delta})$$

$$r\Upsilon(\mu)\alpha(\tau) + r\beta(\mu)\Gamma(\tau) = 0 , \quad (\mu, \tau, r \in \hat{\delta})$$

Using (4), we have

$$r\beta(\mu)\Gamma(\tau) = 0 , \quad (\mu, \tau, r \in \hat{\delta}) \tag{6}$$

Since $r \neq 0$ as well as $\hat{\delta}$ be a prime semi-ring, hence (6) gives that $\Gamma(\tau) = 0$.

Theorem 2.4: Consider to $\hat{\delta}$ be a prime semi-ring, ϕ become a $\hat{\delta}$-module, where α and β are two mappings for $\hat{\delta}$. If $\hat{\delta}$ admitting $(\alpha, \beta) - \hat{\delta}$ −derivation pair so that $\Upsilon(\mu)\Gamma(\tau) = 0$ for an $\mu, r \in \hat{\delta}$, then $\Upsilon(\mu) = 0$ and $\Gamma(\mu) = 0$.

Proof: Allow $\mu, r \in \hat{\delta}$, then

$$\Upsilon(\mu)\Gamma(\tau) = 0 , \quad (\mu, \tau \in \hat{\delta}) \tag{7}$$

Replacing τ by $\tau\mu$ in (7), we obtain

$$\Upsilon(\mu)\Gamma(\tau\mu) = 0 , \quad (\mu, \tau \in \hat{\delta}) \tag{8}$$

Because $\hat{\delta}$ we admit $(\alpha, \beta) - \hat{\delta}$ −derivation pair, we are covered by Definition 2.2

$$\Upsilon(\mu)(\Gamma(\tau)\alpha(\mu) + \beta(\tau)\Upsilon(\mu)) = 0 , \quad (\mu, \tau \in \hat{\delta})$$

$$\Upsilon(\mu)\Gamma(\tau)\alpha(\mu) + \Upsilon(\mu)\beta(\tau)\Upsilon(\mu) = 0 , \quad (\mu, \tau \in \hat{\delta})$$

Using (7), we obtain

$$\Upsilon(\mu)\beta(\tau)\Upsilon(\mu) = 0 , \quad (\mu, \tau \in \hat{\delta}) \tag{9}$$

Since $\hat{\delta}$ be a prime semi-ring, hence (9) gives that $\Upsilon(\mu) = 0$. Similarly, we can prove $\Gamma(\mu) = 0$.

3. CONCLUSIONS

The investigation of Von Neumann's regular and Bourne's regular within the Semi ring is discussed. The present work concluded by presenting the notation of $(\alpha, \beta) - \hat{\delta}$ −derivation pair of a semi-ring as well as $J - (\alpha, \beta) - \hat{\delta}$ −derivation pair of a semi-ring. Furthermore, it has been demonstrated that any $(\alpha, \beta) - \hat{\delta}$ −derivation pair is a $J - (\alpha, \beta) - \hat{\delta}$ −derivation pair but not the other way around. Additionally, the product of two $(\alpha, \beta) - \hat{\delta}$ −derivation pair is a $(\alpha, \beta) - \hat{\delta}$ −derivation. Furthermore, we demonstrated that if (Υ, Γ) is a $J - (\alpha, \beta) - \hat{\delta}$ −derivation pair, subsequently (Υ, Γ) is a $(\alpha, \beta) - \hat{\delta}$ −derivation pair.

References

[1] B. Zalar. 1995. Jordan-von Neumann theorem for Saworotnow's generalized Hilbert space. Act. Math. Hungar, vol. 69, pp. 301-325.

[2] B. Zalar. 1997. Jordan ∗-derivation pairs and quadratic functionals on modules over ∗-rings. Aequ. Math. vol.54, pp. 31 43.

[3] M. Bresar and J. Vukman, 1989. On some additive mappings in ring with involution, Aequ. Math., vol. 38 pp. 178-185.

[4] M. I. Hameed, B. N. Shihab and K. A. Jassim. (2021, September). Certain Geometric Properties of Meromorphic Functions Defined by a New Linear Differential Operator, In Journal of Physics: Conference Series (Vol. 1999, No. 1, p. 012090). IOP Publishing.

[5] M. K.Shahoodh. 2022. ($\theta1$, $\theta2$)-derivation pair on rings. Ibn al hithem journal for pure and applied sciences, vol. 23, no. 2.pp.108-116.

[6] S. B. Yass. 2010. Strongly derivation pairs on prime and semi-prime rings. Msc. Thesis University of Baghdad- College of sciences.

[7] M. I. Hameed and B. N. Shihab. (2022). On Differential Subordination and Superordination for Univalent Function Involving New Operator. Iraqi Journal For Computer Science and Mathematics, 3(1), 22-31.

[8] A. K. Kadhim, H. Sulaiman and A. R. H. Majeed. 2015. Γ ∗-derivation pair and Jordan Γ ∗-derivation pair on Γ-ring M with involution. Annals of pure and applies mathematics, vol. 10, no.2, pp.169-177.

[9] V.Thiruveni, S. P. Nirmala devi and M. Chandramouleeswaran. 2017. Jordan derivation pairs on semi-rings. Journal of Indian Academy of mathematics, vol. 39, no. 1.

[10] M. I. Hameed and M. H. Ali. (2021, July). Some Classes Of Analytic Functions For The Third Hankel Determinant, In Journal of Physics: Conference Series (Vol. 1963, No. 1, p. 012080). IOP Publishing.

[11] V. Thiruveni, S. P. Nirmala devi and M. Chandramouleeswaran. 2017. S-derivation pairs and Jordan S-derivation pairs on semi-rings. International journal of pure and applied mathematics, vol. 112, no. 5, pp. 1-8.

[12] S. P. Nirmala devi. 2013. Right derivations on semi-rings. International mathematical Formu. Vol. 8, no. 32, pp. 1569-1576.

[13] M. I. Hameed, B. N. Shihab and K. A. Jassim. (2022, October). An application of subclasses of Goodman-Salagean-type harmonic univalent functions involving hypergeometric function, In AIP Conference Proceedings (Vol. 2398, No. 1, p. 060012). AIP Publishing LLC.