Migration Letters

Volume: 21, No: 8 (2024), pp. 1-13

ISSN: 1741-8984 (Print) ISSN: 1741-8992 (Online)

www.migrationletters.com

Shift Detection Comparison For Sized Biased And Area Biased Exponential Distributions Based Control Charts

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Abstract

Probability distribution models are very helpful to solving many real-life problems. The development of new or extensions to an existing model is a very attractive area of research. Among many types of probability distribution models, weighted distributions are becoming more important for modeling data. Weighted distributions can help discover defects in manufacturing and outliers by weighting significant observations. Control charts are the most important tools to monitor process efficiency. In recent decades, researchers have focused on weighted distributions, especially those with size and area biased exponential distribution. This research introduces a novel control chart for a characteristic following a size-biased and area-biased exponential distribution to emphasize the relevance of weighted distributions. Charts are effectively determines size-biased and area-biased exponential distribution control chart limits also Monte Carlo simulations are used to assess different Average Run Length (ARL) values.

KEYWORDS *Shift Detection, Size-Biased Exponential Distribution, Area-Biased Exponential Distribution, Statistical Process Control, Detection Sensitivity, Comparative Analysis***.**

1. Introduction

The most crucial part of statistical quality control is identifying process parameter changes. This ensures optimal efficiency and reduces errors. This study assesses shift detection strategies for exponential distributions with size and area bias. Applying shift detection to exp[1]onential distributions to do this. These biased distributions assistance in comprehending process changes beneath the surface. They may do this because they typically depict real-world events more realistically. Comparing the two approaches to establish which has more sensitivity and accuracy in monitoring processes. This comparison is done to compare. These discoveries are extremely relevant in medicine, where precise process control is critical for patient safety and treatment effectiveness. Overall, the study's findings are relevant. This comparative research study may shed light on the development of advanced monitoring systems in the healthcare industry and other relevant areas.Shewhart control charts were important early 20th-century statistical process control (SPC). Use Shewhart control charts for developing SPC. Monitoring data points against control limits set around a central average allowed them to discover process deviations. This was done via data monitoring. These charts tracked processes, identified issues, and indicated when changes were required to maintain quality and reduce variability[1]. In an economic-statistical context, parameter estimation affected control chart performance. It examined control charts' behaviour when crucial parameters were produced from sample data

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and their implications on statistical correctness and economic issues. Control chart operations were stressed. Trade-offs between these components were examined in this study. The study studied how parameter estimates affected charts' ability to detect out-of-control events and false alarm or process change costs. It also investigated the company's false alarm costs. Estimating parameters may increase variability, evidence shows. The results also suggested that control chart design must include corporate and industrial uncertainty [2].This research examined control chart in-control resilience. These control charts were tested for stability and false alarm resistance at constant settings. Multiple charts were compared to see which worked best when operations were under control[3]. Studies analyzed quality and production management using Shewhart control charts. A study found these charts beneficial for industrial site monitoring. The charts identified process abnormalities, enhancing quality control and production efficiency [4].

Many techniques to enhance Shewhart control charts were studied, emphasizing high sensitivity and accuracy. To track the Weibull mean. Design changes may enhance Weibull mean oscillation detection, improving process monitoring and quality control [5].Process mean control charts were compared to assess how effectively they identify changes. The research tested control chart change detection. Showing chart reliability and sensitivity assistances identify the best process monitoring systems [6]. This research compared highly competent process control charts statistically. Analysis happened throughout the project. This study assessed how successfully control charts identify and stabilize changes. The report recommended effective monitoring charts for high-performance situations [7].

This study examined intelligent manufacturing control charts. The research examined these control charts' efficacy in automated, technologically sophisticated settings. This study assessed charts for smart manufacturing continuous monitoring and quality control[8]. Shewhart time-between-events control charts are compared and analyzed in a renewal process to measure their effectiveness. The optimum control chart designs for time interval monitoring were investigated. Emphasizing control chart design contrasts accomplished this purpose[9]. Shewhart control charts' Average Run Length (ARL) was used to assess their efficiency when process data were obtained from a much smaller population. This study how this characteristic affects the control chart's efficacy and reliability to better understand these elements. Performance fluctuated owing to the limited population sample [10-]. This study considered run-length for detecting structural change in time series data. Modification identification speed was considered throughout the experiment. Many detection approaches were tested for their sensitivity to time series data alterations and structural change analysis. This tested detecting system sensitivity[11].This study suggests that statistical process control ARLs should be affected by non-normally distributed data. This research examined how distributions influenced ARLs using computer modelling. The research found control chart reliability and robustness [12].

Which side-sensitive synthetic chart designs worked best was studied. We stressed the coefficient of variation and used median run length and expected median run length. These designs were tested for coefficient of variation detection. Improvements in control chart topologies boosted dependability [13]. A novel Rayleigh-distributed system control chart was created from this study. Glass fiber strength was tracked using this chart. This control chart was tested for Rayleigh-distributed data variation detection [14]. The progressive adaptation of linear profile parameters was the primary objective throughout this research. The study explored several industrial linear profile monitoring approaches. Since process configurations fluctuate, the research prioritized precision and efficiency. Many strategies were tested to enhance linear profile monitoring and management in complex industrial situations [15].

Industrial quality assurance is being improved via a weighted exponential distribution-based control chart. The monitoring process utilizes this method since it detects production irregularities better than common control charts. Production quality control methods are predicted to improve with this innovation [16].

2 Materials and Methods

2.1 ARL Curves Under Size Biased and Area Biased Exponential Distributions

The pdf of the size-biased exponential distribution is given.

$f(x;\theta) = x\theta^2 e^{-\theta x} \qquad x > 0, \theta > 0$

The cdf is.

$F(x) = 1 - (1 + \theta x)e^{-\theta x}$

The mean and the variance SBED are.

$$E(\theta) = \frac{2}{\theta}$$

$$var(\theta) = \frac{2}{\theta^2}$$

The pdf of the Area-biased exponential distribution is given.

$$f(x,\theta) = \frac{x^2\theta^3 e^{-\theta x}}{2} \qquad ; \; x > 0, \theta > 0$$

The cdf is.

$$F(x) = 1 - \frac{(\theta^2 x^2 + 2\theta x + 2)e^{-x\theta}}{2}$$

The mean and the variance of ABED are.

$$E(\theta) = \frac{3}{\theta}$$

$$var(\theta) = \frac{3}{\theta^2}$$

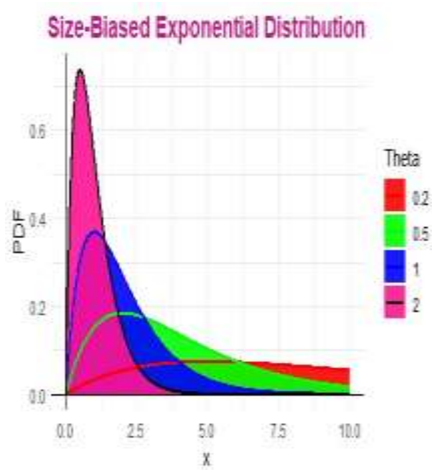

Figure: 1 PDF of Size Biased Exponential Distribution

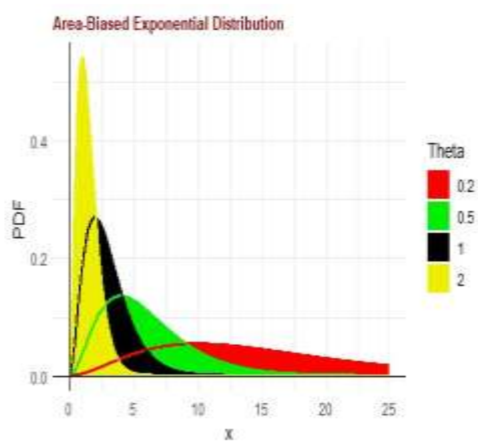

Figure: 2 PDF of Area Biased Exponential Distribution

The underlying principle of the weighted variance technique involves dividing a skewed distribution into two phases at its mean and then using each section to establish a new symmetric distribution. The control chart limits in the weighted variance approach are determined using symmetric distributions. The weighted variance approach shifts control limits for skewed distribution based on population skewness without assumptions.

$$UCL = \mu + 3\frac{\sigma}{\sqrt{n}}\sqrt{2P_x} \quad (2.1)$$

$$LCL = \mu - 3\frac{\sigma}{\sqrt{n}}\sqrt{2(1-P_x)} \quad (2.2)$$

The likelihood that a $r.v$ (x) will be smaller than or equal to its mean is represented by "P_x" Note that when P_x = 0.5, the weighted variance $\overline{X}$chart reduces to the standard $\overline{X}$chart. In this case, (n) is the sample size and σ is the S.D of X.

Size-biased exponential distribution mean and standard deviation are used to derive its theoretical control limits, which are as follows.

$$UCL = E(\theta)\left(1 + \frac{k}{\sqrt{2}} \times \sqrt{2P_x}\right) \quad (2.3)$$

$$CL = E(\theta) \quad (4.16)$$

$$LCL = E(\theta)\left(1 - \frac{k}{\sqrt{2}} \times \sqrt{2(1-P_x)}\right) \quad (2.4)$$

The variable k is used to denote the extent of the control limits, where as θ_0 represents the in-control value of the scale parameter θ. In the case of an out-of-control process, the parameter undergoes a shift to a different value, denoted as $\theta_0 = \delta\theta_0$.

The control limits refer to the statistical limits that are established to determine if a process is within acceptable bounds or whether it shows variations that are outside the expected range. The parameters of the size-biased exponential distribution chart are the (LCL), (UCL), and (CL). If the (UCL) and (LCL) demonstrate random behavior, it can be inferred that the process is stable. The phenomenon is commonly referred to as a "shifted process". The suggested study utilizes the ARL (Average Run Length) and ΔARL (Change in Average Run Length) performance indicators.

2.2.1 Simulation study

i. The predetermined value for the average run length under control conditions ARL_0 has been established at 200, 300, and 370.
ii. The control limits are determined by using a known parameter of the size-biased exponential distribution.
iii. A total of 30,000 samples, each with a sample size of $n = 1$, are drawn from a distribution that follows a size-biased and Area biased exponential distribution.
iv. Each sample is analyzed until it is found to be outside of a set of control limits.
v. Samples outside control limits are recorded as the value of random variable RL, and the loop repeats step 2.
vi. The above method is iterated 50,000 times, resulting in the random variable RL having a total of 50,000 values.
vii. The last procedure is to determine RL mean, median, standard deviation, minimum, and maximum. The 25^{th}, 75^{th} and 99^{th} percentiles of RL and the percentage decline in ARL are calculated.

To further understand the (ARL) pattern, other useful measures of (RL) should be applied. The metrics include SDRL, MRL, MinRL, MaxRL, and various RL percentiles. RL curves may be used to get more information about chart performance. Monte Carlo simulation was utilized to evaluate all chart metrics.

3: Performance Evaluation of ARL Curves under Size Biased Exponential Distribution

To thoroughly evaluate ARL's performance for the chart, other useful RL metrics should be employed. SDRL, MRL, MinRL, MaxRL, and RL percentiles are utilized in this category. This section also includes (MinRL), (MaxRL), and (SDRL) metrics. The Monte Carlo simulation approach was utilized alone and with the RL curves to discover the best ARL curves for optimum outcomes. As illustrated in Tables 4.5–4.7, size-biased and area-biased exponential distribution ARL curves follow this trend. This pattern is seen in the curves. Data from both distributions indicates this pattern.

Table 3.1 RL measures for shifted process at $\mathbf{ARL_0 = 370}$, $\mathbf{K_{SBED} = 3.74}$, $\boldsymbol{\theta = 1}$

Shift $\boldsymbol{\delta}$	ARL	M-RL	SD-RL	MinRL	Max-RL	P_{25}	P_{75}	P_{99}
1	371.31	258	368.49	1	4531	108	518	1677
0.9	156.44 (57.87)	109	155.21	1	1365	45	216	721
0.8	74.19 (80.02)	52	73.55	1	869	22	103	344
0.7	37.15 (89.99)	26	36.43	1	331	11	51	168
0 .6	19.10 (94.86)	13	18.67	1	201	6	26	87
0.5	9.99 (97.31)	7	9.41	1	93	3	14	44
0.4	5.48 (98.52)	4	4.93	1	49	2	7	23
0.3	3.10 (99.17)	2	2.54	1	25	1	4	12
0.2	1.85 (99.05)	1	1.26	1	13	1	2	7
0.1	1.22 (99.67)	1	0.52	1	6	1	1	3

Table 3.2: RL measures for shifted process at $\mathbf{ARL_0 = 370}$, $\mathbf{K_{ABED} = 2.92}$, $\boldsymbol{\theta = 1}$

Shift $\boldsymbol{\delta}$	ARL	M-RL	SD-RL	Min-RL	Max-RL	$\mathbf{P_{25}}$	$\mathbf{P_{75}}$	$\mathbf{P_{99}}$
1	369.64	259	369.58	1	3346	106	511	1687
0.9	142.64 (61.41)	98	142.89	1	1524	41	198	653
0.8	61.77 (83.29)	43	61.33	1	652	18	85	285
0.7	28.89 (92.18)	20	28.34	1	313	9	40	130
0 .6	12.82 (96.53)	9	12.46	1	137	4	18	58
0.5	7.18 (98.06)	5	6.73	1	69	2	10	32
0.4	3.88 (98.95)	3	3.31	1	36	1	5	16
0.3	2.18 (99.41)	2	1.59	1	15	1	3	8
0.2	1.44 (99.61)	1	0.79	1	9	1	2	4
0.1	1.08 (99.71)	1	0.30	1	4	1	1	2

Any control chart's effectiveness is corresponding to its limits, both in identifying changing processes and maintaining the average run length (ARL). This occurs distinct of control chart type. To function properly, the chart must include all of these properties. Lower shifted process indicator values correspond with more quick responses. Therefore, utilize a control chart with decreased Average Run Length $\mathrm{ARL_1}$ values to identify process changes.

4: Comparative Study

The proposed control chart technique aims to reduce Average Run Length (ARL) to detect process changes quickly while maintaining a fixed $\mathrm{ARL_0}$during control. Reduce the ARL. Reduce the ARL to do this. To ensure the required $\mathrm{ARL_0}$ remains constant, the control limit width (K) is adjusted. To prevent changes to $\mathrm{ARL_0}$, this step is performed. The quantity of false alarms and their detection rate are balanced this way. Comparing $\mathrm{ARL_1}$ for process shifts (δ) across control chart configurations evaluates strategy success. Comparing this strategy helps determine its efficacy. We may assess the chart's responsiveness and ability to spot process changes by focusing on distributions like the Size-Biased Exponential and Area-Biased exponential distributions. We can assess the chart's process change detection using this information. With this study, practitioners may study how chart features impact detection speed. This allows them to choose control charts which satisfy their operational demands, which is convenient. Control limit widths, established by a specified $\mathrm{ARL_0}$, enable early process problem detection. Thus, quality control is ensured and the risk of non-compliant products is reduced.

Table 4.1 RL measures for shifted process at $ARL_0 = 200$, $K_{SBED} = 3.392$, $K_{ABED} = 2.659$, $\theta = 1$

Shift δ	ARL_{SBED}	ARL_{ABED}
1	201.39	201.81
0.9	98.28 (51.20)	88.84 (55.98)
0.8	49.74 (75.30)	41.93 (79.22)
0.7	26.65 (86.76)	20.9 (89.64)
0 .6	14.52 (92.79)	10.99 (94.55)
0.5	8.11 (95.97)	5.91 (97.07)
0.4	4.65 (97.69)	3.37 (98.33)
0.3	2.76 (98.62)	2.05 (98.88)
0.2	1.73 (99.14)	1.37 (99.32)
0.1	1.20 (99.40)	1.07 (99.47)

Table 4.2 RL measures for shifted process at $ARL_0 = 300$, $K_{SBED} = 3.635$, $K_{ABED} = 2.839$, $\theta = 1$

Shift δ	ARL_{SBED}	ARL_{SBED}
1	299.77	299.75
0.9	135.77 (54.71)	122.73 (59.96)
0.8	66.08 (77.96)	54.55 (81.80)
0.7	33.24 (88.91)	25.94 (91.35)
0 .6	17.52 (94.16)	12.99 (95.67)
0.5	9.44 (96.85)	6.72 (97.67)
0.4	5.17 (98.28)	3.71 (98.76)
0.3	2.99	2.17

	(99.00)	(99.28)
0.2	1.81 (99.40)	1.42 (99.58)
0.1	1.22 (99.59)	1.08 (99.64)

Table 4.3 RL measures for shifted process at $\mathbf{ARL_0 = 370}$, $\mathbf{K_{SBED} = 3.74}$, $\mathbf{K_{ABED} =}$

Shift δ	$\mathbf{ARL_{SBED}}$	$\mathbf{ARL_{ABED}}$
1	371.31	369.64
0.9	156.44 (57.87)	142.64 (61.41)
0.8	74.19 (80.02)	61.77 (83.29)
0.7	37.15 (89.99)	28.89 (92.18)
0 .6	19.10 (94.86)	12.82 (96.53)
0.5	9.99 (97.31)	7.18 (98.06)
0.4	5.48 (98.52)	3.88 (98.95)
0.3	3.10 (99.17)	2.18 (99.41)
0.2	1.85 (99.05)	1.44 (99.61)
0.1	1.22 (99.67)	1.08 (99.71)

$\mathbf{2.92}$, $\boldsymbol{\theta = 1}$

The following tables 4.1-4.3 compares Size biased exponential (SBED) and Area biased exponential (ABED) Average Run Length (ARL) values. Comparisons are done over various shifts (δ) and ARL targets (200, 300, and 370). Parentheses are used to express detectability percentages, indicating process change recognition accuracy. SBED and ABED have comparable ARL values when the ARL objective is 200. It demonstrates that both methods can detect process changes. However, their differing detectability percentages reflect variable detection accuracy. This is because their detection abilities varied. SBED is 51.20% detectable and ABED 55.98% detectable with a shift of 0.9. ABED seems to be more reliable than other methods for identifying changes at this ARL target. This is because the information provided is now accessible.

SBED and ABED indicate comparable ARL values when examined against an ARL objective of 300, indicating that both approaches may detect process changes. The identical ARL values for both demonstrate this. The detectability percentages reveal that ABED has a considerable advantage in detection measure accuracy. Statistical approaches can show that ABED detects shifts better than SBED. Because SBED detects 88.91% at 0.7 shift, whereas ABED detects 91.35%. This is why.SBED and ABED both detect process changes, as indicated by their equivalent ARL outcomes for an ARL objective of 370. This proves both processes can detect process changes. ABED frequently has higher detectability percentages, indicating that it is more successful for detection changes. With a shift of 0.6, SBED is 94.86% detectable and ABED 96.53%. ABED maintains great detection accuracy at this ARL target, proving its superiority.

The higher detectability rates of ABED imply that it performs better than other methods. ABED is more precise than other approaches. This is the case even though SBED and ABED provide equivalent ARL values across a wide range of shifts. These findings suggest that ABED may be the best method for process monitor detection accuracy.

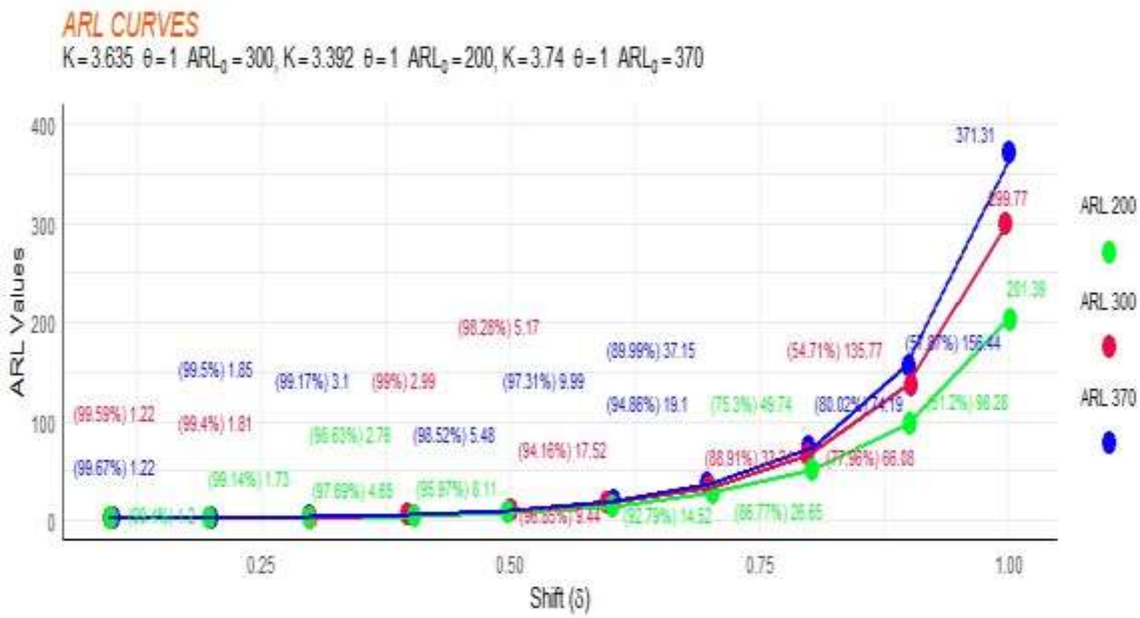

Figure 3: ARL Curves of SBED

In Figure 3 demonstrates that ARL curves are essential for assessing SPC charts. These curves may indicate how fast a control chart can detect a process out of control. The chart's sensitivity to process changes can only be understood by examining K. For an ARL of 200, the Shewhart chart has a 99.40% chance of detecting a shift at 0.1 after the process chances out of control. The ARL value of 300, the Shewhart chart has a 99.59% chance of detecting a shift at 0.1 after the process is out of control. For an ARL of 370, the Shewhart chart has a 99.67% chance of detecting a shift at 0.1 after the process chances out of control. The main goal is to develop SPC charts that can quickly identify minor issues and major changes. Process performance and deviations must be monitored and corrected.

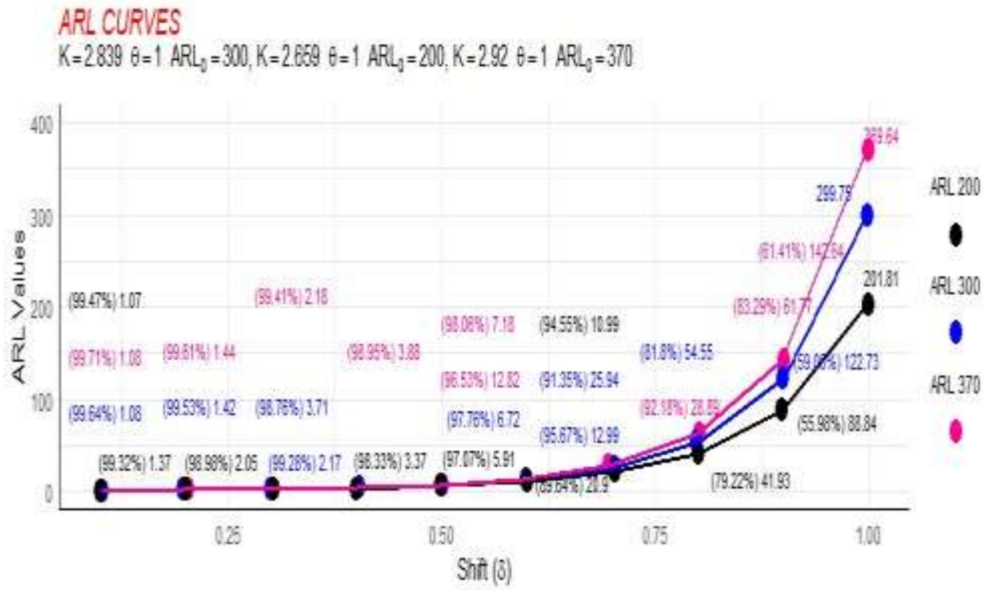

Figure 4: ARL Curves of ABED

In Figure 4 ARL curves are essential for assessing SPC charts. These curves may indicate how fast a control chart can detect a process out of control. The chart's sensitivity to process changes can only be understood by examining K. For an ARL of 200, the shewart chart has a 99.47% chance of detecting a shift at 0.1 after the process chances out of control. The ARL value of 300, the shewart chart has a 99.64% chance of detecting a shift at 0.1 after the process is out of control. For an ARL of 370, the shewart chart has a 99.71% chance of detecting a shift at 0.1 after the process chances out of control. The main goal is to create SPC charts that can quickly identify little issues and major changes. Process performance and deviations must be monitored and corrected.

Data Application

The provided data set consists of relief times, measured in minutes, for 20 patients receiving analgesic treatment[17]. This data reflects the lifetimes or durations between administration and the onset of relief from pain. Using statistical analysis, this data can reveal patterns in how quickly patients respond to the analgesic, offering insights into its effectiveness and consistency. The estimation of scale parameters in biased exponential distributions can give a deeper understanding of this data. In this context, the scale parameter $\widehat{\delta_0}$ for the size-biased exponential distribution was estimated at 1.053, while the corresponding parameter for the area-biased exponential distribution was 1.57. These parameters indicate the rate at which relief occurs and the distribution of relief times among the patients. Such data is invaluable for medical professionals and researchers as it can inform the design of treatment protocols, predict patient outcomes, and highlight potential variations in drug efficacy. Additionally, it can guide the implementation of control charts to monitor and detect any significant shifts in treatment outcomes, ensuring consistent and reliable analgesic effects for patients.

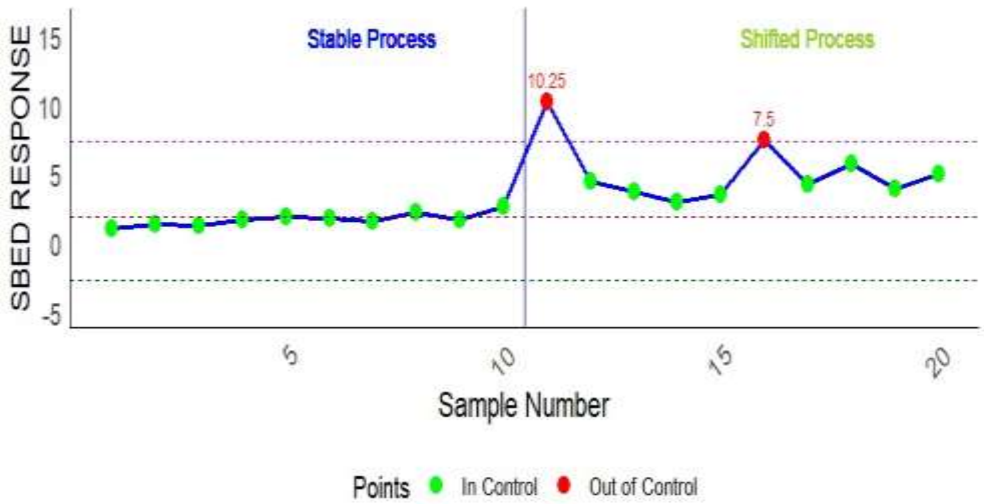

Figure5: The SBED control chart for patients receiving analgesic treatment.

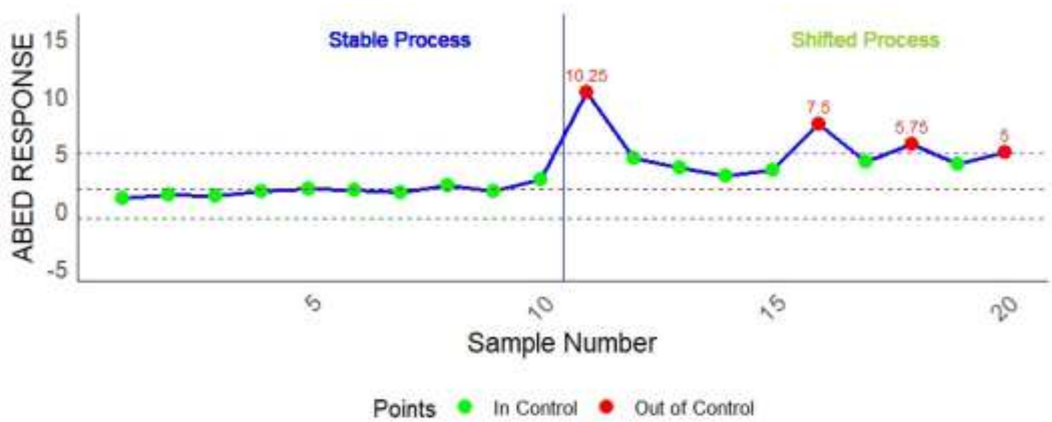

Figure6: The ABED control chart for patients receiving analgesic treatment.

In Figure 5-6 Size-Biased Exponential (SBE) control chart identified shifts in the 20 shifted sample points, particularly in the 11th and 16th samples. However, the Area-Biased Exponential (ABE) control chart recognized the shift at four independent sample point 12th, 16th, 18th, and 20th. Given this, the Area-Biased Exponential control chart revealed is preferable to the Size-Biased Exponential control chart in certain instances.

However, the ABED control chart identified the change earlier than the SBED chart. The ABED-based control chart also had a broader detection range of four points instead of two . ABED control charts aspect to catch deviations from acceptable criteria better. Its more thorough detecting abilities may explain this. This makes it useful for quality control and process monitoring.

Results and discussions

The purpose of this study was to develop a new size-biased exponential distribution control chart and compare it to an area-biased exponential distribution control chart. Since it incorporates these changes into its calculations, the new control chart for the size-biased exponential distribution and area-biased exponential distribution effectively describes the process. The new control chart has been simulated extensively to determine its performance. According to the results, the recently developed control chart had a shorter Average Run Length (ARL), indicating quicker process change detection.

The current research demonstrated that the Area-biased exponential distribution control chart is more effective than the Size-biased exponential distribution control chart. Since this is the case, better quality control and process monitoring should be implemented. Thus, abnormalities would be identified more quickly and corrective initiatives more responsive, improving output quality and efficiency. In medical applications, this increased response to anomalies could contribute to early diagnosis and treatment. Thus, patient outcomes will improve and unforeseen effects will decrease.

REFRENCES

1. Goedhart, R. (2018). Statistical control of Shewhart control charts. IBIS UvA
2. Alsaid, M., Kamal, R. M., & Rashwan, M. M. (2021). The performance of control charts with economic-statistical design when parameters are estimated. Review of Economics and Political Science, 6(2), 142-160.
3. Abid, M., Nazir, H. Z., Riaz, M., & Lin, Z. (2018). In-control robustness comparison of different control charts. Transactions of the Institute of Measurement and Control, 40(13), 3860-3871.
4. Malindzakova, M., Čulková, K., & Trpčevská, J. (2023). Shewhart control charts implementation for quality and production management. Processes, 11(4), 1246.
5. Ho, L. L., Fernandes, F. H., Quinino, R. C., & Bourguignon, M. (2021). Improving Shewhart control chart performance for monitoring the Weibull mean. Quality and Reliability Engineering International, 37(3), 984-996.
6. Woon, Y. H. (2021). A Comparative Analysis of Control Charts for Monitoring Process Mean. Turkish Journal of Computer and Mathematics Education (TURCOMAT), 12(3), 2616-2622.
7. arošová, E., & Noskievičová, D. (2019). Comparative statistical analysis of selected control charts for highly capable processes.
8. Smajdorová, T., & Noskievičová, D. (2022). Analysis and application of selected control charts suitable for smart manufacturing processes. Applied Sciences, 12(11), 5410.
9. Ali, S., Shah, I., Wang, L., & Yue, Z. (2020). A comparison of shewhart-type time-between-events control charts based on the renewal process. IEEE Access, 8, 113683-113701.
10. Nidsunkid, S., & Chomtee, B. (2022). The Average Run Length Performance of Shewhart Control Chart when the Process Data are Sampled from Finite Population. International Journal of Mathematics and Computer Science, 17, 509-524.
11. Prihartanti, W., Rasyid, D. A., & Iriawan, N. (2019, December). On the run-length of the structural change in time series data. In AIP Conference Proceedings (Vol. 2194, No. 1). AIP Publishing.
12. Durtka, S. (2018). Simulation Study of Average Run Lengths in Statistical Process Control and Effect of Non-Normally Distributed Data on ARLs (Doctoral dissertation, Montana State University).
13. Yeong, W. C., Lee, P. Y., Lim, S. L., Ng, P. S., & Khaw, K. W. (2021). Optimal designs of the side sensitive synthetic chart for the coefficient of variation based on the median run length and expected median run length. Plos one, 16(7), e0255366.
14. Hossain, M. P., Omar, M. H., Riaz, M., & Arafat, S. Y. (2022). On designing a new control chart for Rayleigh distributed processes with an application to monitor glass fiber strength. Communications in Statistics-Simulation and Computation, 51(6), 3168-3184.
15. Saeed, U., Mahmood, T., Riaz, M., & Abbas, N. (2018). Simultaneous monitoring of linear profile parameters under progressive setup. Computers & Industrial Engineering, 125, 434-450.

16. Iqbal, T., Iqbal, M. Z., Kashif, M., & Farid, G. (2024). Advancing Quality Assurance In Manufacturing: A Weighted Exponential Distribution Control Chart For Enhanced Production Monitoring. Migration Letters, 21(S5), 1344-1362.
17. Gross, A. J., & Clark, V. (1975). Survival distributions: reliability applications in the biomedical sciences.